4.6 SECTION EXERCISES

VERBAL

1. How can an exponential equation be solved?

2. When does an extraneous solution occur? How can an extraneous solution be recognized?

3. When can the one-to-one property of logarithms be used to solve an equation? When can it not be used?

ALGEBRAIC

For the following exercises, use like bases to solve the exponential equation.

4. $4^{-3v-2} = 4^{-v}$

5. $64 \cdot 4^{3x} = 16$

6. $3^{2x+1} \cdot 3^x = 243$

7. $2^{-3n} \cdot \frac{1}{4} = 2^{n+2}$

8. $625 \cdot 5^{3x+3} = 125$

9. $\frac{36^{3b}}{36^{2b}} = 216^{2-b}$

10. $\left(\frac{1}{64}\right)^{3n} \cdot 8 = 2^6$

For the following exercises, use logarithms to solve.

11. $9^{x-10} = 1$

12. $2e^{6x} = 13$

13. $e^{r+10} - 10 = -42$

14. $2 \cdot 10^{9a} = 29$

15. $-8 \cdot 10^{p+7} - 7 = -24$

16. $7e^{3n-5} + 5 = -89$

17. $e^{-3k} + 6 = 44$

18. $-5e^{9x-8} - 8 = -62$

19. $-6e^{9x+8} + 2 = -74$

20. $2^{x+1} = 5^{2x-1}$

21. $e^{2x} - e^x - 132 = 0$

22. $7e^{8x+8} - 5 = -95$

23. $10e^{8x+3} + 2 = 8$

24. $4e^{3x+3} - 7 = 53$

25. $8e^{-5x-2} - 4 = -90$

26. $3^{2x+1} = 7^{x-2}$

27. $e^{2x} - e^x - 6 = 0$

28. $3e^{3-3x} + 6 = -31$

For the following exercises, use the definition of a logarithm to rewrite the equation as an exponential equation.

29. $\log\left(\frac{1}{100}\right) = -2$

30. $\log_{324}(18) = \frac{1}{2}$

For the following exercises, use the definition of a logarithm to solve the equation.

31. $5\log_7(n) = 10$

32. $-8\log_9(x) = 16$

33. $4 + \log_2(9k) = 2$

34. $2\log(8n + 4) + 6 = 10$

35. $10 - 4\ln(9 - 8x) = 6$

For the following exercises, use the one-to-one property of logarithms to solve.

36. $\ln(10 - 3x) = \ln(-4x)$

37. $\log_{13}(5n - 2) = \log_{13}(8 - 5n)$

38. $\log(x + 3) - \log(x) = \log(74)$

39. $\ln(-3x) = \ln(x^2 - 6x)$

40. $\log_4(6 - m) = \log_4 3(m)$

41. $\ln(x - 2) - \ln(x) = \ln(54)$

42. $\log_9(2n^2 - 14n) = \log_9(-45 + n^2)$

43. $\ln(x^2 - 10) + \ln(9) = \ln(10)$

For the following exercises, solve each equation for x.

44. $\log(x + 12) = \log(x) + \log(12)$

45. $\ln(x) + \ln(x - 3) = \ln(7x)$

46. $\log_2(7x + 6) = 3$

47. $\ln(7) + \ln(2 - 4x^2) = \ln(14)$

48. $\log_8(x + 6) - \log_8(x) = \log_8(58)$

49. $\ln(3) - \ln(3 - 3x) = \ln(4)$

50. $\log_3(3x) - \log_3(6) = \log_3(77)$

GRAPHICAL

For the following exercises, solve the equation for x, if there is a solution. Then graph both sides of the equation, and observe the point of intersection (if it exists) to verify the solution.

51. $\log_9(x) - 5 = -4$

52. $\log_3(x) + 3 = 2$

53. $\ln(3x) = 2$

54. $\ln(x - 5) = 1$

55. $\log(4) + \log(-5x) = 2$

56. $-7 + \log_3(4 - x) = -6$

57. $\ln(4x - 10) - 6 = -5$

58. $\log(4 - 2x) = \log(-4x)$

59. $\log_{11}(-2x^2 - 7x) = \log_{11}(x - 2)$

60. $\ln(2x + 9) = \ln(-5x)$

61. $\log_9(3 - x) = \log_9(4x - 8)$

62. $\log(x^2 + 13) = \log(7x + 3)$

63. $\frac{3}{\log_2(10)} - \log(x - 9) = \log(44)$

64. $\ln(x) - \ln(x + 3) = \ln(6)$

Section 4.4

1. Since the functions are inverses, their graphs are mirror images about the line $y = x$. So for every point (a, b) on the graph of a logarithmic function, there is a corresponding point (b, a) on the graph of its inverse exponential function. **3.** Shifting the function right or left and reflecting the function about the y-axis will affect its domain. **5.** No. A horizontal asymptote would suggest a limit on the range, and the range of any logarithmic function in general form is all real numbers.

