Miss Weisse's Own Physics Textbook

An Introduction to Momentum

Momentum
is
MASS in Motion

The equation for momentum follows the definition.

momentum is mass in motion

$p = m \times \vec{v}$

The variable for momentum is "p".
Sometimes you'll see Capital P,
I am going to use little p. You choose!

Now that we've identified the variable, let's talk about the unit of momentum.

$$p = \text{mass} \times \text{velocity}$$

$$\frac{kg \cdot m}{s} = kg \times m/s$$

Yes, the unit of momentum is $\frac{kg \cdot m}{s}$, OR...

Another unit of momentum is N·s. WHY?

Let's think about a Newton (N).

→ A Newton is the unit of Force

→ Force = mass × acceleration ← I hope you notice how similar this is to the momentum equation...

$$N = kg \cdot m/s^2$$

$$N = \frac{kg \cdot m}{s^2}$$

$$N = \frac{kg \cdot m}{s} \cdot \frac{1}{s}$$

PAUSE! Don't look at the next sheet yet!

Can you turn this $\boxed{\text{last statement}}$ into the units of momentum and show that another unit for momentum is N·s?

HINT! If you think this is simple, IT IS!
If you think this is difficult, use Algebra!

What does momentum mean?

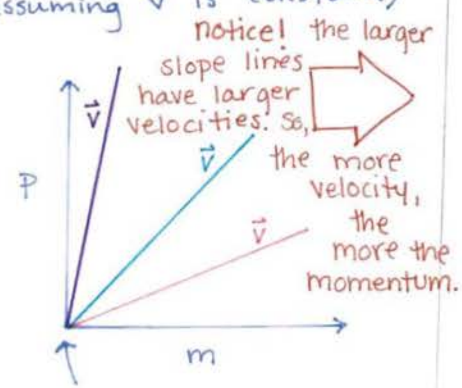
This is a difficult question to answer succinctly. So, I'll use math!

$$p \propto m$$

momentum is directly proportional to mass

- If $\uparrow p$, $\uparrow m$
- If $\downarrow p$, $\downarrow m$

(assuming $\vec{v}$ is constant)



if there is no mass, there is no momentum!

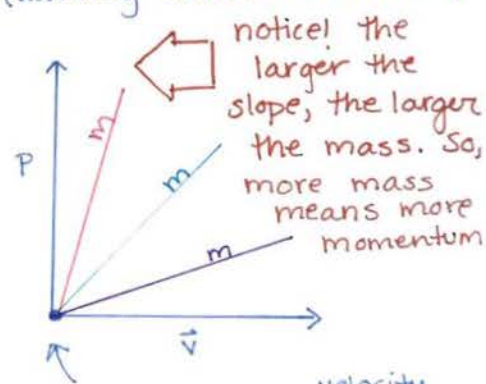
(aka, there is no mass to be in motion)

$$p \propto \vec{v}$$

momentum is directly proportional to velocity

- If $\uparrow p$, $\uparrow \vec{v}$
- If $\downarrow p$, $\downarrow \vec{v}$

(assuming mass is constant)



if there is no velocity ~~mass~~, there is no momentum! (i.e., the ~~no~~ mass is not moving)

finally, when it comes to describing momentum

$$m \propto \frac{1}{\vec{v}}$$

mass is INVERSELY proportional to velocity.

- If $\uparrow m$, $\downarrow \vec{v}$
- If $\downarrow m$, $\uparrow \vec{v}$

(assuming momentum is constant)

Let's Try Some Problems (try to answer & justify each yourself)

1. A truck and a bee are moving at the same velocity, 20 m/s.

a) Which has more momentum? Why?

b) How much momentum does each have if their masses are 10,000 kg and 0.5 kg?

c) How slow would the truck have to move to have the same momentum as the bee? The bee is still travelling at 20 m/s.

