Example 5 Using Newton's Law of Cooling

A cheesecake is taken out of the oven with an ideal internal temperature of 165°F, and is placed into a 35°F refrigerator. After 10 minutes, the cheesecake has cooled to 150°F. If we must wait until the cheesecake has cooled to 70°F before we eat it, how long will we have to wait?

Solution Because the surrounding air temperature in the refrigerator is 35 degrees, the cheesecake's temperature will decay exponentially toward 35, following the equation

$$T(t) = Ae^{kt} + 35$$

We know the initial temperature was 165, so $T(0) = 165$.

$$165 = Ae^{k0} + 35 \qquad \text{Substitute } (0, 165).$$

$$A = 130 \qquad \text{Solve for } A.$$

We were given another data point, $T(10) = 150$, which we can use to solve for k.

$$150 = 130e^{k10} + 35 \qquad \text{Substitute } (10, 150).$$

$$115 = 130e^{k10} \qquad \text{Subtract 35.}$$

$$\frac{115}{130} = e^{10k} \qquad \text{Divide by 130.}$$

$$\ln\left(\frac{115}{130}\right) = 10k \qquad \text{Take the natural log of both sides.}$$

$$k = \frac{\ln\left(\frac{115}{130}\right)}{10} \approx -0.0123 \qquad \text{Divide by the coefficient of } k.$$

This gives us the equation for the cooling of the cheesecake: $T(t) = 130e^{-0.0123t} + 35$.

Now we can solve for the time it will take for the temperature to cool to 70 degrees.

$$70 = 130e^{-0.0123t} + 35 \qquad \text{Substitute in 70 for } T(t).$$

$$35 = 130e^{-0.0123t} \qquad \text{Subtract 35.}$$

$$\frac{35}{130} = e^{-0.0123t} \qquad \text{Divide by 130.}$$

$$\ln\left(\frac{35}{130}\right) = -0.0123t \qquad \text{Take the natural log of both sides}$$

$$t = \frac{\ln\left(\frac{35}{130}\right)}{-0.0123} \approx 106.68 \qquad \text{Divide by the coefficient of } t.$$

It will take about 107 minutes, or one hour and 47 minutes, for the cheesecake to cool to 70°F.

Try It #17

A pitcher of water at 40 degrees Fahrenheit is placed into a 70 degree room. One hour later, the temperature has risen to 45 degrees. How long will it take for the temperature to rise to 60 degrees?

Using Logistic Growth Models

Exponential growth cannot continue forever. Exponential models, while they may be useful in the short term, tend to fall apart the longer they continue. Consider an aspiring writer who writes a single line on day one and plans to double the number of lines she writes each day for a month. By the end of the month, she must write over 17 billion lines, or one-half-billion pages. It is impractical, if not impossible, for anyone to write that much in such a short period of time. Eventually, an exponential model must begin to approach some limiting value, and then the growth is forced to slow. For this reason, it is often better to use a model with an upper bound instead of an exponential growth model, though the exponential growth model is still useful over a short term, before approaching the limiting value.

The **logistic growth model** is approximately exponential at first, but it has a reduced rate of growth as the output approaches the model's upper bound, called the **carrying capacity**. For constants a, b, and c, the logistic growth of a population over time x is represented by the model

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

The graph in **Figure 6** shows how the growth rate changes over time. The graph increases from left to right, but the growth rate only increases until it reaches its point of maximum growth rate, at which point the rate of increase decreases.

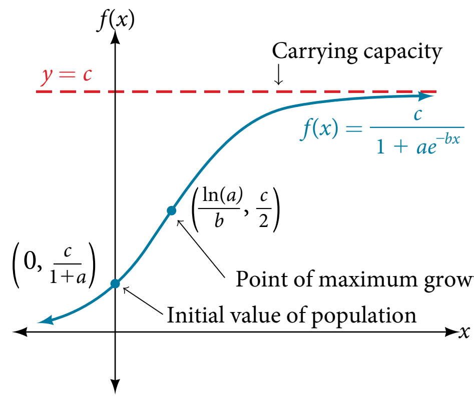


Figure 6

> ***logistic growth***
>
> The logistic growth model is
>
> $$f(x) = \frac{c}{1 + ae^{-bx}}$$
>
> where
>
> - $\frac{c}{1+a}$ is the initial value
> - c is the *carrying capacity, or limiting value*
> - b is a constant determined by the rate of growth.

Example 6 **Using the Logistic-Growth Model**

An influenza epidemic spreads through a population rapidly, at a rate that depends on two factors: The more people who have the flu, the more rapidly it spreads, and also the more uninfected people there are, the more rapidly it spreads. These two factors make the logistic model a good one to study the spread of communicable diseases. And, clearly, there is a maximum value for the number of people infected: the entire population.

For example, at time $t = 0$ there is one person in a community of 1,000 people who has the flu. So, in that community, at most 1,000 people can have the flu. Researchers find that for this particular strain of the flu, the logistic growth constant is $b = 0.6030$. Estimate the number of people in this community who will have had this flu after ten days. Predict how many people in this community will have had this flu after a long period of time has passed.