7. Domain: $\left(-\infty, \frac{1}{2}\right)$; range: $(-\infty, \infty)$

9. Domain: $\left(-\frac{17}{4}, \infty\right)$; range: $(-\infty, \infty)$

11. Domain: $(5, \infty)$; vertical asymptote: $x = 5$

13. Domain: $\left(-\frac{1}{3}, \infty\right)$; vertical asymptote: $x = -\frac{1}{3}$

15. Domain: $(-3, \infty)$; vertical asymptote: $x = -3$

17. Domain: $\left(\frac{3}{7}, \infty\right)$; vertical asymptote: $x = \frac{3}{7}$; end behavior: as $x \to \left(\frac{3}{7}\right)^{+}, f(x) \to -\infty$ and as $x \to \infty, f(x) \to \infty$

19. Domain: $(-3, \infty)$; vertical asymptote: $x = -3$; end behavior: as $x \to -3^{+}, f(x) \to -\infty$ and as $x \to \infty, f(x) \to \infty$

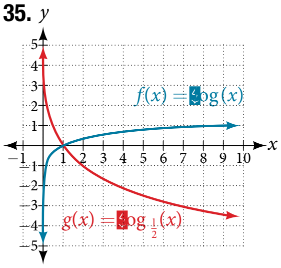
21. Domain: $(1, \infty)$; range: $(-\infty, \infty)$; vertical asymptote: $x = 1$; x-intercept: $\left(\frac{5}{4}, 0\right)$; y-intercept: DNE

23. Domain: $(-\infty, 0)$; range: $(-\infty, \infty)$; vertical asymptote: $x = 0$; x-intercept: $(-e^2, 0)$; y-intercept: DNE

25. Domain: $(0, \infty)$; range: $(-\infty, \infty)$ vertical asymptote: $x = 0$; x-intercept: $(e^3, 0)$; y-intercept: DNE

27. B **29.** C **31.** B **33.** C

35.



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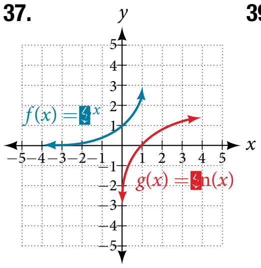
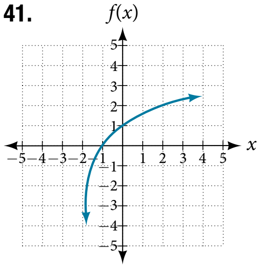


39. C

41.



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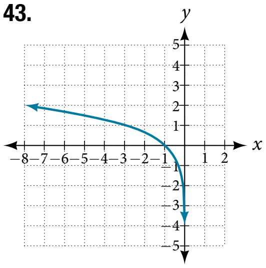

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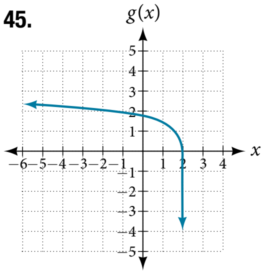


47. $f(x) = \log_2(-(x-1))$

49. $f(x) = 3\log_4(x+2)$

51. $x = 2$

53. $x \approx 2.303$

55. $x \approx -0.472$

57. The graphs of $f(x) = \log_{\frac{1}{2}}(x)$ and $g(x) = -\log_2(x)$ appear to be the same; conjecture: for any positive base $b \neq 1$, $\log_b(x) = -\log_{\frac{1}{b}}(x)$.

59. Recall that the argument of a logarithmic function must be positive, so we determine where $\frac{x+2}{x-4} > 0$. From the graph of the function $f(x) = \frac{x+2}{x-4}$, note that the graph lies above the x-axis on the interval $(-\infty, -2)$ and again to the right of the vertical asymptote, that is $(4, \infty)$. Therefore, the domain is $(-\infty, -2) \cup (4, \infty)$.

Section 4.5

1. Any root expression can be rewritten as an expression with a rational exponent so that the power rule can be applied, making the logarithm easier to calculate. Thus, $\log_b(x^{\frac{1}{n}}) = \frac{1}{n}\log_b(x)$.

