Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

May 4-8, 2020
Course: 10 Precalculus
Teacher(s): Mr. Simmons

Weekly Plan:

Monday, May 4
- ☐ Story time!
- ☐ Problems 11-14 and 16 from "Relationship between Trig Functions"

Tuesday, May 5
- ☐ Read "Radian measure."

Wednesday, May 6
- ☐ Problems 1-14

Thursday, May 7
- ☐ Read "Introduction to the Polar Plane

Friday, May 8
- ☐ Attend office hours
- ☐ Catch up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 4

1. Story time! If technologically feasible, email me with a story!

Also, please email me if you have questions about trigonometry. BUT. Please make your questions specific. At least give me a page number where you got confused. Better yet, tell me exactly what words you read that confused you.

For today, we're going to do a few more problems from "Relationships between Trig Functions" before moving on to radians:

2. Complete Problems 11-14 and 16 on pp. 122-123.

Tuesday, May 5

1. Read "Radian Measure" on pp. 124-133.

Wednesday, May 6

1. Complete Problems 1-14 on pp. 133-135.

Thursday, May 7

1. Read "Introduction to the Polar Plane" on pp. 135-143.

2.) Let us complete Example 1c. If $\sin\alpha = a$, what are the six Trig ratios?

3.) Now let $\tan\alpha = d$.
   - (A) What are the six Trig ratios given this?
   - (B) Compare this answer to the previous.

4.) In Example 1c, we let the length of the hypotenuse of the triangle to equal 1. Is that OK? Why don't we let the hypotenuse equal c, to allow for any and all possibilities? Now suppose $\sin\alpha = \frac{x}{c}$, and the lengths of your triangle are a, b, and c, with the hypotenuse equaling c.
   - (A) What are the six Trig ratios?
   - (B) Compare this with the previous two results.

5.) Given the Trig functions and angle measure, write the equivalent cofunction.
   - (A) $\sin 30°$
   - (B) $\cos 10°$
   - (C) $\cot 7°$
   - (D) $\sec 64°$
   - (E) $\cos 31°$
   - (F) $\tan 14°$
   - (G) $\csc 47°$
   - (H) $\sin 25°$
   - (I) $\tan(\beta + \gamma)$
   - (J) $\sin\beta$

6.) Write out all of the cofunction identities, including the ones we discovered in the reading. Hint: There are six of them.

7.) Now write out all of the reciprocal identities. Hint: There are six of them.

8.) Evaluate the following.
   - (A) $\sin^2 30°$
   - (B) $\cos^2 30°$
   - (C) $\tan^2 30°$
   - (D) $\cos^2 45°$
   - (E) $\tan^2 45°$
   - (F) $\sin^2 60°$
   - (G) $\cos^2 60°$
   - (H) $\tan^2 60°$
   - (I) $\sin(30°^2)$
   - (J) $\cos^2\alpha$

9.) Write out a table of values for $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$, starting at $\alpha = 0$, going up by 5° each row, and ending at $\alpha = 90°$. You will need a calculator for this Exercise.

10.) Use your results from the previous Exercise to answer the following questions.
   - (A) What is the maximum value of $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$?
   - (B) What is the minimum value of $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$?
   - (C) Are there any similarities or differences between $\sin^2\alpha$ and $\sin\alpha$? Compare your results from the previous section.

11.) One of the most important relationships in Trigonometry is the Pythagorean Identity we discussed in the reading. Write this identity down now.

12.) Evaluate the following.
   - (A) $\sin^2 60° + \cos^2 60°$
   - (B) $\cos^2 30° + \sin^2 30°$
   - (C) $\sin^2\left(\frac{\alpha}{2}\right) + \cos^2\left(\frac{\alpha}{2}\right)$
   - (D) $\sin^2(3\alpha + \pi) + \cos^2(3\alpha + \pi)$

13.) It is often helpful to rewrite $\sin^2\alpha$ or $\cos^2\alpha$. Use the Pythagorean Identity to rewrite $\sin^2\alpha$ and $\cos^2\alpha$.