Solution We substitute the given data into the logistic growth model

$$f(x) = \frac{c}{1 + ae^{-bx}}$$

Because at most 1,000 people, the entire population of the community, can get the flu, we know the limiting value is $c = 1000$. To find a, we use the formula that the number of cases at time $t = 0$ is $\frac{c}{1+a} = 1$, from which it follows that $a = 999$. This model predicts that, after ten days, the number of people who have had the flu is $f(x) = \frac{1000}{1 + 999e^{-0.6030x}} \approx 293.8$. Because the actual number must be a whole number (a person has either had the flu or not) we round to 294. In the long term, the number of people who will contract the flu is the limiting value, $c = 1000$.

Analysis *Remember that, because we are dealing with a virus, we cannot predict with certainty the number of people infected. The model only approximates the number of people infected and will not give us exact or actual values. The graph in* ***Figure 7*** *gives a good picture of how this model fits the data.*

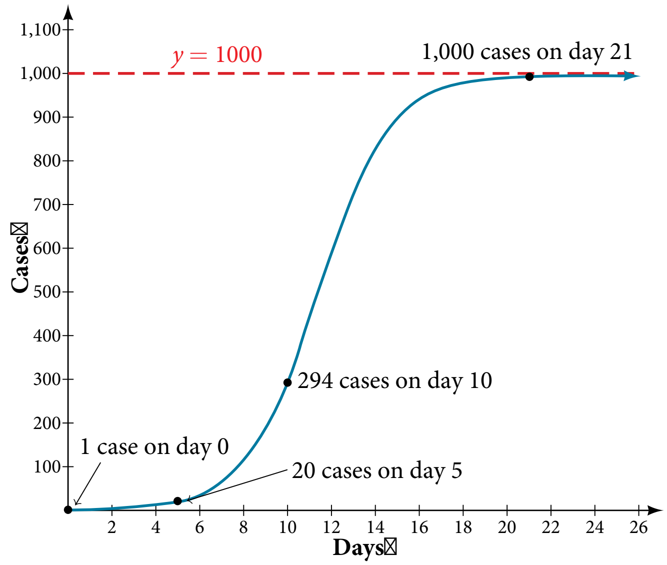


Figure 7 The graph of $f(x) = \dfrac{1000}{1 + 999e^{-0.6030x}}$

Try It #18

Using the model in **Example 6**, estimate the number of cases of flu on day 15.

Choosing an Appropriate Model for Data

Now that we have discussed various mathematical models, we need to learn how to choose the appropriate model for the raw data we have. Many factors influence the choice of a mathematical model, among which are experience, scientific laws, and patterns in the data itself. Not all data can be described by elementary functions. Sometimes, a function is chosen that approximates the data over a given interval. For instance, suppose data were gathered on the number of homes bought in the United States from the years 1960 to 2013. After plotting these data in a scatter plot, we notice that the shape of the data from the years 2000 to 2013 follow a logarithmic curve. We could restrict the interval from 2000 to 2010, apply regression analysis using a logarithmic model, and use it to predict the number of home buyers for the year 2015.

Three kinds of functions that are often useful in mathematical models are linear functions, exponential functions, and logarithmic functions. If the data lies on a straight line, or seems to lie approximately along a straight line, a linear model may be best. If the data is non-linear, we often consider an exponential or logarithmic model, though other models, such as quadratic models, may also be considered.

In choosing between an exponential model and a logarithmic model, we look at the way the data curves. This is called the concavity. If we draw a line between two data points, and all (or most) of the data between those two points lies above that line, we say the curve is concave down. We can think of it as a bowl that bends downward and therefore cannot hold water. If all (or most) of the data between those two points lies below the line, we say the curve is concave up. In this case, we can think of a bowl that bends upward and can therefore hold water. An exponential curve, whether rising or falling, whether representing growth or decay, is always concave up away from its horizontal asymptote. A logarithmic curve is always concave away from its vertical asymptote. In the case of positive data, which is the most common case, an exponential curve is always concave up, and a logarithmic curve always concave down.

A logistic curve changes concavity. It starts out concave up and then changes to concave down beyond a certain point, called a point of inflection.

After using the graph to help us choose a type of function to use as a model, we substitute points, and solve to find the parameters. We reduce round-off error by choosing points as far apart as possible.

Example 7 Choosing a Mathematical Model

Does a linear, exponential, logarithmic, or logistic model best fit the values listed in **Table 1**? Find the model, and use a graph to check your choice.

x	1	2	3	4	5	6	7	8	9
y	0	1.386	2.197	2.773	3.219	3.584	3.892	4.159	4.394

Table 1

Solution First, plot the data on a graph as in **Figure 8**. For the purpose of graphing, round the data to two significant digits.

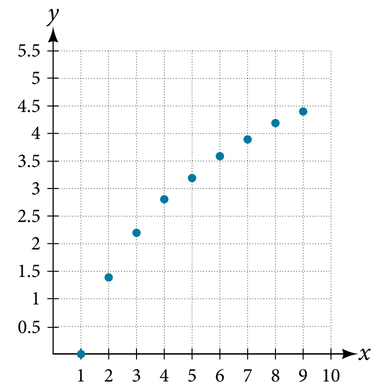


Figure 8

Clearly, the points do not lie on a straight line, so we reject a linear model. If we draw a line between any two of the points, most or all of the points between those two points lie above the line, so the graph is concave down, suggesting a logarithmic model. We can try $y = a\ln(bx)$. Plugging in the first point, (1,0), gives $0 = a\ln b$.