3. $\log_b(2) + \log_b(7) + \log_b(x) + \log_b(y)$

5. $\log_b(13) - \log_b(17)$ **7.** $-k\ln(4)$ **9.** $\ln(7xy)$

11. $\log_b(4)$ **13.** $\log_b(7)$ **15.** $15\log(x) + 13\log(y) - 19\log(z)$

17. $\frac{3}{2}\log(x) - 2\log(y)$ **19.** $\frac{8}{3}\log(x) + \frac{14}{3}\log(y)$ **21.** $\ln(2x^7)$

23. $\log\left(\frac{xz^3}{\sqrt{y}}\right)$ **25.** $\log_7(15) = \frac{\ln(15)}{\ln(7)}$

27. $\log_{11}(5) = \frac{1}{b}$ **29.** $\log_{11}\left(\frac{6}{11}\right) = \frac{a-b}{b}$ or $\frac{a}{b} - 1$ **31.** 3

33. ≈ 2.81359 **35.** ≈ 0.93913 **37.** ≈ -2.23266

39. $x = 4$, By the quotient rule:

$\log_6(x+2) - \log_6(x-3) = \log_6\left(\frac{x+2}{x-3}\right) = 1$

Rewriting as an exponential equation and solving for x:

$$6^1 = \frac{x+2}{x-3}$$
$$0 = \frac{x+2}{x-3} - 6$$
$$0 = \frac{x+2}{x-3} - \frac{6(x-3)}{(x-3)}$$
$$0 = \frac{x+2-6x+18}{x-3}$$
$$0 = \frac{x-4}{x-3}$$
$$x = 4$$

Checking, we find that $\log_6(4+2) - \log_6(4-3) = \log_6(6) - \log_6(1)$ is defined, so $x = 4$.

41. Let b and n be positive integers greater than 1. Then, by the change-of-base formula, $\log_b(n) = \frac{\log_n(n)}{\log_n(b)} = \frac{1}{\log_n(b)}$.

Section 4.6

1. Determine first if the equation can be rewritten so that each side uses the same base. If so, the exponents can be set equal to each other. If the equation cannot be rewritten so that each side uses the same base, then apply the logarithm to each side and use properties of logarithms to solve. **3.** The one-to-one property can be used if both sides of the equation can be rewritten as a single logarithm with the same base. If so, the arguments can be set equal to each other, and the resulting equation can be solved algebraically. The one-to-one property cannot be used when each side of the equation cannot be rewritten as a single logarithm with the same base. **5.** $x = -\frac{1}{3}$ **7.** $n = -1$ **9.** $b = \frac{6}{5}$

11. $x = 10$ **13.** No solution **15.** $p = \log\left(\frac{17}{8}\right) - 7$

17. $k = -\frac{\ln(38)}{3}$ **19.** $x = \frac{\ln\left(\frac{38}{3}\right) - 8}{9}$ **21.** $x = \ln(12)$

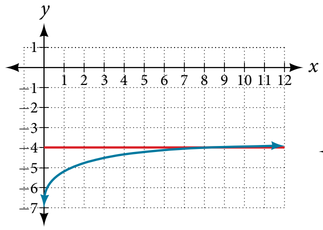
23. $x = \dfrac{\ln\left(\frac{3}{5}\right) - 3}{8}$ **25.** No solution **27.** $x = \ln(3)$

29. $10^{-2} = \dfrac{1}{100}$ **31.** $n = 49$ **33.** $k = \dfrac{1}{36}$ **35.** $x = \dfrac{9 - e}{8}$

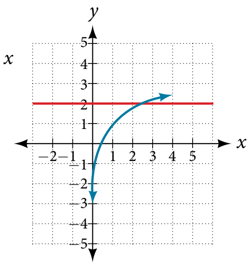
37. $n = 1$ **39.** No solution **41.** No solution

43. $x = \pm\dfrac{10}{3}$ **45.** $x = 10$ **47.** $x = 0$ **49.** $x = \dfrac{3}{4}$