14.) Simplify the following.
   - (A) $\tan\alpha \cdot \csc\alpha$
   - (B) $(\sin\alpha + \cos\alpha)^2$

15.) Are there any other Pythagorean Identities? To find this out, use a calculator and try the following for different values of α.
   - (A) $\sec^2\alpha + \csc^2\alpha$
   - (B) $\tan^2\alpha + \cot^2\alpha$
   - (C) There are two other Pythagorean Identities. First, using the previous two, guess what they might be. Then, if you can't figure it out, look them up and write them down now. We'll discover how to arrive at these results when we have some better tools.

16.) Answer True or False.
   - (A) $\sin\alpha = \cos(\alpha - 90°)$
   - (B) $\sin^2\alpha + \cos^2\beta = 1$ iff $\alpha + \beta = 90°$
   - (C) $\sin^2\alpha = \sin\alpha \cdot \alpha$
   - (D) $\sin^2\alpha$ is sometimes negative.[vi]

[vi] Assume $\alpha \in \mathbb{R}$.

Unit five

Radians and the Unit Circle

"Degrees are fine for everyday measurements. But Trigonometry marks a turning point in math, when the student lifts his gaze from the everyday towards larger, more distant ideas. You begin exploring basic relationships, deep symmetries, the kinds of patterns that make the universe tick. And to navigate that terrain, you need a notion of angles that's more natural, more fundamental, than slicing up the circle into an arbitrary number of pieces. The number π, strange though it may seem, lies at the heart of mathematics. The number 360 doesn't. Clinging to that Babylonian artifact will only distract you and obscure the elegant truths you're searching for."

Ben Orlin

Up to this point, we've measured all of our angles using degrees. In this unit, we'll endeavor to find a different and perhaps better method of measuring angles. Then we'll use that to graph points in a new type of plane. Finally, after this, we introduce perhaps the most important thing in Trigonometry: The Unit Circle.

§1 Radian measure

Degrees were invented millennia ago, perhaps by the ancient peoples living in modern day Iraq. Knowing the origins of this unit could shed some light on its usefulness, and whether there isn't a more useful unit to use.

There are various theories as to why degrees were used and why they are the way that are. Almost certainly, however, it has to do with a circle. As with anything, it's often useful to consider portions or fractions of the whole.[i] The ancients chose to chop the circle up into 360 equal portions, calling the angle created by each portion a degree, as (partially) shown in Figure 47.

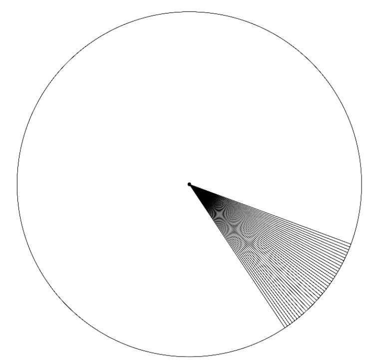


Figure 47

Each of the individual spokes above measures a single degree. If we were to continue creating these spokes, there would be 360 of them.

Why 360? Perhaps because it is a nice number with many factors. So cutting a circle in half gives you a nice number of 180°, in thirds 120°, fourths 90°, and so on. This means

[i] This is why, for example, we have yards. Could you imagine measuring things if the smallest unit we could get was miles? And even that isn't enough, which is why continue to subdivide the units smaller and smaller.

that commonly used ratios are left with a whole number. This wouldn't be the case if the number, say, 10, was used. Then only a half-circle and a fifth-of-a-circle would have whole numbers. Another supposition is that there are approximately 360 days in a year. And since, each year, seasons repeat themselves, a circle makes a nice representation of a calendar.

Whatever the reason, however, we want to see if there is a better way of measuring angles. Of course, "better" is relative, and different situations might call for different units. So when we say "better," perhaps what we should say is more appropriate for our work in Trigonometry.

Consider the circle shown in Figure 48. What is the length of the **arc** from A to B?

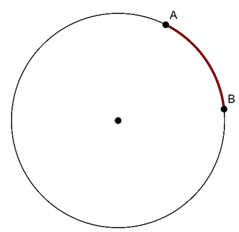


Figure 48

There are a few ways we could answer this question. One is to measure it the old-fashioned way. That, however, leaves room for error, and wouldn't help us to measure an arc from a different circle. Another way we could do it is to find the circumference of the circle, then multiply by the fraction of the outside of the circle represented by $\widehat{AB}$.[ii] This isn't the worst thing in the world, but then… How will we measure the angle which will allow us to find the fraction of the outside of the circle that $\widehat{AB}$ takes up? As you can see, we have a bit of an issue.

