GreatHearts
Irving

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

April 20 - 24, 2020
Course: 11 Precalculus
Teacher(s): Mr. Simmons

Weekly Plan:

Monday, April 20
☐ "The Unit Circle: Sine and Cosine" handout

Tuesday, April 21
☐ "The Unit Circle: Sine and Cosine" handout answer key

Wednesday, April 22
☐ "The Other Trigonometric Functions" handout

Thursday, April 23
☐ "The Other Trigonometric Functions" handout answer key

Friday, April 24
☐ Vocabulary

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____
Parent Signature

Monday, April 20

Read through carefully and complete each of the problems and exercises in the handout entitled "The Unit Circle: Sine and Cosine."

Tuesday, April 21

Check your answers on the "Unit Circle" handout with my answer key. Make corrections as needed.

Wednesday, April 22

Read through carefully and complete each of the problems and exercises in the handout entitled "The Other Trigonometric Functions."

Thursday, April 23

Check your answers on the "Other Trig Functions" handout with my answer key. Make corrections as needed.

Friday, April 24

Spend all of today reviewing vocabulary from the last few handouts, starting with the handout entitled "Angles." The expectation is that you be able to give, without looking at notes, a complete mathematical definition of each word whose formal definition is stated in these handouts.

The Unit Circle: Sine and Cosine

Precalculus
Mr. Simmons

Read through this handout carefully and pause to think and respond when instructed.

Definition (UNIT CIRCLE)**.** The unit circle is the circle centered at $(0, 0)$ with a radius of 1.

Problem 1. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{2}$. Label the point at which this angle's terminal side intersects the unit circle (x, y). Calculate x and y. That is, find their exact values.

Problem 2. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{4}$. Label the point at which this angle's terminal side intersects the unit circle (x, y). Calculate x and y. (Hint: draw a triangle with corners at $(0, 0)$, (x, y), and $(x, 0)$.)

Problem 3. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{6}$. Label the point at which this angle's terminal side intersects the unit circle (x, y). Calculate x and y. (Hint: after drawing a triangle with corners at $(0, 0)$, (x, y), and $(x, 0)$, draw that triangle's reflection across the x-axis as well.)

For each of the preceding two problems, you were given (as an input) an angle measure θ, and you found (as outputs) a value x and a value y. Notice how I'm phrasing this in the language of functions. We can put θ through two different functions, one of which puts out a horizontal distance x—the horizontal leg of the right triangle with its hypotenuse going from the origin to (x, y)—the other of which puts out a vertical distance y—the vertical leg of that same right triangle.

Let's give names to each of these functions:

Definition (SINE AND COSINE—UNIT CIRCLE DEFINITION)**.** Let a line through the origin, making an angle of θ with the positive half of the x-axis, intersect the unit circle. The x- and y-coordinates of this point of intersection are equal to $\cos(\theta)$ and $\sin(\theta)$, respectively.

Exercise 4. Draw a diagram of the unit circle (in green pencil if you have one) and label the point $(\cos(\theta), \sin(\theta))$ for each of

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}.$$

For example, to label $(\cos(\theta), \sin(\theta))$ for $\theta = \frac{\pi}{2}$, you would plot the point $(0, 1)$ and label it "$(0, 1)$." You would *not* label it "$\left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right)\right)$."

Exercise 5. Draw coodinate axes, label the horizontal axis θ, and label the vertical axis $\sin(\theta)$. On the $\sin(\theta)$-axis, draw and label tick marks at 1 and -1. On the θ-axis, draw and label tick marks at $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Plot (in red pencil if you have one) the point $(\theta, \sin(\theta))$ for each of

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi.$$

You needn't label each point.

Then connect the dots as smoothly as possible.