We reject the case that $a = 0$ (if it were, all outputs would be 0), so we know $\ln(b) = 0$. Thus $b = 1$ and $y = a\ln(x)$. Next we can use the point (9,4.394) to solve for a:

$$y = a\ln(x)$$

$$4.394 = a\ln(9)$$

$$a = \frac{4.394}{\ln(9)}$$

Because $a = \dfrac{4.394}{\ln(9)} \approx 2$, an appropriate model for the data is $y = 2\ln(x)$.

To check the accuracy of the model, we graph the function together with the given points as in **Figure 9**.

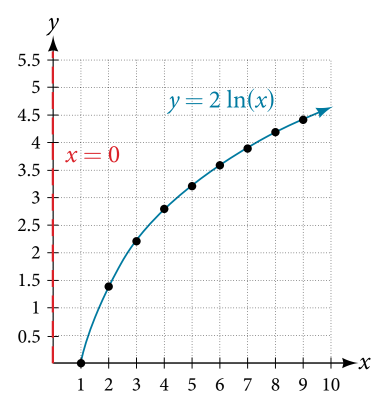


Figure 9 The graph of $y = 2\ln x$.

We can conclude that the model is a good fit to the data.

Compare **Figure 9** to the graph of $y = \ln(x^2)$ shown in **Figure 10**.

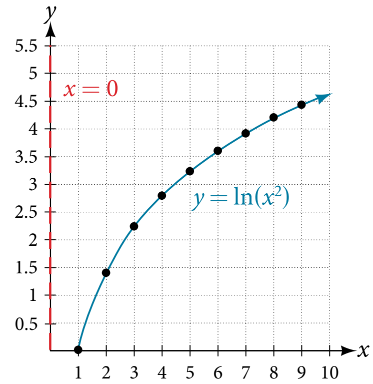


Figure 10 The graph of $y = \ln(x^2)$

The graphs appear to be identical when $x > 0$. A quick check confirms this conclusion: $y = \ln(x^2) = 2\ln(x)$ for $x > 0$.

However, if $x < 0$, the graph of $y = \ln(x^2)$ includes a "extra" branch, as shown in **Figure 11**. This occurs because, while $y = 2\ln(x)$ cannot have negative values in the domain (as such values would force the argument to be negative), the function $y = \ln(x^2)$ can have negative domain values.

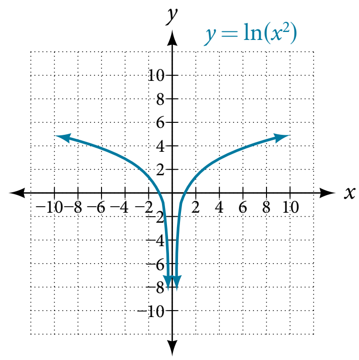


Figure 11

Try It #19

Does a linear, exponential, or logarithmic model best fit the data in **Table 2**? Find the model.

x	1	2	3	4	5	6	7	8	9
y	3.297	5.437	8.963	14.778	24.365	40.172	66.231	109.196	180.034

Table 2

Expressing an Exponential Model in Base *e*

While powers and logarithms of any base can be used in modeling, the two most common bases are 10 and e. In science and mathematics, the base e is often preferred. We can use laws of exponents and laws of logarithms to change any base to base e.

How To…

Given a model with the form $y = ab^x$, change it to the form $y = A_0e^{kx}$.

1. Rewrite $y = ab^x$ as $y = ae^{\ln(b^x)}$.
2. Use the power rule of logarithms to rewrite y as $y = ae^{x\ln(b)} = ae^{\ln(b)x}$.
3. Note that $a = A_0$ and $k = \ln(b)$ in the equation $y = A_0e^{kx}$.

Example 8 Changing to base e

Change the function $y = 2.5(3.1)^x$ so that this same function is written in the form $y = A_0 e^{kx}$.

Solution

The formula is derived as follows

$$
\begin{aligned}
y &= 2.5(3.1)^x \\
&= 2.5e^{\ln(3.1^x)} && \text{Insert exponential and its inverse.} \\
&= 2.5e^{x\ln 3.1} && \text{Laws of logs.} \\
&= 2.5e^{(\ln 3.1)x} && \text{Commutative law of multiplication}
\end{aligned}
$$

Try It #20

Change the function $y = 3(0.5)^x$ to one having e as the base.

Access these online resources for additional instruction and practice with exponential and logarithmic models.

- Logarithm Application – pH (http://openstaxcollege.org/l/logph)
- Exponential Model – Age Using Half-Life (http://openstaxcollege.org/l/expmodelhalf)
- Newton's Law of Cooling (http://openstaxcollege.org/l/newtoncooling)
- Exponential Growth Given Doubling Time (http://openstaxcollege.org/l/expgrowthdbl)
- Exponential Growth – Find Initial Amount Given Doubling Time (http://openstaxcollege.org/l/initialdouble)

4.7 SECTION EXERCISES

VERBAL

1. With what kind of exponential model would *half-life* be associated? What role does half-life play in these models?

2. What is carbon dating? Why does it work? Give an example in which carbon dating would be useful.

3. With what kind of exponential model would *doubling time* be associated? What role does doubling time play in these models?

4. Define Newton's Law of Cooling. Then name at least three real-world situations where Newton's Law of Cooling would be applied.

5. What is an order of magnitude? Why are orders of magnitude useful? Give an example to explain.

NUMERIC

6. The temperature of an object in degrees Fahrenheit after t minutes is represented by the equation $T(t) = 68e^{-0.0174t} + 72$. To the nearest degree, what is the temperature of the object after one and a half hours?