As we've done a few times in this course, we should go back to what we know for certain. We know that the circumference of a circle is

$$C = 2\pi r,$$

[ii] For example, if the arc were half of the outside of the circle, you would multiply the circumference of the circle by $\frac{1}{2}$, right?

where r is the radius of a circle and π is the mathematical constant approximately equal to 3.14. We also know that every radius in a circle is congruent. And that's about it. But this does show us that if we're trying to figure stuff out about a circle, it is usually a good idea to involve a radius. That's what we'll do in Figure 49.

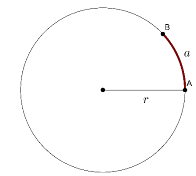


Figure 49

In keeping with our tradition, we've used the Latin letters r and a for the lengths of the radius and arc respectively.

Now, let's see what happens when we relate the radius to the arc length. Let's assume for a moment that $r = a$, i.e., that the radius is the same length as $\widehat{AB}$. This would allow us to create that angle seen in Figure 50, right?

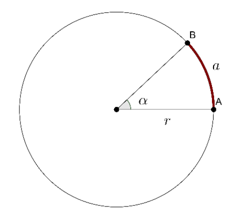


Figure 50

This angle, which we'll call α, is unique. In other words, there is one and only one angle for which the radius is the same length as $\widehat{AB}$. Figure 51 shows this to be true.

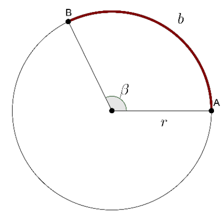


Figure 51

Here, $b = 1.5r$. As a consequence of this increased arc size, the angle is larger, and therefore $\alpha \neq \beta$.

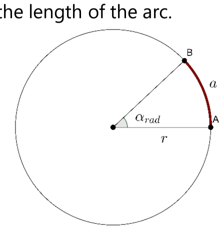
This is interesting for a couple of reasons. First of all, notice that there is one and only one angle that comes out as a consequence of the comparison to the radius and arc length. Secondly, and perhaps more importantly, the size of the circle (and, by extension), the lengths of the radius and arc, won't matter.[iii] Thus we make the following definition.

Radian measure

The angle formed by the ratio of the arc length a to the radius r in a circle. Symbolically,

$$\alpha_{rad} = \frac{a}{r},$$

where r is the radius and a is the length of the arc.



Example 1a

What does an angle measure of 1 radian look like?

This is an excellent question, and a great starting point to see the intuition behind radians. A radian is a ratio between the arc length and the radius, or $\frac{a}{r}$, where r is the radius of a

[iii] We'll show this explicitly in the forthcoming Examples.

circle and a is the length of the arc we're concerned with. Using our definition, then, we have

$$1 = \frac{a}{r}.$$

But this will only be true if $a = r$. Therefore, a measure of 1 radian will look like Figure 52, where the radius and $\widehat{AB}$ are equal.

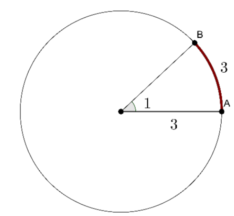


Figure 52

We've chosen a radius and arc length of 3, but we could have easily chosen any other length so long as $r = a$.

Notice that our angle measure is 1? You might be wondering what the units of this angle measure are, but there aren't any! You could say 1 radian, but if an angle measure is reported with no unit, it is assumed to be measured in radians.[iv]

Did you notice that our circle had a radius and arc length of 3? We wanted an angle of 1, and that is only true when $r = a$. So we could have also chosen $r = a = 5$, or $r = a = 100$, (and so on) if we wanted to. Do you see why?

Example 1b

What does an angle of 2 look like?

This is a similar question, so we again go back to the definition. The equation

$$2 = \frac{a}{r}$$

must be true. There is an infinite amount of possibilities for both a and r, such as $a = 6, r = 3$ (which we show in Figure 53).

[iv] This is another reason to prefer radians.

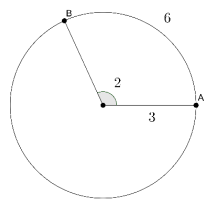


Figure 53

As you can see, the ratio is what's important. The fact that the arc length is twice the length of the radius is what tells us we have an angle measure of 2. Appreciate, also, how the angle is clearly different from the previous Example.