Solutions

a)

Truck	Bee
p =	p =
m =	m =
$\vec{v} = 20\,m/s$	$\vec{v} = 20\,m/s$

1 way to think about it

Because the velocity is the same (constant velocity) we should compare their masses. We're not told their masses, but we KNOW a truck has more mass than a bee.

$$p \propto m$$

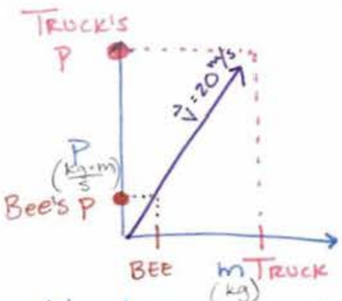
$$\uparrow m,\ \uparrow p$$

$\therefore$ The TRUCK has more momentum bc

$$m_{truck} > m_{bee}$$

2nd way to think about it

Again, the velocities are constant so we're looking at the p vs. m graph



We know the truck has more mass (look @ the mass-axis) and the graph shows that the truck then has more momentum (look at the p-axis)

b)

Truck	Bee
p =	p =
m = 10,000 kg	m = 0.5 kg
$\vec{v} = 20\,m/s$	$\vec{v} = 20\,m/s$

$$p_{truck} = m\cdot\vec{v}$$
$$= (10{,}000\,kg)(20\,m/s)$$
$$= 200{,}000\ \frac{kg\cdot m}{s}$$
$$= 200{,}000\ N\cdot s$$

$$p_{bee} = m\cdot\vec{v}$$
$$= (0.5\,kg)(20\,m/s)$$
$$= 10\ \frac{kg\cdot m}{s}$$
$$= 10\ N\cdot s$$

c)

Truck		Bee
p = 10 N·s	SAME!	p = 10 N·s
m = 10,000 kg		m = 0.5 kg
$\vec{v}$ =		$\vec{v} = 20\,m/s$

$$\frac{p}{m} = \frac{m\cdot\vec{v}}{m}$$

$$\vec{v} = \frac{p}{m}$$

$$\vec{v} = \frac{10\,kg\cdot m/s}{10{,}000\,kg} = 0.001\,m/s$$

An Introduction to Impulse

Impulse (J)

Momentum (mass in motion) is $p = mv$.

I made a side comment in yesterday's notes suggesting the equation $F = ma$ is similar to $p = mv$. Do you see why?

→momentum and force are both descriptions of the motion of massive objects.

$$\left.\begin{aligned} P &= m \times \vec{v} \\ F_{net} &= m \times \vec{a} \end{aligned}\right\}$$

→ $\vec{a}$ is the $\Delta\vec{v}$ over time

→ both $\vec{a}$ and $\vec{v}$ describe the motion of objects (masses!)

→Force describes changes in motion.

→Both momentum and force are directly proportional to mass.

→Momentum describes motion at an instant.

→Both increase/decrease with increased/decreased mass

→Both are directly proportional to mass and something about that mass's motion.

→ Fore both p and F the mass of the object affects the motion of the object

Furthermore, their units are SO similar!

$$P = m \times \vec{v}$$

$$\frac{kg \cdot m}{s} = kg \times \frac{m}{s}$$

$$N \cdot s = \frac{kg \cdot m}{s^2} \cdot s = \frac{kg \cdot m}{s}$$

$$F_{net} = m \times \vec{a}$$

$$N = kg \times \frac{m}{s^2}$$

$$N = \frac{kg \times m}{s^2}$$

Now, you may be wondering why the title of these notes is "Impulse." Let's get to it —

As just shown with the units, the difference between momentum and force is a factor of time.

$$p = F \cdot (s)^{*}$$

→ Side Note! This is NOT an official equation! Do not use it on assignments!

$$(N \cdot s) = (N) \cdot (s) \quad \checkmark$$

$$\frac{kg \cdot m}{s} = \left(\frac{kg \cdot m}{s^2}\right) \cdot (s)$$

$$= \frac{kg \cdot m}{s} \quad \checkmark$$

Also mentioned was the fact that $\vec{a}$ is $\frac{\Delta \vec{v}}{\Delta t}$.
Do you see what I see???

A FACTOR OF TIME!!