For the following exercises, use the logistic growth model $f(x) = \dfrac{150}{1 + 8e^{-2x}}$.

7. Find and interpret $f(0)$. Round to the nearest tenth.

8. Find and interpret $f(4)$. Round to the nearest tenth.

9. Find the carrying capacity.

10. Graph the model.

11. Determine whether the data from the table could best be represented as a function that is linear, exponential, or logarithmic. Then write a formula for a model that represents the data.

x	–2	–1	0	1	2	3	4	5
$f(x)$	0.694	0.833	1	1.2	1.44	1.728	2.074	2.488

12. Rewrite $f(x) = 1.68(0.65)^x$ as an exponential equation with base e to five significant digits.

TECHNOLOGY

For the following exercises, enter the data from each table into a graphing calculator and graph the resulting scatter plots. Determine whether the data from the table could represent a function that is linear, exponential, or logarithmic.

13.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	2	4.079	5.296	6.159	6.828	7.375	7.838	8.238	8.592	8.908

14.

x	1	2	3	4	5	6	7	8	9	10
$f(x)$	2.4	2.88	3.456	4.147	4.977	5.972	7.166	8.6	10.32	12.383

15.

x	4	5	6	7	8	9	10	11	12	13
$f(x)$	9.429	9.972	10.415	10.79	11.115	11.401	11.657	11.889	12.101	12.295

16.

x	1.25	2.25	3.56	4.2	5.65	6.75	7.25	8.6	9.25	10.5
$f(x)$	5.75	8.75	12.68	14.6	18.95	22.25	23.75	27.8	29.75	33.5

For the following exercises, use a graphing calculator and this scenario: the population of a fish farm in t years is modeled by the equation $P(t) = \dfrac{1000}{1 + 9e^{-0.6t}}$.

17. Graph the function.

18. What is the initial population of fish?

19. To the nearest tenth, what is the doubling time for the fish population?

20. To the nearest whole number, what will the fish population be after 2 years?

21. To the nearest tenth, how long will it take for the population to reach 900?

22. What is the carrying capacity for the fish population? Justify your answer using the graph of P.

EXTENSIONS

23. A substance has a half-life of 2.045 minutes. If the initial amount of the substance was 132.8 grams, how many half-lives will have passed before the substance decays to 8.3 grams? What is the total time of decay?

24. The formula for an increasing population is given by $P(t) = P_0 e^{rt}$ where P_0 is the initial population and $r > 0$. Derive a general formula for the time t it takes for the population to increase by a factor of M.

25. Recall the formula for calculating the magnitude of an earthquake, $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$. Show each step for solving this equation algebraically for the seismic moment S.

26. What is the y-intercept of the logistic growth model $y = \frac{c}{1 + ae^{-rx}}$? Show the steps for calculation. What does this point tell us about the population?

27. Prove that $b^x = e^{x\ln(b)}$ for positive $b \neq 1$.

REAL-WORLD APPLICATIONS

For the following exercises, use this scenario: A doctor prescribes 125 milligrams of a therapeutic drug that decays by about 30% each hour.

28. To the nearest hour, what is the half-life of the drug?

29. Write an exponential model representing the amount of the drug remaining in the patient's system after t hours. Then use the formula to find the amount of the drug that would remain in the patient's system after 3 hours. Round to the nearest milligram.

30. Using the model found in the previous exercise, find $f(10)$ and interpret the result. Round to the nearest hundredth.

For the following exercises, use this scenario: A tumor is injected with 0.5 grams of Iodine-125, which has a decay rate of 1.15% per day.

31. To the nearest day, how long will it take for half of the Iodine-125 to decay?

32. Write an exponential model representing the amount of Iodine-125 remaining in the tumor after t days. Then use the formula to find the amount of Iodine-125 that would remain in the tumor after 60 days. Round to the nearest tenth of a gram.

33. A scientist begins with 250 grams of a radioactive substance. After 250 minutes, the sample has decayed to 32 grams. Rounding to five significant digits, write an exponential equation representing this situation. To the nearest minute, what is the half-life of this substance?

34. The half-life of Radium-226 is 1590 years. What is the annual decay rate? Express the decimal result to four significant digits and the percentage to two significant digits.

35. The half-life of Erbium-165 is 10.4 hours. What is the hourly decay rate? Express the decimal result to four significant digits and the percentage to two significant digits.

36. A wooden artifact from an archeological dig contains 60 percent of the carbon-14 that is present in living trees. To the nearest year, about how many years old is the artifact? (The half-life of carbon-14 is 5730 years.)

37. A research student is working with a culture of bacteria that doubles in size every twenty minutes. The initial population count was 1350 bacteria. Rounding to five significant digits, write an exponential equation representing this situation. To the nearest whole number, what is the population size after 3 hours?

For the following exercises, use this scenario: A biologist recorded a count of 360 bacteria present in a culture after 5 minutes and 1,000 bacteria present after 20 minutes.

38. To the nearest whole number, what was the initial population in the culture?

39. Rounding to six significant digits, write an exponential equation representing this situation. To the nearest minute, how long did it take the population to double?

For the following exercises, use this scenario: A pot of boiling soup with an internal temperature of 100° Fahrenheit was taken off the stove to cool in a 69° F room. After fifteen minutes, the internal temperature of the soup was 95° F.