The previous two examples were there to help you get a grasp on radians, but we still haven't seen its best feature. We explore that now.

Example 2a

What is the length of the radius given Figure 54?

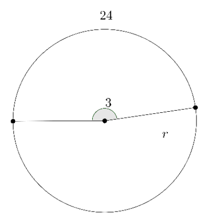


Figure 54

Now this is interesting. We are given an angle and an arc length, and are told to find the length of the radius. Using the definition of a radian, we can work backward and easily get the answer. Since, according to our definition, we have

$$3 = \frac{24}{r},$$

we simply solve the previous equation for r and get

$$r = 8.$$

Easy! But to really appreciate radians, consider Figure 55, where we have used degrees instead.

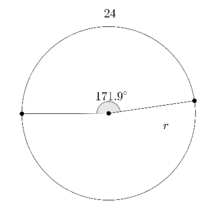


Figure 55

Could we find r? The answer is no – degrees are a measurement found completely independent of the size of a circle. Thus it offers us no help at all. Radians, therefore, give us free information!

Example 2b

What is the length of the arc subtended[v] by an angle of 6 and a radius of 10?

No picture is provided, and it would be helpful for you draw one, but it is not necessary. We simply use the definition:

$$6 = \frac{a}{10}.$$

Hence

$$a = 60.$$

There are a few more important questions which must be asked if we are to succeed with radians. For example, how many radians are in a full rotation? We know there are 360° in a full rotation, but what about radians?

Let us answer this question with a specific circle, and then generalize afterwards. Consider a circle with a radius of 1.[vi] Since the definition of a radian tells us that

$$\alpha_{rad} = \frac{a}{r},$$

And we have $r = 1$, we have

[v] This is a fancy, perhaps old-fashioned word which means formed or created by. So the arc is created by the angle.
[vi] We could have chosen any value for the radius, but we chose 1. Any thoughts on why we would choose this number and not, say, 23?

$$\alpha_{rad} = a.$$

If we are considering a full rotation, however, we are not looking at an arc, but the full circumference of the circle. Therefore, $a = 2\pi$ and hence

$$\alpha_{rad} = 2\pi.$$

This is an important fact, and we list it below for your convenience.

Radians in various rotations
Full rotation: 2π Half rotation: π Quarter rotation: $\frac{\pi}{2}$

Thus, there are 2π radians in a full rotation.

This allows us to answer our next most important question: How do radians relate to degrees? In other words, how does one convert from one to the other?

To find this answer, let us find out how many degrees are in one radian. To do this, we just need to convert. We will use dimensional analysis to help us do this, as we show below.

$$\frac{360°}{1\ \text{rotation}}\left|\frac{1\ \text{rotation}}{2\pi\ \text{radians}}\right| = \left|\frac{180°}{\pi\ \text{radians}}\right| \approx \left|\frac{57.30°}{1\ \text{radian}}\right.$$

The rotations cancel, and leave us with our result of approximately 57.3° for every 1 radian.

This is a strange number, and we will rarely use it. Instead, the fraction $\frac{180}{\pi}$ is what you should memorize and become comfortable with. That said, it is helpful to know how much 1 radian is in degrees, since it will help you get a picture of what you're working with.

Example 3a

Convert 6 radians into degrees.

Since there are $\frac{180}{\pi}$ degrees for every one radian, we simply multiply this number by 6. Thus 6 radians is

$$343.8°.$$

Example 3b

Convert 75° into radians.

This is the opposite of the previous problem. Thus, we need a new conversion factor. We apply the same principle to obtain a conversion factor:

$$\frac{2\pi}{1\ \text{rotation}}\left|\frac{1\ \text{rotation}}{360°}\right| = \left|\frac{\pi}{180}\right.$$

This tells us that one degree is $\frac{\pi}{180}$ radians. And since we want to find out how many radians 75° is, we just multiply the previous by $\frac{\pi}{180}$. We get

$$75 \cdot \frac{\pi}{180} = \frac{75\pi}{180} = \frac{5\pi}{12}.$$

When working with radians, we never want an approximation.

Did you notice how similar the conversion factors were? You should have them memorized, as you'll need them in this section and beyond.

