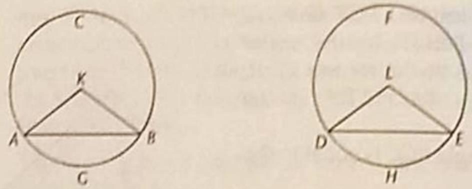


therefore the two sides AK, KB are equal to the two sides DL, LE;
and the base AB is equal to the base DE;
therefore the angle AKB is equal to the angle DLE. [I. 8]

But equal angles stand on equal circumferences, when they are at the centres; [III. 26]
therefore the circumference AGB is equal to DHE.

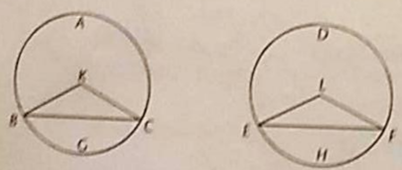
And the whole circle ABC is equal to the whole circle DEF;
therefore the circumference ACB which remains is also equal to the circumference DFE which remains.

Therefore etc.

Q.E.D.

Proposition 29

In equal circles equal circumferences are subtended by equal straight lines.



Let ABC, DEF be equal circles, and in them let equal circumferences BGC, EHF be cut off; and let the straight lines BC, EF be joined;
I say that BC is equal to EF.

For let the centres of the circles be taken, and let them be K, L; let BK, KC, EL, LF be joined.

Now, since the circumference BGC is equal to the circumference EHF,
the angle BKC is also equal to the angle ELF. [III. 27]

And, since the circles ABC, DEF are equal,
the radii are also equal;
therefore the two sides BK, KC are equal to the two sides EL, LF;
and they contain equal angles;
therefore the base BC is equal to the base EF. [I. 4]

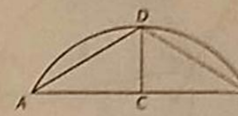
Therefore etc.

Q.E.D.

Proposition 30

To bisect a given circumference.

Let ADB be the given circumference;
thus it is required to bisect the circumference ADB.



Let AB be joined and bisected at C; from the point C let CD be drawn at right angles to the straight line AB, and let AD, DB be joined.

Then, since AC is equal to CB, and CD is common,
the two sides AC, CD are equal to the two sides BC, CD;
and the angle ACD is equal to the angle BCD, for each is right;
therefore the base AD is equal to the base DB. [I. 4]

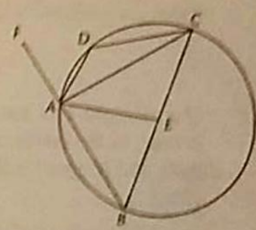
But equal straight lines cut off equal circumferences, the greater equal to the greater, and the less to the less; [III. 28]
and each of the circumferences AD, DB is less than a semicircle;
therefore the circumference AD is equal to the circumference DB.

Therefore the given circumference has been bisected at the point D.

Q.E.F.

Proposition 31

In a circle the angle in the semicircle is right, that in a greater segment less than a right angle, and that in a less segment greater than a right angle; and further the angle of the greater segment is greater than a right angle, and the angle of the less segment less than a right angle.



Let $ABCD$ be a circle, let BC be its diameter, and E its centre, and let BA, AC, AD, DC be joined;
I say that the angle BAC in the semicircle BAC is right,
the angle ABC in the segment ABC greater than the semicircle is less than a right angle,
and the angle ADC in the segment ADC less than the semicircle is greater than a right angle.

Let AE be joined, and let BA be carried through to F.

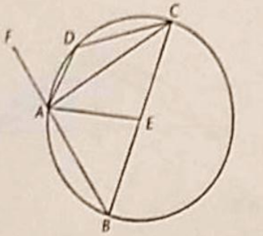
Then, since BE is equal to EA,
the angle ABE is also equal to the angle BAE. [I. 5]

Again, since CE is equal to EA,
the angle ACE is also equal to the angle CAE. [I. 5]

Therefore the whole angle BAC is equal to the two angles ABC, ACB.

But the angle FAC exterior to the triangle ABC is also equal to the two angles ABC, ACB; [I. 32]
therefore the angle BAC is also equal to the angle FAC;
therefore each is right; [I. Def. 10]
therefore the angle BAC in the semicircle BAC is right.