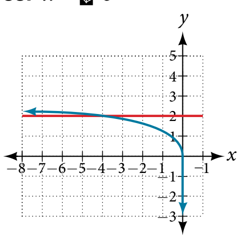
51. $x = 9$



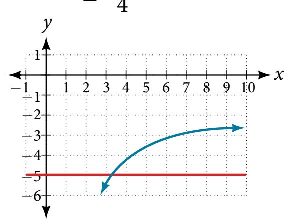
53. $x = \dfrac{e^2}{3} \approx 2.5$



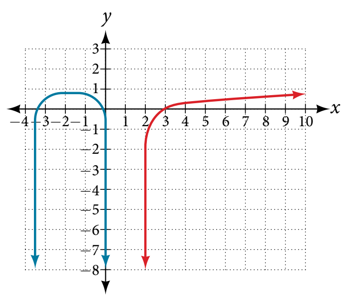
55. $x = -5$



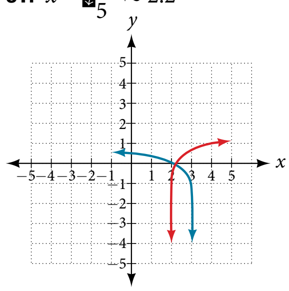
57. $x = \dfrac{e + 10}{4} \approx 3.2$



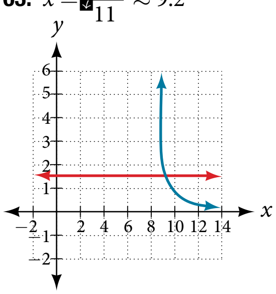
59. No solution



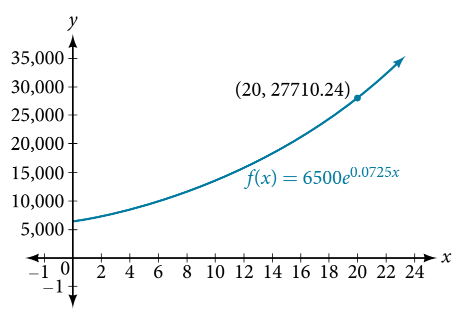
61. $x = \dfrac{11}{5} \approx 2.2$



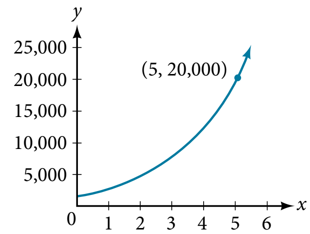
63. $x = \dfrac{101}{11} \approx 9.2$



65. About $27,710.24



67. About 5 years



69. ≈ 0.567 **71.** ≈ 2.078

73. ≈ 2.2401

75. ≈ -44655.7143

77. About 5.83

79. $t = \ln\left(\left(\dfrac{y}{A}\right)^{\frac{1}{k}}\right)$

81. $t = \ln\left(\left(\dfrac{T - T_s}{T_0 - T_s}\right)^{-\frac{1}{k}}\right)$

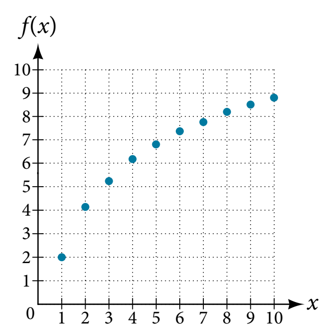
Section 4.7

1. Half-life is a measure of decay and is thus associated with exponential decay models. The half-life of a substance or quantity is the amount of time it takes for half of the initial amount of that substance or quantity to decay. **3.** Doubling time is a measure of growth and is thus associated with exponential growth models. The doubling time of a substance or quantity is the amount of time it takes for the initial amount of that substance or quantity to double in size. **5.** An order of magnitude is the nearest power of ten by which a quantity exponentially grows. It is also an approximate position on a logarithmic scale; Sample response: Orders of magnitude are useful when making comparisons between numbers that differ by a great amount. For example, the mass of Saturn is 95 times greater than the mass of Earth. This is the same as saying that the mass of Saturn is about 10^2 times, or 2 *orders of magnitude* greater, than the mass of Earth.

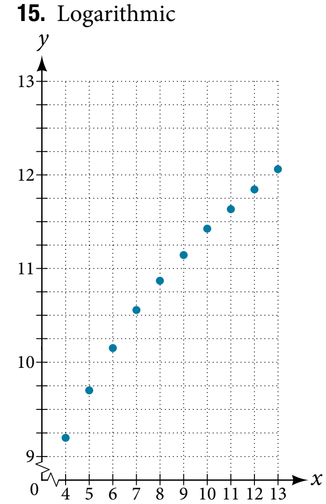
7. $f(0) \approx 16.7$; the amount initially present is about 16.7 units.

9. 150 **11.** Exponential; $f(x) = 1.2^x$

13. Logarithmic



15. Logarithmic



17.