40. Use Newton's Law of Cooling to write a formula that models this situation.

41. To the nearest minute, how long will it take the soup to cool to 80° F?

42. To the nearest degree, what will the temperature be after 2 and a half hours?

For the following exercises, use this scenario: A turkey is taken out of the oven with an internal temperature of 165° Fahrenheit and is allowed to cool in a 75° F room. After half an hour, the internal temperature of the turkey is 145° F.

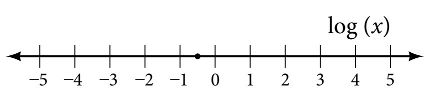
43. Write a formula that models this situation.

44. To the nearest degree, what will the temperature be after 50 minutes?

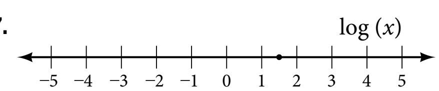
45. To the nearest minute, how long will it take the turkey to cool to 110° F?

For the following exercises, find the value of the number shown on each logarithmic scale. Round all answers to the nearest thousandth.

46.



47.



48. Plot each set of approximate values of intensity of sounds on a logarithmic scale: Whisper: $10^{-10}\frac{W}{m^2}$, Vacuum: $10^{-4}\frac{W}{m^2}$, Jet: $10^{2}\frac{W}{m^2}$

49. Recall the formula for calculating the magnitude of an earthquake, $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$. One earthquake has magnitude 3.9 on the MMS scale. If a second earthquake has 750 times as much energy as the first, find the magnitude of the second quake. Round to the nearest hundredth.

For the following exercises, use this scenario: The equation $N(t) = \frac{500}{1 + 49e^{-0.7t}}$ models the number of people in a town who have heard a rumor after t days.

50. How many people started the rumor?

51. To the nearest whole number, how many people will have heard the rumor after 3 days?

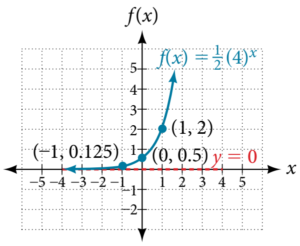
52. As t increases without bound, what value does $N(t)$ approach? Interpret your answer.

For the following exercise, choose the correct answer choice.

53. A doctor and injects a patient with 13 milligrams of radioactive dye that decays exponentially. After 12 minutes, there are 4.75 milligrams of dye remaining in the patient's system. Which is an appropriate model for this situation?

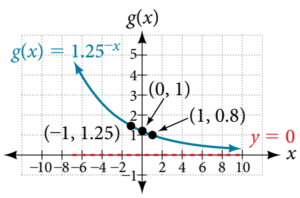
a. $f(t) = 13(0.0805)^t$ **b.** $f(t) = 13\, e^{0.9195t}$ **c.** $f(t) = 13\, e^{(-0.0839t)}$ **d.** $f(t) = \frac{4.75}{1 + 13e^{-0.83925t}}$

4.



The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.

5.



The domain is $(-\infty, \infty)$; the range is $(0, \infty)$; the horizontal asymptote is $y = 0$.

6. $f(x) = -\frac{1}{3}e^x - 2$; the domain is $(-\infty, \infty)$; the range is $(-\infty, 2)$; the horizontal asymptote is $y = 2$.

Section 4.3

1. a. $\log_{10}(1{,}000{,}000) = 6$ is equivalent to $10^6 = 1{,}000{,}000$ **b.** $\log_5(25) = 2$ is equivalent to $5^2 = 25$ **2. a.** $3^2 = 9$ is equivalent to $\log_3(9) = 2$ **b.** $5^3 = 125$ is equivalent to $\log_5(125) = 3$ **c.** $2^{-1} = \frac{1}{2}$ is equivalent to $\log_2\left(\frac{1}{2}\right) = -1$

3. $\log_{121}(11) = \frac{1}{2}$ (recalling that $\sqrt{121} = 121^{\frac{1}{2}} = 11$)

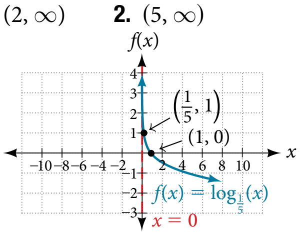
4. $\log_2\left(\frac{1}{32}\right) = -5$ **5.** $\log(1{,}000{,}000) = 6$ **6.** $\log(123) \approx 2.0899$

7. The difference in magnitudes was about 3.929. **8.** It is not possible to take the logarithm of a negative number in the set of real numbers.

Section 4.4

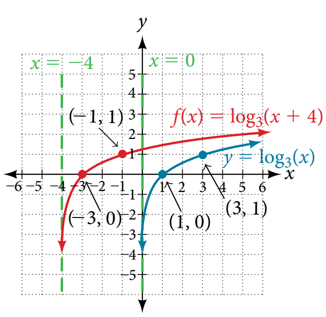
1. $(2, \infty)$ **2.** $(5, \infty)$

3.



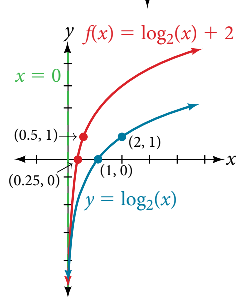
The domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

4.



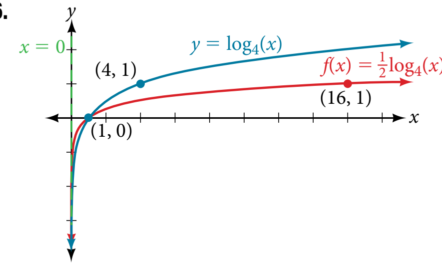
The domain is $(-4, \infty)$, the range $(-\infty, \infty)$, and the asymptote $x = -4$.