Exercise 6. Draw coodinate axes, label the horizontal one θ, and label the vertical one $\cos(\theta)$. On the $\sin(\theta)$-axis, draw and label tick marks at 1 and -1. On the θ-axis, draw and label tick marks at $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Plot (in blue pencil if you have one) the point $(\theta, \cos(\theta))$ for each of

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}, 2\pi.$$

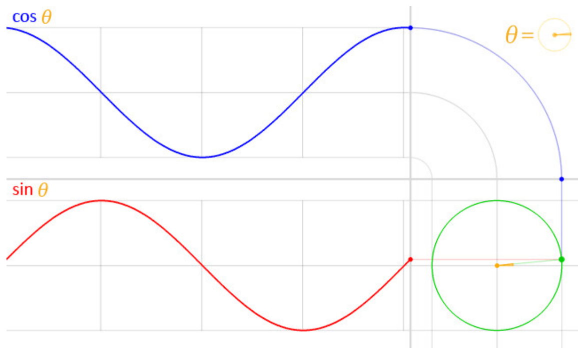
You needn't label each point.

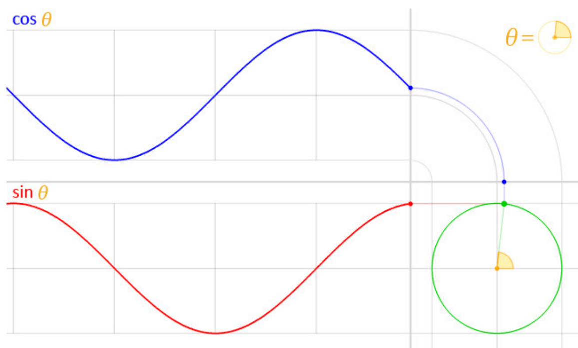
Then connect the dots as smoothly as possible.

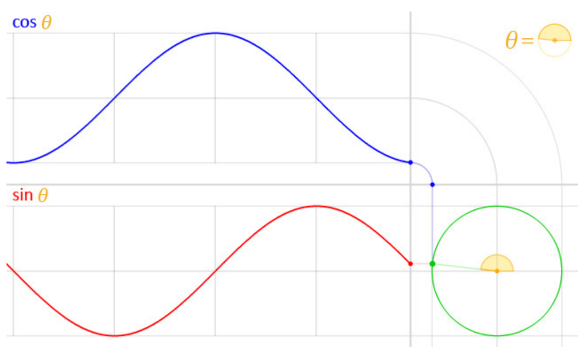
Exercise 7. If you are able, go to

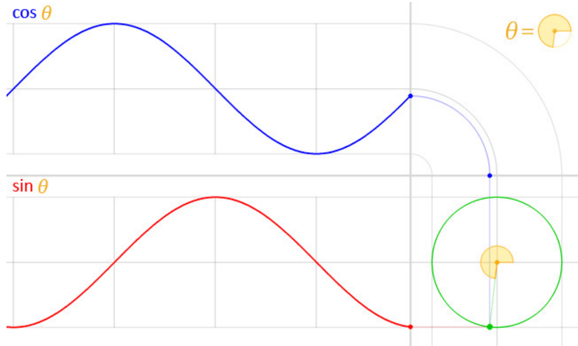
https://upload.wikimedia.org/wikipedia/commons/3/3b/Circle_cos_sin.gif

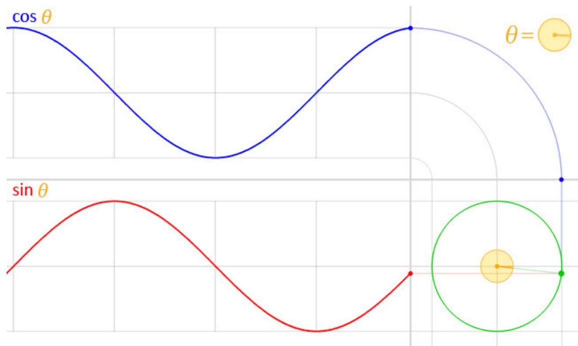
and observe the animation. Here are a few frames from the animation for those who are not able to view it:

1. 

2. 

3. 

4. 

5. 