§1 Exercises

1.) Determine the measure of α (in radians) given the following radii and arc lengths.
   (A) $r = 10, a = 20$
   (B) $r = 30, a = 10$
   (C) $r = 15, a = 100$
   (D) $r = \frac{3}{4}, a = 16$

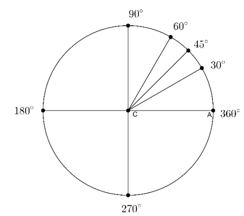
2.) Determine the length of $\widehat{AB}$ given the following radii and angles. Then sketch a circle with the given information and the length of the arc.
   (A) $r = 5, \alpha = 2$
   (B) $r = 3, \alpha = 3$
   (C) $r = \frac{5}{3}, \alpha = \frac{3}{4}$
   (D) $r = 10, \alpha = \frac{1}{2}$

3.) Determine the length of the radius given the following arc lengths and angles.
   (A) $\widehat{AB} = 35, \alpha = 7$
   (B) $\widehat{AB} = 5, \alpha = 5$
   (C) $\widehat{AB} = 10, \alpha = 3$
   (D) $\widehat{AB} = 6, \alpha = \frac{1}{2}$

4.) Sketch the following angles.
   (A) 1
   (B) 2
   (C) 3
   (D) 4
   (E) 5
   (F) 6
   (G) 0.5
   (H) 6.28

5.) Convert the following angle measure from angles to radians or vice versa.
   (A) 200°
   (B) 100°
   (C) 50°
   (D) 1.5
   (E) $\frac{\pi}{12}$
   (F) 720°
   (G) $\frac{\pi}{5}$
   (H) π

6.) The following Figure is a circle with various rotations on it. Assume that C is located at the origin, and that there are four Quadrants, as normally defined on a coordinate plane. Each angle begins with $\overline{AC}$, then rotates counter-clockwise up to the next point.



(A) Write each of the angle measures in radians.[vii]
(B) Which is the larger angle, $\frac{\pi}{4}$ or $\frac{\pi}{2}$?
(C) Assume that we continue this pattern, so that in Quadrant II the next angle is 30° more than 90°, then 45° more than 90°, and so on (including Quadrants III and IV). Write each of these angles in degrees.
(D) Now write each of the angles you found in (C) in radians.

7.) One of the problems that students have with radians is that they're terrible with fractions. Accordingly, let us practice our fraction intuition.
(A) Which number is larger, $\frac{1}{4}$ or $\frac{1}{2}$? How can you tell without having to divide the numerator and denominator?
(B) Make an argument for why $\frac{1}{4}$ is less that $\frac{1}{2}$. (Hint: Try using money!)
(C) Likewise, which is larger: $\frac{\pi}{3}$ or $\frac{\pi}{6}$?
(D) When comparing a whole number to a fraction, it's often useful to convert the whole number into a fraction. For example, which is larger, 2 or $\frac{5}{3}$? To see, let's convert 2 into a fraction that has the same denominator as $\frac{5}{3}$. Now answer the question: Which is larger, 2 or $\frac{5}{3}$?
(E) Which is larger: π or $\frac{5\pi}{6}$?
(F) Which is larger: $\frac{15\pi}{4}$ or 2π?

8.) How can you tell that $\frac{5\pi}{6}$ is less than a half-rotation? (Hint: Try using common denominators in your fractions.)

9.) Which Quadrant is $\frac{11\pi}{6}$ in? How can you quickly tell? (Hint: Use the Figure from 2.) to help you visualize.)

10.) Of course, you can always convert the radians into degrees to check which one is larger. Convert $\frac{7\pi}{4}$ and $\frac{5\pi}{3}$ into degrees and then determine which one is larger.

11.) Now take $\frac{7\pi}{4}$ and $\frac{5\pi}{3}$ and get common denominators. Which one is larger?

[vii] And just to reiterate: Your answers must be in exact form.

12.) A wheel has a radius of 12 inches.[viii]
(A) If the wheel makes a full rotation, how far has the wheel traveled from its starting point?
(B) If the wheel makes a half rotation, how far has the wheel traveled from its starting point?
(C) Suppose the wheel has rotated 10π. How far has it traveled?
(D) If the wheel has traveled 36 inches, how much has it rotated?

