GreatHearts
Irving

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

April 6 - April 10, 2020

Course: 10 Precalculus

Teacher: Mr Simmons michael.simmons@greatheartsirving.org

Weekly Plan:

Monday, March 30

☐ "Exponential and Logarithmic Models II" worksheet answer key

Tuesday, March 31

☐ Read about logistic growth models and work example problems

Wednesday, April 1

☐ Read "Choosing an Appropriate Model for Data" and work example problems

Thursday, April 2

☐ Read "Expressing an Exponential Model in Base *e*" and work example problems

Friday, April 3

☐ Problem set 4.7: 6-12

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

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Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____

Parent Signature

Monday, March 30

Read through, in detail, the answer key to the "Exponential and Logarithmic Models II" worksheet that you already completed. If you haven't already completed it, do that first.

Tuesday, March 31

Read, on pages 407-409 of the textbook, the section about logistic growth. Complete the example problem and the "try it" problem.

Wednesday, April 1

Check your answer to the "try it" problem in the back of the book.
Read, on pages 409-411, the section on "choosing an appropriate model for data," and complete the example problem and the "try it" problem.

Thursday, April 2

Check your answer to the "try it" problem in the back of the book.
Read, on pages 411-412, the section on "Expressing an Exponential Model in Base *e*," and complete the example and "try it" problems.

Friday, April 3

Check your answer to the "try it" problem in the back of the book.
Complete the problem set 4.7: 6-12.

Exponential and Logarithmic Models II

Mr. Simmons
Precalculus

Newton's Law of Cooling (which you derived in the previous worksheet) states that the temperature of an object, T, in surrounding air with temperature T_s, will behave according to the formula

$$T(t) = Ae^{kt} + T_s,$$

where

- t is time,
- A is the difference between the initial temperature of the object and the surroundings, and
- k is a constant, the continuous rate of cooling of the object.

Use Newton's Law of Cooling to answer the following questions:

1. A cheesecake is taken out of the oven with an ideal internal temperature of 165°F, and is placed into a 35°F refrigerator. After 10 minutes, the cheesecake has cooled to 150°F. If we must wait until thecheesecake has cooled to 70°F before we eat it, how long will we have to wait?

2. A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

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Mr. Simmons
Precalculus

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   Solution. Plugging in known values and solving for k, we get

   $$\begin{aligned} 150 &= \left(165 - 35\right) e^{k(10)} + 35 \\ 115 &= 130e^{10k} \\ \frac{23}{26} &= e^{10k} \\ 10k &= \ln\left(\frac{23}{26}\right) \\ k &= \frac{\ln\left(\frac{23}{26}\right)}{10}. \end{aligned}$$

   So

   $$T\left(t\right) = 130e^{\frac{\ln\left(\frac{23}{26}\right)}{10}t} + 35.$$

   We want to know what t will be when $T\left(t\right)$ is 70, so we plug in this known point and solve for t:

$$\begin{aligned}
70 &= 130e^{\frac{\ln\left(\frac{23}{26}\right)}{10}t} + 35 \\
35 &= 130e^{\frac{\ln\left(\frac{23}{26}\right)}{10}t} \\
\frac{7}{26} &= e^{\frac{\ln\left(\frac{23}{26}\right)}{10}t} \\
\frac{\ln\left(\frac{23}{26}\right)}{10}t &= \ln\left(\frac{7}{26}\right) \\
t &= 10\frac{\ln\left(\frac{7}{26}\right)}{\ln\left(\frac{23}{26}\right)} \\
&\approx 107.
\end{aligned}$$

So we need to wait about 107 minutes, or 1 hour and 47 minutes.

2. A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

   Solution. Plugging in known values and solving for unknown values, we have

$$\begin{aligned}
T(t) &= Ae^{kt} \\
(45) &= (40-70)\,e^{k(1)} + 70 \\
-25 &= -30e^{k} \\
\frac{5}{6} &= e^{k} \\
k &= \ln\left(\frac{5}{6}\right); \\
60 &= -30e^{\ln\left(\frac{5}{6}\right)t} + 70 \\
-10 &= -30e^{\ln\left(\frac{5}{6}\right)t} \\
\frac{1}{3} &= e^{\ln\left(\frac{5}{6}\right)t} \\
\ln\left(\frac{5}{6}\right)t &= \ln\left(\frac{1}{3}\right) \\
t &= \frac{\ln\left(\frac{1}{3}\right)}{\ln\left(\frac{5}{6}\right)} \\
&\approx 6.03;
\end{aligned}$$

   so it will take about 6.03 hours for the pitcher to warm to 60 degrees.