In completing the problems earlier in this packet, you probably used the Pythagorean Theorem:

Theorem (Pythagorean Theorem)**.** *For any right triangle with legs of lengths a and b and a hypotenuse of length c,*

$$a^2 + b^2 = c^2.$$

Problem 8. Let a line through the origin, making an angle of θ with the positive half of the x-axis, intersect the unit circle at point (x, y). Rephrase the Pythagorean Theorem as a statement about the right triangle formed by the points $(0, 0)$, (x, y), and $(x, 0)$ completely in terms of θ.

The equation you just derived is known as the **Pythagorean Identity**.

The Unit Circle: Sine and Cosine – Answer Key

Precalculus
Mr. Simmons

Read through this handout carefully and pause to think and respond when instructed.

Definition (UNIT CIRCLE)**.** The unit circle is the circle centered at $(0,0)$ with a radius of 1.

Problem 1. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{2}$. Label the point at which this angle's terminal side intersects the unit circle (x,y). Calculate x and y. That is, find their exact values.

Solution. Since the angle's terminal side intersects the unit circle at $(0,1)$, there are hardly any calculations to be done: $x=0$ and $y=1$.

Problem 2. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{4}$. Label the point at which this angle's terminal side intersects the unit circle (x,y). Calculate x and y. (Hint: draw a triangle with corners at $(0,0)$, (x,y), and $(x,0)$.)

Solution. Since x and y are the legs of a right triangle (the one hinted at), we can use the Pythagorean Theorem to get

$$x^2+y^2=1^2,$$

since the hypotenuse of this triangle is the radius of the unit circle, which is 1. In a right triangle with one acute angle of $\frac{\pi}{4}$, we know the other acute angle is also $\frac{\pi}{4}$, since all three angles have to add up to π (180°). Since, then, $x=y$, we have

$$\begin{aligned} x^2+x^2 &= 1 \\ 2x^2 &= 1 \\ x^2 &= \frac{1}{2} \\ x &= \sqrt{\frac{1}{2}} \\ &= \frac{1}{\sqrt{2}} \\ &= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{2}}{2}, \end{aligned}$$

and since $x=y$, we then know $y=\frac{\sqrt{2}}{2}$ as well.

Problem 3. Draw a diagram of the unit circle with a central angle of measure $\frac{\pi}{6}$. Label the point at which this angle's terminal side intersects the unit circle (x, y). Calculate x and y. (Hint: after drawing a triangle with corners at $(0,0)$, (x, y), and $(x, 0)$, draw that triangle's reflection across the x-axis as well.)

Solution. The triangle with corners at $(0,0)$, (x, y), and $(x, -y)$ has an angle of $\frac{\pi}{3}$ ($60°$), and the sides touching that angle each have length 1—so it's an equilateral triangle. Therefore the third side also has length 1. That is, the distance between (x, y) and $(x, -y)$ is 1, telling us that $y = \frac{1}{2}$.

What about x? Consider only the top half of that equilateral triangle. It is a right triangle, so we can use the Pythagorean Theorem as before to get

$$\begin{aligned} x^2 + y^2 &= 1 \\ x^2 + \left(\frac{1}{2}\right)^2 &= 1 \\ x^2 + \frac{1}{4} &= 1 \\ x^2 &= \frac{3}{4} \\ x &= \sqrt{\frac{3}{4}} \\ &= \frac{\sqrt{3}}{2}. \end{aligned}$$

For each of the preceding two problems, you were given (as an input) an angle measure θ, and you found (as outputs) a value x and a value y. Notice how I'm phrasing this in the language of functions. We can put θ through two different functions, one of which puts out a horizontal distance x—the horizontal leg of the right triangle with its hypotenuse going from the origin to (x, y)—the other of which puts out a vertical distance y—the vertical leg of that same right triangle.