13.) Suppose a wheel is 24 inches around.
(A) If the wheel has rotated $\frac{5\pi}{6}$, how far has it traveled?
(B) If the wheel has traveled 10 feet, how much has it rotated?

14.) Suppose a wheel has traveled 10 feet.
(A) If it has rotated $\frac{13\pi}{6}$, what is its radius?
(B) If it has rotated $\frac{3\pi}{4}$, what is its circumference?

§2 Introduction to the Polar Plane

After a brief hiatus, we now return to our Trig functions. We will use what you learned in Unit four extensively in this section; you will need to be able to calculate Trig ratios very quickly.[i] We will also begin to work with Trig functions where our angles are in radians.

To help us practice and memorize these Trig functions, we will now introduce a new way to graph.

Before we do this, a few words on what makes the coordinate plane so effective. There are numerous ways one could set up a graphing system, but, ideally, we would like it be simple to use and effective. The coordinate plane is great because it requires just two components (an x- and y-value) to plot any point. So it's effective and easy to use.

So if we're going to come up with a new way to plot points, it should be just as simple and effective. In other words, we should come up with a system that only requires two components to plot any point. Anything more than that will render our system far less effective.

So let us consider a system that uses circles instead of rectangular gridlines, as shown in Figure 56.

[viii] Assume that the wheel is a perfect circle.
[i] Or, as we suggested, you should simply memorize them.

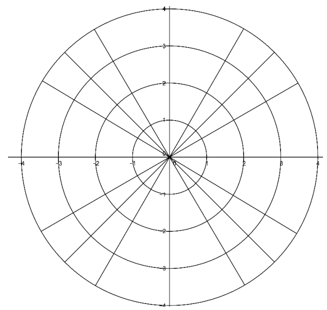


Figure 56

Let us emphasize that we have circles here, not rectangular gridlines. So using an x- and y-value will not suffice. We need two different components entirely. How about we use an angle measure and a radius? That should allow us to plot any point on this plane using only two components.

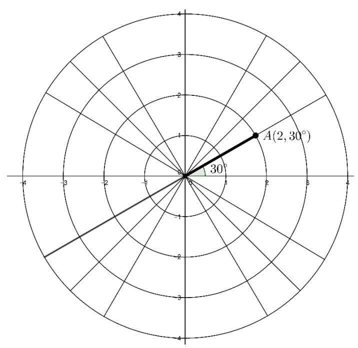


Figure 57

Consider Figure 57. We have two components, 2 and 30°. Let's start with the 2. Formally, it's a radius – that is, it is the distance from the center to the point. Another way of looking

at it is that our point must be on the second circle.[ii] The second component, 30°, tells us where upon that second circle we must put our point. So we place our point 2 away from the origin,[iii] then rotate the point 30° up. We show this process in Figures 58 a and b.

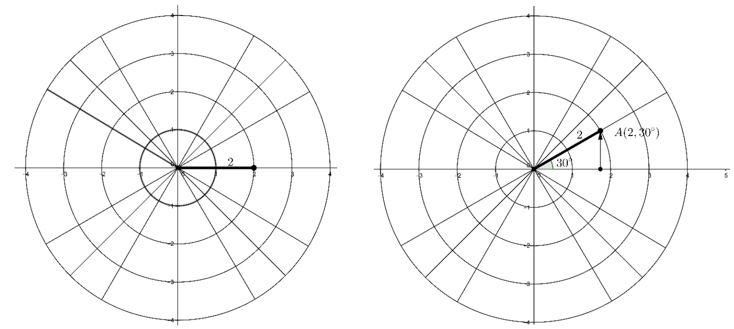


Figure 58a and b

We started by making a segment of length 2, then rotating that segment up 30°. Note that the segment only helps to place the point; what we really care about is the point.

This process works very well, so let us now define our new way of graphing

Graphing on the Polar Plane
A point $A(r, \alpha)$ is plotted by creating a line segment of length r, and then rotating that segment α.

Just like with the coordinate plane, we should label our circles. You may have noticed lines extruding from the **pole**, or origin of the Polar Plane. These lines form angles with positive x-axis, and look awfully similar to a problem from the previous section. We'll label them below in Figure 59.