5.



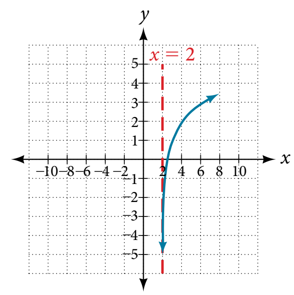
The domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

6.



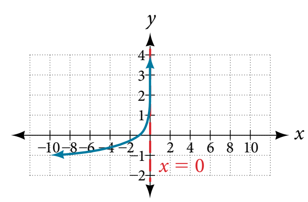
The domain is $(0, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

7.



The domain is $(2, \infty)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 2$.

8.



The domain is $(-\infty, 0)$, the range is $(-\infty, \infty)$, and the vertical asymptote is $x = 0$.

9. $x \approx 3.049$ **10.** $x = 1$

11. $f(x) = 2\ln(x + 3) - 1$

Section 4.5

1. $\log_b(2) + \log_b(2) + \log_b(2) + \log_b(k) = 3\log_b(2) + \log_b(k)$

2. $\log_3(x + 3) - \log_3(x - 1) - \log_3(x - 2)$ **3.** $2\ln(x)$

4. $-2\ln(x)$ **5.** $\log_3(16)$ **6.** $2\log(x) + 3\log(y) - 4\log(z)$

7. $\frac{2}{3}\ln(x)$ **8.** $\frac{1}{2}\ln(x - 1) + \ln(2x + 1) - \ln(x + 3) - \ln(x - 3)$

9. $\log\left(\frac{3 \cdot 5}{4 \cdot 6}\right)$; can also be written $\log\left(\frac{5}{8}\right)$ by reducing the fraction to lowest terms. **10.** $\log\left(\frac{5(x-1)^3\sqrt{x}}{(7x-1)}\right)$

11. $\log\frac{x^{12}(x+5)^4}{(2x+3)^4}$; this answer could also be written $\log\left(\frac{x^3(x+5)}{(2x+3)}\right)^4$.

12. The pH increases by about 0.301. **13.** $\frac{\ln(8)}{\ln(0.5)}$

14. $\frac{\ln(100)}{\ln(5)} \approx \frac{4.6051}{1.6094} = 2.861$

Section 4.6

1. $x = -2$ **2.** $x = -1$ **3.** $x = \frac{1}{2}$

4. The equation has no solution. **5.** $x = \frac{\ln(3)}{\ln\left(\frac{2}{3}\right)}$

6. $t = 2\ln\left(\frac{11}{3}\right)$ or $\ln\left(\frac{11}{3}\right)^2$

7. $t = \ln\left(\frac{1}{\sqrt{2}}\right) = -\frac{1}{2}\ln(2)$ **8.** $x = \ln(2)$ **9.** $x = e^4$

10. $x = e^5 - 1$ **11.** $x \approx 9.97$ **12.** $x = 1$ or $x = -1$

13. $t = 703{,}800{,}000 \times \frac{\ln(0.8)}{\ln(0.5)}$ years $\approx$ 226,572, 993 years.

Section 4.7

1. $f(t) = A_0e^{-0.0000000087t}$ **2.** Less than 230 years; 229.3157 to be exact

3. $f(t) = A_0e^{\left(\frac{\ln(2)}{3}\right)t}$ **4.** 6.026 hours **5.** 895 cases on day 15

6. Exponential. $y = 2e^{0.5x}$ **7.** $y = 3e^{(\ln 0.5)x}$

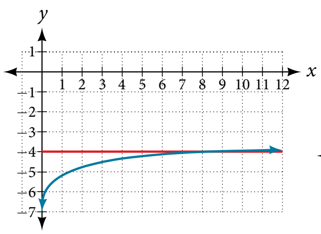
23. $x = \dfrac{\ln\left(\frac{3}{5}\right) - 3}{8}$ **25.** No solution **27.** $x = \ln(3)$

29. $10^{-2} = \dfrac{1}{100}$ **31.** $n = 49$ **33.** $k = \dfrac{1}{36}$ **35.** $x = \dfrac{9 - e}{8}$

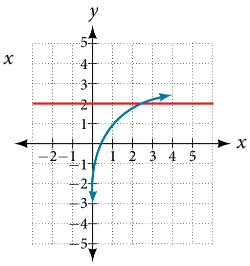
37. $n = 1$ **39.** No solution **41.** No solution

43. $x = \pm\dfrac{10}{3}$ **45.** $x = 10$ **47.** $x = 0$ **49.** $x = \dfrac{3}{4}$