Let's give names to each of these functions:

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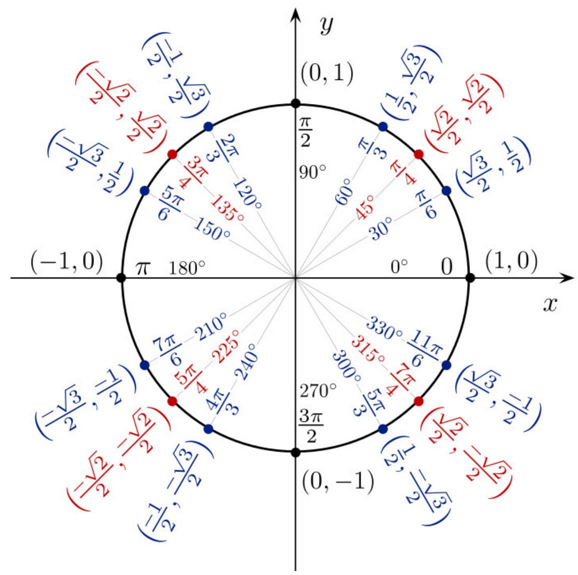
Exercise 4. Draw a diagram of the unit circle (in green pencil if you have one) and label the point $(\cos(\theta), \sin(\theta))$ for each of

$$\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{5\pi}{6}, \pi, \frac{7\pi}{6}, \frac{5\pi}{4}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}, \frac{7\pi}{4}, \frac{11\pi}{6}.$$

For example, to label $(\cos(\theta), \sin(\theta))$ for $\theta = \frac{\pi}{2}$, you would plot the point $(0, 1)$ and label it "$(0, 1)$." You would *not* label it "$\left(\cos\left(\frac{\pi}{2}\right), \sin\left(\frac{\pi}{2}\right)\right)$."

Solution. Your diagram should look something like this (but with only the points labeled, not necessarily the angle measures):



Exercise 5. Draw coodinate axes, label the horizontal axis θ, and label the vertical axis $\sin(\theta)$. On the $\sin(\theta)$-axis, draw and label tick marks at 1 and -1. On the θ-axis, draw and label tick marks at $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, and 2π. Plot (in red pencil if you have one) the point $(\theta, \sin(\theta))$ for each of

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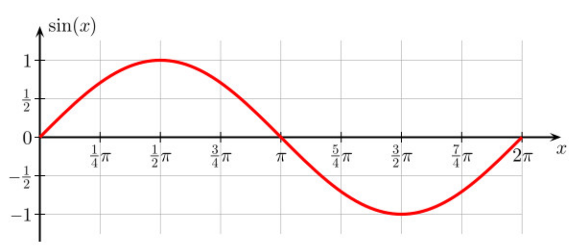
You needn't label each point.

Then connect the dots as smoothly as possible.

Solution. Your graph should look something like this (but with θ instead of x):



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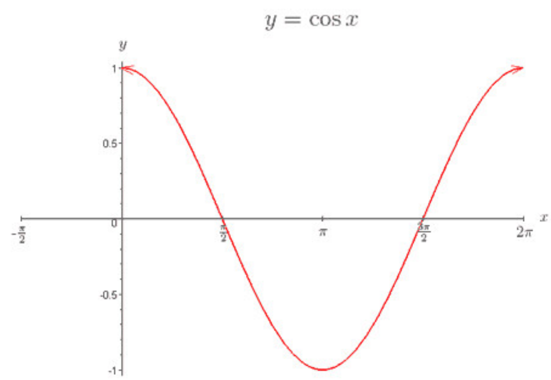