[ii] There's a disadvantage to this perspective, since we could also have 1.5 as the radius. In this case, we need to draw a circle halfway between the first and second circle. So it's not too much of a stretch to us this logic.
[iii] Which we'll give a different name shortly.

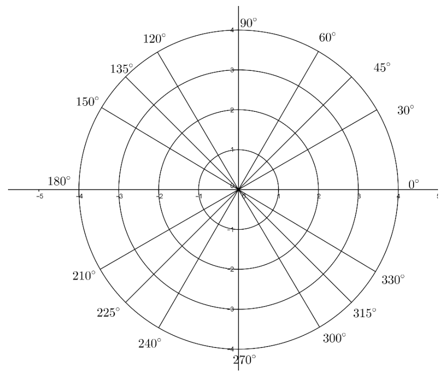


Figure 59

As with the coordinate plane, we don't *have* to choose these angles. However, there is good reason to choose these particular angles, and we'll reveal that answer shortly.[iv] Also of important note: The angle measures always start on what we normally call the positive x-axis and rotate up from there.[v]

Example 1a

Plot the point $A(2,45°)$.

To do this, we simply go to our second circle, then rotate up 45°. We show the plotted point in Figure 60.

[iv] If you've not already figured it out yourself.

[v] This is by convention. Someone somewhere decided that they were going to do it that way, and we've all followed suit.

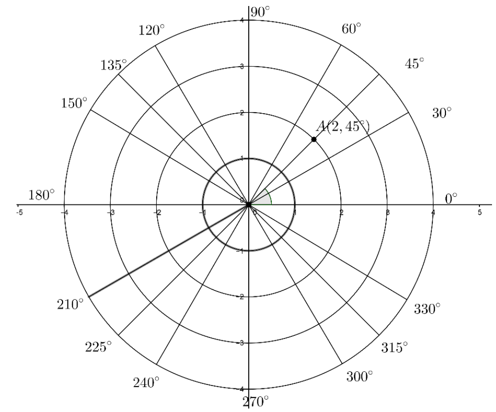


Figure 60

Example 1b

Graph the point $B(3.5,10°)$.

Neither of the two components in B is on a line, but, like the coordinate plane, we can easily approximate their locations. We show this in Figure 61.

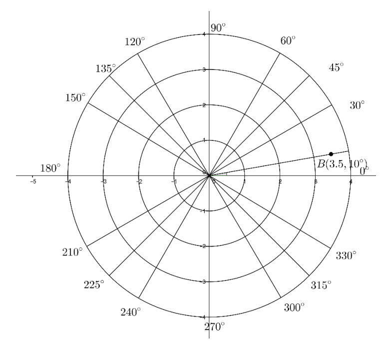


Figure 61

It might be helpful to draw a line for a 10° angle. It might also be helpful to draw a circle halfway between the third and fourth circle.

Example 1c

Graph the point $C\left(2.2, \frac{\pi}{2}\right)$.

In this example, we are using radians to measure our angles and not degrees. Recalling that 90° and $\frac{\pi}{2}$ are equivalent, we can easily graph Figure 62.

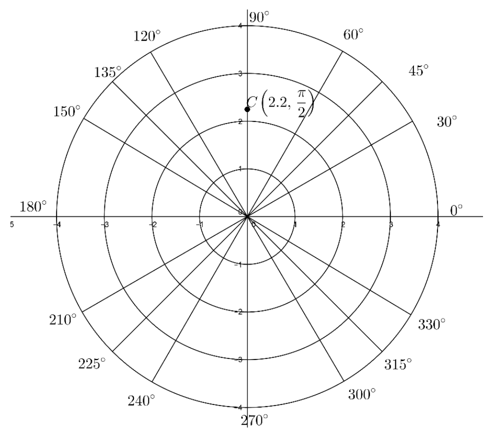


Figure 62

Many of your Exercises will use radians, so the above graph we've provided may not be the most helpful. You will want to create a Polar Plane with the radians listed and not degrees. But don't worry, this will be one of your Exercises.

We have not yet discussed negative angles. Up to this point, you might think that angles, like lengths, can only be positive. But angles (unlike lengths) have a direction. So far we've always rotated "up", which has amounted to a counter-clockwise rotation. Nothing is stopping us from rotating in the opposite direction, i.e., clockwise, but we haven't had a good way to label this other than spelling it out entirely. Let us therefore agree that a negative angle measure tells us to rotate clockwise, while a positive angle measure tells us to rotate counter-clockwise.