Next, since in the triangle ABC the two angles ABC, BAC are less than two right angles, [I. 17]
and the angle BAC is a right angle,
the angle ABC is less than a right angle;



and it is the angle in the segment ABC greater than the semicircle.

Next, since $ABCD$ is a quadrilateral in a circle,
and the opposite angles of quadrilaterals in circles are equal to two right angles, [III. 22]
while the angle ABC is less than a right angle,
therefore the angle ADC which remains is greater than a right angle;
and it is the angle in the segment ADC less than the semicircle.

I say further that the angle of the greater segment, namely that contained by the circumference ABC and the straight line AC, is greater than a right angle;
and the angle of the less segment, namely that contained by the circumference ADC and the straight line AC, is less than a right angle.

This is at once manifest.

For, since the angle contained by the straight lines BA, AC is right,
the angle contained by the circumference ABC and the straight line AC is greater than a right angle.

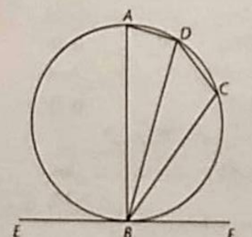
Again, since the angle contained by the straight lines AC, AF is right,
the angle contained by the straight line CA and the circumference ADC is less than a right angle.

Therefore etc.

Q.E.D.

Proposition 32

If a straight line touch a circle, and from the point of contact there be drawn across, in the circle, a straight line cutting the circle, the angles which it makes with the tangent will be equal to the angles in the alternate segments of the circle.



For let a straight line EF touch the circle $ABCD$ at the point B, and from the point B let there be drawn across, in the circle $ABCD$, a straight line BD cutting it;
I say that the angles which BD makes with the tangent EF will be equal to the angles in the alternate segments of the circle, that is, that the angle FBD is equal to the angle constructed in the segment BAD, and the angle EBD is equal to the angle constructed in the segment DCB.

For let BA be drawn from B at right angles to EF,
let a point C be taken at random on the circumference BD,
and let AD, DC, CB be joined.

Then, since a straight line EF touches the circle $ABCD$ at B,

and BA has been drawn from the point of contact at right angles to the tangent,
the centre of the circle $ABCD$ is on BA. [III. 19]

Therefore BA is a diameter of the circle $ABCD$;
therefore the angle ADB, being an angle in a semicircle, is right. [III. 31]

Therefore the remaining angles BAD, ABD are equal to one right angle. [I. 32]

But the angle ABF is also right;
therefore the angle ABF is equal to the angles BAD, ABD.

Let the angle ABD be subtracted from each;
therefore the angle DBF which remains is equal to the angle BAD in the alternate segment of the circle.

Next, since $ABCD$ is a quadrilateral in a circle,
its opposite angles are equal to two right angles. [III. 22]

But the angles DBF, DBE are also equal to two right angles;
therefore the angles DBF, DBE are equal to the angles BAD, BCD,
of which the angle BAD was proved equal to the angle DBF;
therefore the angle DBE which remains is equal to the angle DCB in the alternate segment DCB of the circle.

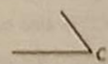
Therefore etc.

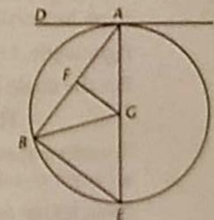
Q.E.D.

Proposition 33

On a given straight line to describe a segment of a circle admitting an angle equal to a given rectilineal angle.

Let AB be the given straight line, and the angle at C the given rectilineal angle;
thus it is required to describe on the given straight line AB a segment of a circle admitting an angle equal to the angle at C.





The angle at C is then acute, or right, or obtuse.

First, let it be acute, and, as in the first figure, on the straight line AB, and at the point A, let the angle BAD be constructed equal to the angle at C;
therefore the angle BAD is also acute.

Let AE be drawn at right angles to DA, let AB be bisected at F, let FG be drawn from the point F at right angles to AB, and let GB be joined.

Then, since AF is equal to FB,
and FG is common,
the two sides AF, FG are equal to the two sides BF, FG;
and the angle AFG is equal to the angle BFG;
therefore the base AG is equal to the base BG. [I. 4]

Therefore the circle described with centre G and distance GA will pass through B also.