

## Remote Learning Packet

*NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.*

**March 30 - April 3, 2020**

**Course:** 11 Precalculus

**Teacher:** Mr. Simmons michael.simmons@greatheartsirving.org

### Weekly Plan:

Monday, March 30

“Exponential and Logarithmic Models II” worksheet answer key

Tuesday, March 31

Read about logistic growth models and work example problems

Wednesday, April 1

Read “Choosing an Appropriate Model for Data” and work example problems

Thursday, April 2

Read “Expressing an Exponential Model in Base  $e$ ” and work example problems

Friday, April 3

Problem set 4.7: 6-12

### Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

---

Student Signature

---

Parent Signature

## **Monday, March 30**

Dearest students, hello! I hope you've enjoyed your leisure over the past two weeks. While I am heartbroken that I can't be with you in person to guide you through the wonderful beauties of mathematics over these next few weeks, I see some good coming from the circumstances, as each of you will be challenged - and surely will rise to meet the challenge - to engage in mathematical inquiry in a more independent way. The frustrations you will feel in your isolated struggle for mathematical understanding, no matter your success, will doubtless sow seeds of true mathematical thought - and winter always ends in a fruitful spring. The essentially collaborative nature of our class has unfortunately (though necessarily) been temporarily suspended, so take courage as you embark on this lonely adventure, and look forward to our reunion in good time. Have fortitude, and I look forward to seeing you all again.

In the meantime, your individual adventures in math will be guided by written instructions contained in packets like this one. Your first instruction is simply to read through, in detail, the answer key to the "Exponential and Logarithmic Models II" worksheet that you already completed. If you haven't already completed it, do that first.

## **Tuesday, March 31**

Read, on pages 407-409 of the textbook, the section about logistic growth. Complete the example problem and the "try it" problem.

## **Wednesday, April 1**

Check your answer to the "try it" problem in the back of the book.

Read, on pages 409-411, the section on "choosing an appropriate model for data," and complete the example problem and the "try it" problem.

## **Thursday, April 2**

Check your answer to the "try it" problem in the back of the book.

Read, on pages 411-412, the section on "Expressing an Exponential Model in Base  $e$ ," and complete the example and "try it" problems.

## **Friday, April 3**

Check your answer to the "try it" problem in the back of the book.

Complete the problem set 4.7: 6-12.

## Exponential and Logarithmic Models II

*Mr. Simmons*  
*Precalculus*

Newton's Law of Cooling (which you derived in the previous worksheet) states that the temperature of an object,  $T$ , in surrounding air with temperature  $T_s$ , will behave according to the formula

$$T(t) = Ae^{kt} + T_s,$$

where

- $t$  is time,
- $A$  is the difference between the initial temperature of the object and the surroundings, and
- $k$  is a constant, the continuous rate of cooling of the object.

Use Newton's Law of Cooling to answer the following questions:

1. A cheesecake is taken out of the oven with an ideal internal temperature of  $165^\circ\text{F}$ , and is placed into a  $35^\circ\text{F}$  refrigerator. After 10 minutes, the cheesecake has cooled to  $150^\circ\text{F}$ . If we must wait until the cheesecake has cooled to  $70^\circ\text{F}$  before we eat it, how long will we have to wait?

2. A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

## Exponential and Logarithmic Models II

*Mr. Simmons*

*Precalculus*

Newton's Law of Cooling (which you derived in the previous worksheet) states that the temperature of an object,  $T$ , in surrounding air with temperature  $T_s$ , will behave according to the formula

$$T(t) = Ae^{kt} + T_s,$$

where

- $t$  is time,
- $A$  is the difference between the initial temperature of the object and the surroundings, and
- $k$  is a constant, the continuous rate of cooling of the object.

Use Newton's Law of Cooling to answer the following questions:

1. A cheesecake is taken out of the oven with an ideal internal temperature of  $165^\circ\text{F}$ , and is placed into a  $35^\circ\text{F}$  refrigerator. After 10 minutes, the cheesecake has cooled to  $150^\circ\text{F}$ . If we must wait until the cheesecake has cooled to  $70^\circ\text{F}$  before we eat it, how long will we have to wait?

**Solution.** Plugging in known values and solving for  $k$ , we get

$$150 = (165 - 35)e^{k(10)} + 35$$

$$115 = 130e^{10k}$$

$$\frac{23}{26} = e^{10k}$$

$$10k = \ln\left(\frac{23}{26}\right)$$

$$k = \frac{\ln\left(\frac{23}{26}\right)}{10}.$$

So

$$T(t) = 130e^{\frac{\ln\left(\frac{23}{26}\right)}{10}t} + 35.$$

We want to know what  $t$  will be when  $T(t)$  is 70, so we plug in this known point and solve

for  $t$ :

$$70 = 130e^{\frac{\ln(\frac{23}{26})}{10}t} + 35$$

$$35 = 130e^{\frac{\ln(\frac{23}{26})}{10}t}$$

$$\frac{7}{26} = e^{\frac{\ln(\frac{23}{26})}{10}t}$$

$$\frac{\ln(\frac{23}{26})}{10}t = \ln\left(\frac{7}{26}\right)$$

$$t = 10 \frac{\ln\left(\frac{7}{26}\right)}{\ln\left(\frac{23}{26}\right)}$$

$$\approx 107.$$

So we need to wait about 107 minutes, or 1 hour and 47 minutes.

2. A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

**Solution.** Plugging in known values and solving for unknown values, we have

$$\begin{aligned}T(t) &= Ae^{kt} \\(45) &= (40 - 70)e^{k(1)} + 70 \\-25 &= -30e^k \\ \frac{5}{6} &= e^k \\ k &= \ln\left(\frac{5}{6}\right); \\60 &= -30e^{\ln\left(\frac{5}{6}\right)t} + 70 \\-10 &= -30e^{\ln\left(\frac{5}{6}\right)t} \\ \frac{1}{3} &= e^{\ln\left(\frac{5}{6}\right)t} \\ \ln\left(\frac{5}{6}\right)t &= \ln\left(\frac{1}{3}\right) \\ t &= \frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{5}{6}\right)} \\ &\approx 6.03;\end{aligned}$$

so it will take about 6.03 hours for the pitcher to warm to 60 degrees.