Example 2

Graph the point $D(3, -60°)$.

The negative angle tells us to rotate 60° clockwise. Since we know our point must be on the third circle (due to the radius being 3), we just need to determine how to rotate clockwise. Using the same Polar Plane as before, but counting in the opposite direction (and applying appropriate labels), we come up with Figure 63.

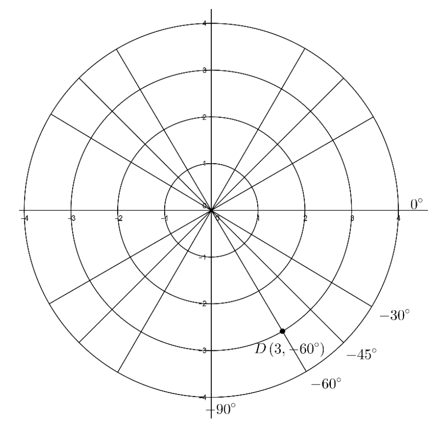


Figure 63

Perhaps you noticed that we did not complete labeling this Polar Plane. As you might have guessed, yes, this will be one of your Exercises.

But there's something quite curious as to the above: This is a point we could have made using our Polar Plane from before. If we take the same point but switch the labels back, we get Figure 64.

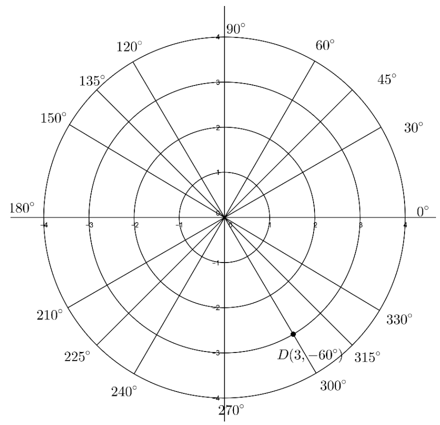


Figure 64

We've left the label from the previous Figure for you to help you compare.

Notice that our point is now located at $(3,300°)$. So it appears as though 300° is equivalent to $-60°$. This is interesting!

Coterminal angles

Two different angles that end up in the same spot are said to be coterminal.

So $-60°$ and $300°$ are coterminal, since they end up in the exact same spot. Another way of looking at this is that the point created by $(r, -60°)$ and $(r, 300°)$ will be the same (where $r \in \mathbb{R} > 0$).

Example 3

Plot the point $E(2, 840°)$.

This problem contains another strange angle. After all, there are only 360° in a rotation. But who's to say that we can only do one rotation? To account for multiple rotations, we can have angles that exceed 360°.[vi] How many rotations is 840°? Well, if 360° is one rotation, then two rotations would be

[vi] Likewise, we can also have angles that are less than −360°.

$$360° + 360° = 720°,$$

right? Likewise, we can see that three rotations would be 1,080°. So it seems as though we have two full rotations and then some left over. To account for this, we'll just subtract two full rotations from what we have, 840°:

$$840° - 720°$$

$$120°.$$

So we have two full rotations and then 120° more. This helps tremendously when we graph 840°, since now all we have to do is identify the 120° angle on our Polar Plane. We show this in Figure 65.

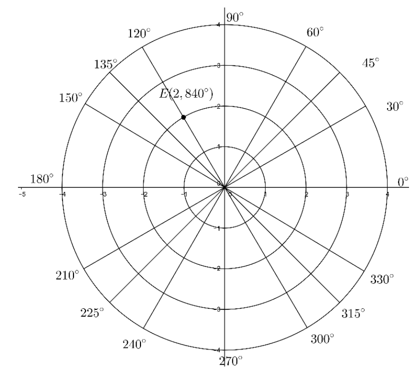


Figure 65

This also tells us that 840° and 120° are coterminal.

And this, in turn, tells us how many coterminal angles each angle has. Can you figure it out?

We'll further explore these ideas in the Exercises, as well as prepare for our more formal introduction to the Polar Plane in Unit seven. The brevity of this section will give you time to practice the basics and, perhaps more importantly, pave the way for success in the last section of this Unit, which is perhaps the most important.