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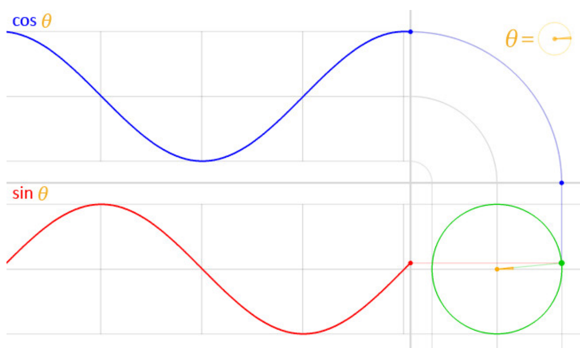
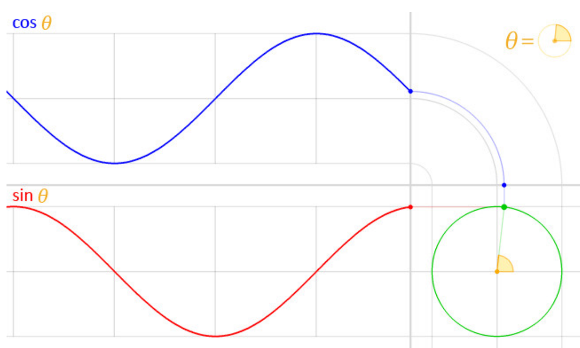
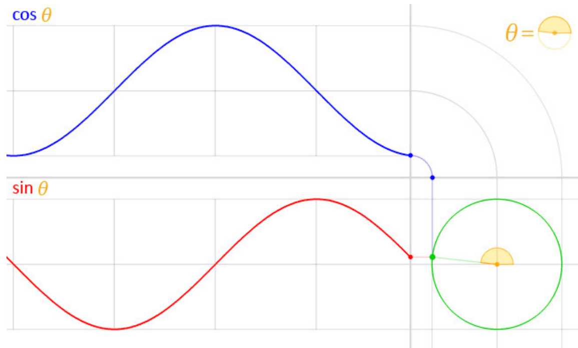
1. 

2. 



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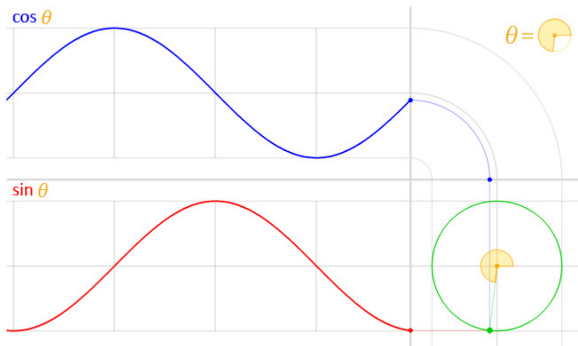

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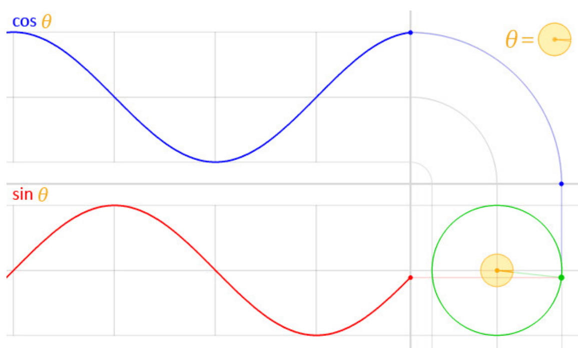

5.

In completing the problems earlier in this packet, you probably used the Pythagorean Theorem:

Theorem (Pythagorean Theorem)**.** *For any right triangle with legs of lengths a and b and a hypotenuse of length c,*

$$a^2 + b^2 = c^2.$$

Problem 8. Let a line through the origin, making an angle of θ with the positive half of the x-axis, intersect the unit circle at point (x, y). Rephrase the Pythagorean Theorem as a statement about the right triangle formed by the points $(0, 0)$, (x, y), and $(x, 0)$ completely in terms of θ.

Solution.

Derivation. Since we have the right triangle with leg lengths x and y and hypotenuse 1 (the radius of the unit circle), the Pythagorean Theorem formula

$$a^2 + b^2 = c^2$$

becomes

$$x^2 + y^2 = 1^2,$$

but x and y can be rewritten in terms of θ using the trigonometric functions, since $x = \cos(\theta)$ and $y = \sin(\theta)$, so our final equation is

$$\sin^2(\theta) + \cos^2(\theta) = 1.$$

$\square$

The equation you just derived is known as the **Pythagorean Identity**.

The Other Trigonometric Functions

Precalculus
Mr. Simmons

Read through this handout carefully and pause to think and respond when instructed.

Definition (TANGENT)**.** If θ is a real number and (x, y) is a point where the terminal side of a central angle of measure θ intersects the unit circle, then

$$\tan(\theta) = \frac{y}{x},$$

provided $x \neq 0$.