Let's rewrite the Force equation with this factor of time!

$$F_{net} = m \cdot \vec{a}$$

$$F_{net} = m \cdot \frac{\Delta \vec{v}}{\Delta t} \qquad \vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Now, if we multiply both sides by Δt...

$$\Delta t \times F_{net} = m \cdot \frac{\Delta \vec{v}}{\cancel{\Delta t}} \cancel{\times \Delta t}$$

We get something that looks like momentum and something that looks like force × time!

$$F_{net} \Delta t = m \Delta \vec{v}$$

$$= \Delta (m \vec{v})$$

$$F_{net} \Delta t = \Delta p = \text{IMPULSE!!}$$

We finally made it! Impulse is a change in momentum OR force × time. And the variable for impulse is J. ("I" is already taken by the measurement of current.)

$$\boxed{J = F_{net} \Delta t = \Delta p}$$

And the units of Impulse? The same as momentum. Try to prove it!

Now, let me remind you (for the millionth time) that a Δ in ANYTHING = final − initial.

$$J = F_{net}(t_f - t_i) = p_f - p_i = mv_f - mv_i = m(v_f - v_i$$

we almost always assume $t_i = 0$ unless told otherwise

EXAMPLE PROBLEMS

1. A 50kg mass is sitting on a frictionless surface. An unknown constant force pushes the mass for 2 seconds until the mass reaches a velocity of 3m/s.

a) Draw a before & after picture. List what you know

$\vec{v}_i = 0$ m/s

$\vec{v}_f = 3$ m/s

$\Delta t = 2$s

$m = 50$kg

$F = ?$

$\Delta p = ?$

b) What is the initial momentum of the mass?

$p_i = mv_i$

$= (50\text{kg})(0\,\text{m/s})$

$\boxed{p_i = 0\,\frac{\text{kg}\cdot\text{m}}{\text{s}}}$

c) What is the final momentum of the mass?

$p_f = mv_f$

$= (50\text{kg})(3\,\text{m/s})$

$\boxed{p_f = 150\,\frac{\text{kg}\cdot\text{m}}{\text{s}}}$

d) & e) on next page ↓

d) What was the force acting on the mass?

$$\frac{F_{net}\cdot\Delta t}{\Delta t} = \frac{\Delta p}{\Delta t}$$

$$F_{net} = \frac{\Delta p}{\Delta t}$$

$$= \frac{p_f - p_i}{\Delta t}$$ ← we know all these pieces of information!

$$= \frac{150\,\frac{\text{kg}\cdot\text{m}}{\text{s}} - 0\,\frac{\text{kg}\cdot\text{m}}{\text{s}}}{2\text{s}}$$

$$F_{net} = 75\,\frac{\text{kg}\cdot\text{m}}{\text{s}^2}$$

$$\boxed{F_{net} = 75\text{ N}}$$

e) What was the impulse on the mass?

$J = F_{net}\Delta t$ OR $J = \Delta p$

$= 75\text{N}\cdot 2\text{s}$ | $= 150\,\frac{\text{kg}\cdot\text{m}}{\text{s}} - 0\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$

$\boxed{J = 150\text{ N}\cdot\text{s}}$ ✓ $J = 150\,\frac{\text{kg}\cdot\text{m}}{\text{s}}$ ✓

either method works!

Momentum, Impulse, And Everything We Know About the Motion of Objects

Momentum, Impulse, And Everything We Know About The Motion of Objects

I'm hoping in studying Monday & Tuesday's notes on momentum and impulse you realized these new measurements of motion are related to all the measurements (and laws) we already know.

So, today we will use THE KINEMATIC EQUATIONS and NEWTON'S LAWS along with momentum and impulse to solve more involved ~~eq~~ problems.

First, let's review:

KINEMATIC EQUATIONS

$V_f = V_i + a\Delta t$

$V_f^2 = V_i^2 + 2a\Delta x$

$\Delta x = \frac{1}{2}(V_i + V_f)\Delta t$

$\Delta x = V_i\Delta t + \frac{1}{2}a(\Delta t)^2$

NEWTON'S LAWS

LAW 1: An object in motion stays in motion, an object at rest stays at rest, unless acted on by an outside force.

LAW 2: Acceleration is directly proportional to Force and inversely proportional to mass

LAW 3: For every action (force) there is an equal and opposite reaction.

Problem 1

A 0.5 kg ball is in free fall from a height of 7.2 m. Determine the Impulse.

1st let's list what is given, AND hidden information.