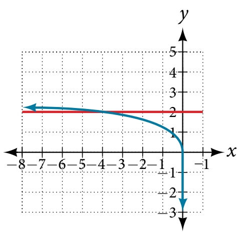
51. $x = 9$



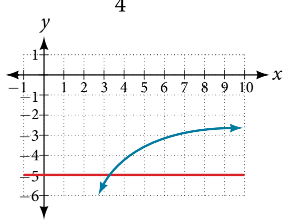
53. $x = \dfrac{e^2}{3} \approx 2.5$



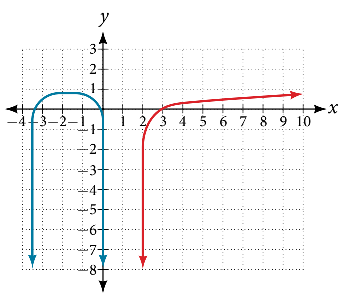
55. $x = -5$



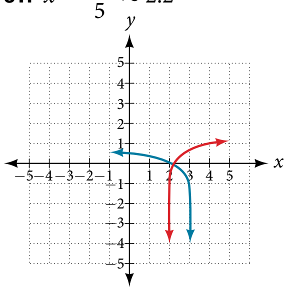
57. $x = \dfrac{e + 10}{4} \approx 3.2$



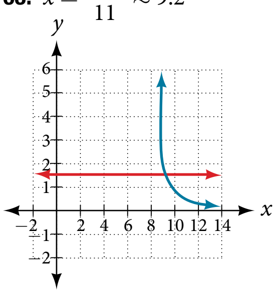
59. No solution



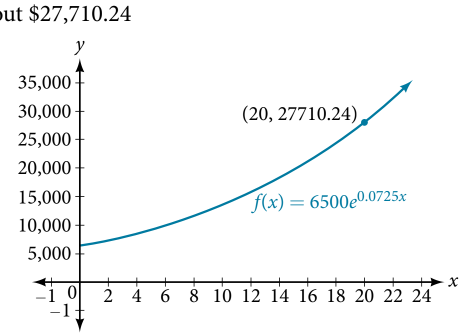
61. $x = \dfrac{11}{5} \approx 2.2$



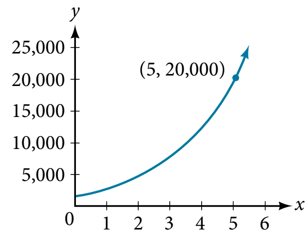
63. $x = \dfrac{101}{11} \approx 9.2$



65. About \$27,710.24



67. About 5 years



69. ≈ 0.567 **71.** ≈ 2.078

73. ≈ 2.2401

75. ≈ -44655.7143

77. About 5.83

79. $t = \ln\left(\left(\dfrac{y}{A}\right)^{\frac{1}{k}}\right)$

81. $t = \ln\left(\left(\dfrac{T - T_s}{T_0 - T_s}\right)^{-\frac{1}{k}}\right)$

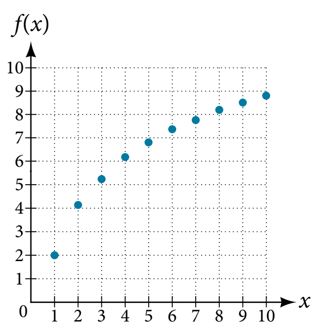
Section 4.7

1. Half-life is a measure of decay and is thus associated with exponential decay models. The half-life of a substance or quantity is the amount of time it takes for half of the initial amount of that substance or quantity to decay. **3.** Doubling time is a measure of growth and is thus associated with exponential growth models. The doubling time of a substance or quantity is the amount of time it takes for the initial amount of that substance or quantity to double in size. **5.** An order of magnitude is the nearest power of ten by which a quantity exponentially grows. It is also an approximate position on a logarithmic scale; Sample response: Orders of magnitude are useful when making comparisons between numbers that differ by a great amount. For example, the mass of Saturn is 95 times greater than the mass of Earth. This is the same as saying that the mass of Saturn is about 10^2 times, or 2 *orders of magnitude* greater, than the mass of Earth.

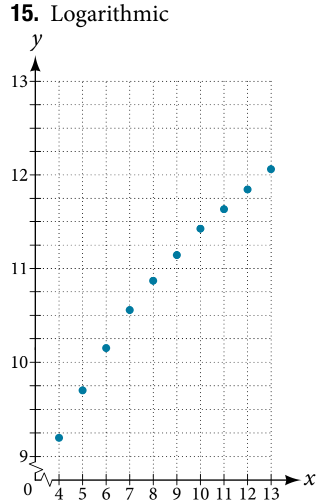
7. $f(0) \approx 16.7$; the amount initially present is about 16.7 units.

9. 150 **11.** Exponential; $f(x) = 1.2^x$

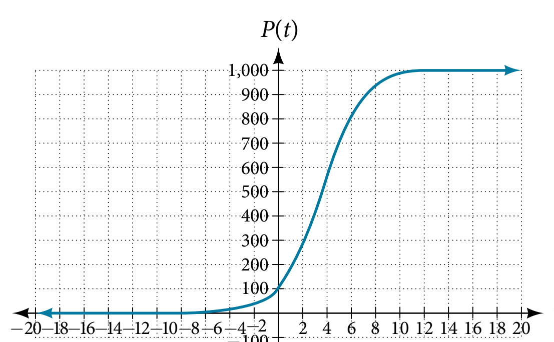
13. Logarithmic



15. Logarithmic



17.