Exercise 1. The terminal side of a central angle of measure θ passes through the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle. Find $\sin\theta$, $\cos(\theta)$, and $\tan(\theta)$.

These values are all fractions. For convenience, mathematicians have defined three more trigonometric functions that are simply the reciprocals of the three we have convered. This helps get rid of ugly fractions. The reciprocal of sine is cosecant, the reciprocal of cosine is secant, and the reciprocal of tangent is cotangent.

Definition (COSECANT, SECANT, AND COTANGENT)**.** If θ is a real number and (x, y) is a point where the terminal side of a central angle of measure θ intersects the unit circle, then

$$\csc(\theta) = \frac{1}{y}, \text{ provided } (y \neq 0),$$

$$\sec(\theta) = \frac{1}{x}, \text{ provided } (x \neq 0),$$

and

$$\cot(\theta) = \frac{x}{y}, \text{ provided } (y \neq 0).$$

Exercise 2. The terminal side of a central angle of measure θ passes through the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle. Find $\csc\theta$, $\sec(\theta)$, and $\cot(\theta)$.

Exercise 3. Fill in the following chart:

Angle	0	$\frac{\pi}{6}$, or 30°	$\frac{\pi}{4}$, or 45°	$\frac{\pi}{3}$, or 60°	$\frac{\pi}{2}$, or 90°
Sine					
Cosine		$\frac{\sqrt{3}}{2}$			
Tangent			1		
Cosecant	Undefined			2	
Secant					
Cotangent					0

The Other Trigonometric Functions – Answer Key

Precalculus
Mr. Simmons

Read through this handout carefully and pause to think and respond when instructed.

Definition (TANGENT)**.** If θ is a real number and (x, y) is a point where the terminal side of a central angle of measure θ intersects the unit circle, then

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Exercise 1. The terminal side of a central angle of measure θ passes through the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle. Find $\sin\theta$, $\cos(\theta)$, and $\tan(\theta)$.

Solution. We have here

$$\sin(\theta) = y = \frac{1}{2},$$

$$\cos(\theta) = x = -\frac{\sqrt{3}}{2},$$

and

$$\tan(\theta) = \frac{y}{x} = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2}\left(-\frac{2}{\sqrt{3}}\right) = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}.$$

These values are all fractions. For convenience, mathematicians have defined three more trigonometric functions that are simply the reciprocals of the three we have convered. This helps get rid of ugly fractions. The reciprocal of sine is cosecant, the reciprocal of cosine is secant, and the reciprocal of tangent is cotangent.

Definition (COSECANT, SECANT, AND COTANGENT)**.** If θ is a real number and (x, y) is a point where the terminal side of a central angle of measure θ intersects the unit circle, then

$$\csc(\theta) = \frac{1}{y}, \text{ provided } (y \neq 0),$$

$$\sec(\theta) = \frac{1}{x}, \text{ provided } (x \neq 0),$$

and

$$\cot(\theta) = \frac{x}{y}, \text{ provided } (y \neq 0).$$

Exercise 2. The terminal side of a central angle of measure θ passes through the point $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ on the unit circle. Find $\csc\theta$, $\sec(\theta)$, and $\cot(\theta)$.

Solution. We have here

$$\csc(\theta) = \frac{1}{y} = \frac{1}{\frac{1}{2}} = 2,$$

$$\sec(\theta) = \frac{1}{x} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3},$$

and

$$\cot(\theta) = \frac{x}{y} = \frac{\frac{-\sqrt{3}}{2}}{\frac{1}{2}} = -\frac{\sqrt{3}}{2}\left(\frac{2}{1}\right) = -\sqrt{3}.$$

Exercise 3. Fill in the following chart:

Angle	0	$\frac{\pi}{6}$, or $30°$	$\frac{\pi}{4}$, or $45°$	$\frac{\pi}{3}$, or $60°$	$\frac{\pi}{2}$, or $90°$
Sine	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
Cosine	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{}}{}$	$\frac{1}{2}$	0
Tangent	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined
Cosecant	Undefined	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1
Secant	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	Undefined
Cotangent	Undefined	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0