Impulse/Momentum Info	Kinematic Eqn Info
$m = 0.5kg$	$v_i = 0\ m/s$
$v_i = 0\ m/s$	$v_f = ?$
$v_f = ?$	$a = -10\ m/s^2$
$\Delta p = ?$	$\Delta y = -7.2m$
$F_{net} = m\vec{g} = (0.5kg)(10m/s^2) = 5N$	$\Delta t = ?$
$\Delta t = ?$	
$\boxed{J = ?}$	

2nd write important equations and determine what information we need to find first

$$J = F_{net} \cdot \underset{?}{\underset{\uparrow}{\Delta t}} = m\Delta v = m(\underset{?}{\underset{\uparrow}{v_f}} - v_i)$$

I either need to find time of final velocity

Let's do both!

Finding time

$\Delta y = v_i \Delta t + \frac{1}{2}a(\Delta t^2)$

$-7.2 = \cancel{0\Delta t}^{0} + \frac{1}{2}(-10)\Delta t^2$

$\sqrt{\frac{-7.5}{-5}} = \sqrt{\Delta t^2}$

$t = 1.2247s$

$v_f^2 = v_i^2 + 2a\Delta y$

$v_f^2 = 0 + 2(-10)(-7.2)$

$= 20(7.2)$

$v_f = \sqrt{144}$

$v_f = 12\ m/s$

3rd Solve for Impulse

$J = F_{net} \cdot \Delta t$

$= (5N)(1.2247s)$

$\boxed{J = 6\ N \cdot s}$ ✓

$J = m(\Delta v)$

$= (0.5kg)(12 - 0)$

$= 6\ N \cdot s$ ✓

Problem 2

A 5g ball in free fall hits the floor and bounces back up. The velocity of the ball before hitting the floor is 6 m/s. The velocity after hitting the floor and bouncing is 4 m/s. Determine the impulse and the force the ball exerted on the ground.

1st make your lists

p/J info	Kinematic info
$m = 5g = 0.005\ kg$	$v_i = -6\ m/s$
$v_i = -6\ m/s$ ← *negative direction*	$v_f = 4\ m/s$
$v_f = 4\ m/s$	$\Delta y = ?$
$F_{net} = ?$	$a = -10\ m/s^2$
$\Delta p = ?$	$\Delta t = ?$
$J = ?$	
$\Delta t = ?$	

2nd determine necessary equations and find necessary information (thinking only about finding J right now)

$$J = F_{net} \cdot \underset{?}{\underset{\downarrow}{\Delta t}} = \underline{\underline{m\Delta v}}$$

We have this information!

3rd solve!

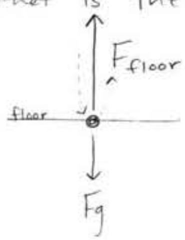
$J = (.005\ kg)(4\ m/s - (-6\ m/s)) = (0.5)(10) = \boxed{0.05\ \frac{kg \cdot m}{s}}$

↓

We still need to find the force the ball exerted on the ground.

$$J = F_{net} \cdot \overset{?}{\Delta t}$$

↑ this F_{net} is the force acting on the ball



$$F_{net} = F_{floor} - F_g$$

$$F_g = m\vec{g}$$
$$= (0.005g)(10\,m/s^2)$$
$$= 0.05\,N$$

But, according to NEWTON'S 3RD LAW, the force of the floor on the ball is equal and opposite to the ball on the floor!

$$F_{Floor \to ball} = -F_{ball \to floor}$$

So we need to find Δt (1) to find F_{net} (2) to find $F_{floor \to ball}$ (3).

(1) Let's just say this information was given...

$$t = 0.0003s$$

(2) Finding F_{net}

$$J = F_{net} \cdot \Delta t$$

$$F_{net} = \frac{J}{\Delta t}$$

$$= \frac{0.05\,\frac{kg \cdot m}{s}}{0.0003s}$$

$$F_{net} = 166.6667\,N$$

(3) Find $F_{floor \to ball}$

$$F_{net} = F_{floor \to ball} - F_g$$

$$166.7\,N = F_{floor \to ball} - 0.05N$$

$$F_{floor \to ball} = 166.667N + 0.05N$$

$$= 166.7\,N$$

$\therefore$

$$\boxed{F_{ball \to floor} = -166.7\,N}$$