19. About 1.4 years **21.** About 7.3 years
23. Four half-lives; 8.18 minutes
25. $M = \frac{2}{3}\log\left(\frac{S}{S_0}\right)$

$\frac{3}{2}M = \log\left(\frac{S}{S_0}\right)$

$10^{\frac{3M}{2}} = \left(\frac{S}{S_0}\right)$

$S_0 10^{\frac{3M}{2}} = S$

27. Let $y = b^x$ for some non-negative real number b such that $b \neq 1$. Then,

$\ln(y) = \ln(b^x)$

$\ln(y) = x\ln(b)$

$e^{\ln(y)} = e^{x\ln(b)}$

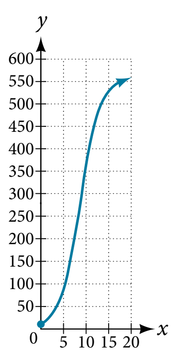
$y = e^{x\ln(b)}$

29. $A = 125e^{(-0.3567t)}$; $A \approx 43$mg **31.** About 60 days
33. $f(t) = 250e^{-0.00914t}$; half-life: about 76 minutes
35. $r \approx -0.0667$; hourly decay rate: about 6.67%
37. $f(t) = 1350\,e^{0.03466573 59t}$; after 3 hours; $P(180) \approx 691{,}200$
39. $f(t) = 256\,e^{(0.068110t)}$; doubling time: about 10 minutes
41. About 88minutes **43.** $T(t) = 90\,e^{(-0.008377t)} + 75$, where t is in minutes **45.** About 113 minutes **47.** $\log_{10}x = 1.5$; $x \approx 31.623$
49. MMS Magnitude: ≈ 5.82 **51.** $N(3) \approx 71$ **53.** C

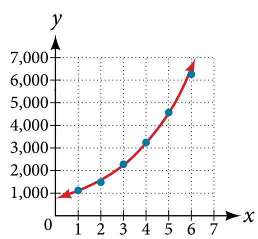
Section 4.8

1. Logistic models are best used for situations that have limited values. For example, populations cannot grow indefinitely since resources such as food, water, and space are limited, so a logistic model best describes populations. **3.** Regression analysis is the process of finding an equation that best fits a given set of data points. To perform a regression analysis on a graphing utility, first list the given points using the STAT then EDIT menu. Next graph the scatter plot using the STAT PLOT feature. The shape of the data points on the scatter graph can help determine which regression feature to use. Once this is determined, select the appropriate regression analysis command from the STAT then CALC menu.
5. The y-intercept on the graph of a logistic equation corresponds to the initial population for the population model.
7. C **9.** B **11.** $P(0) = 22$; 175
13. $p \approx 2.67$ **15.** y-intercept: (0, 15) **17.** 4 koi
19. About 6.8 months.

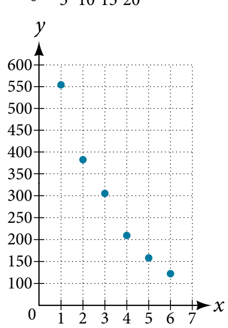
21.



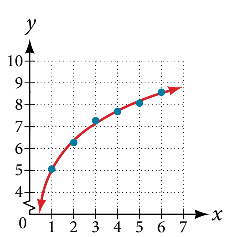
23. About 38 wolves
25. About 8.7 years
27. $f(x) = 776.682(1.426)^x$
29.



31.



33. $f(x) = 731.92e^{-0.3038x}$
35. When $f(x) = 250$, $x \approx 3.6$
37. $y = 5.063 + 1.934\log(x)$
39.



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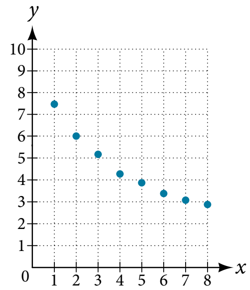
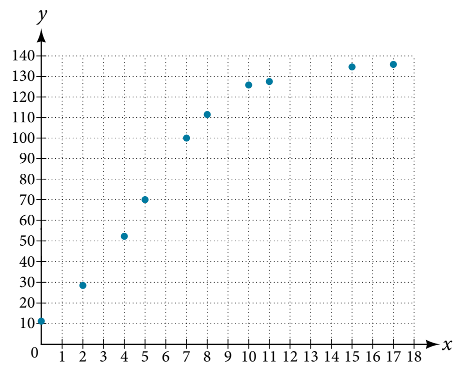


43. $f(10) \approx 2.3$
45. When $f(x) = 8$, $x \approx 0.82$
47. $f(x) = \frac{25.081}{1 + 3.182e^{-0.545x}}$
49. About 25

51.



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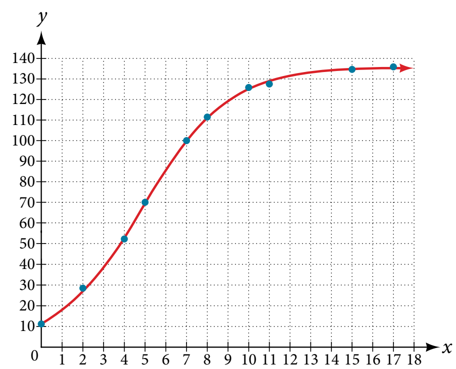

55. When $f(x) = 68$, $x \approx 4.9$ **57.** $f(x) = 1.034341(1.281204)^x$; $g(x) = 4.035510$; the regression curves are symmetrical about $y = x$, so it appears that they are inverse functions.

59. $f^{-1}(x) = \dfrac{\ln(a) - \ln\left(\frac{c}{x} - 1\right)}{b}$

Chapter 4 Review Exercises

1. Exponential decay; the growth factor, 0.825, is between 0 and 1.
3. $y = 0.25(3)^x$ **5.** \$42,888.18 **7.** Continuous decay; the growth rate is negative
9. Domain: all real numbers; range: all real numbers strictly greater than zero; y-intercept: (0, 3.5)