# 11th Grade Lesson Plan Packet

5/4/2020-5/8/2020

GreatHearts Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: 11 Art (Art I)
**Teacher(s)**: Ms. Clare Frank

**Weekly Plan**:

Monday, May 4
☐ Watch instructional video "The Window"
☐ Read through and look at the pictures from the NY Times article.

Tuesday, May 5
☐ Draw "the world outside" from three places in your home.

Wednesday, May 6
☐ Draw "the world outside" from one place in your home.
☐ Write notes about the view, and what you see, hear and smell.

Thursday, May 7
☐ Compositional sketches for project
☐ Begin a full page drawing of your view of the world outside as seen through the window (or door, porch or balcony).

Friday, May 8
☐ attend office hours
☐ catch-up or review the week's work

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

## Monday, May 4

In places throughout the United States and around the world there are people who have not left their home in many weeks. The view of the world outside our home, as seen through a window, has occupied a special place in the human imagination likely since the first windows, but when confined to our homes it acquires a heightened significance. Throughout this week and into next week you will be contemplating your view of the world through the windows of your home. You may also use the view through a door or from a porch or balcony if you wish, but make sure to show something of the vantage point.

You have two assignments today:

1. Watch the instructional video "The Window", found as a Material for Monday, May 4 in Google Classroom.
2. Read through the excerpted New York Times article "17 Artists Capture a Surreal New York From Their Windows", by Antonio de Luca, Sasha Portis and Adriana Ramic, April 16, 2020. As you read, look thoughtfully at the artworks, taking part in the view in your imagination.

## Tuesday, May 5

1. 15 minutes: Select three locations in your home from which you have distinctly different views of the world outside - perhaps difference in height, direction, or type of space.
   - Make a full page, 5-minute drawing from each of those three places.
   - Label each drawing with your name, the date, the window or aperture through which you have the view, and what type of view it is.
   - For example, one of my drawings would be inscribed "Clare Frank, May 5, 2020, view of the neighborhood street from the kitchen window".

   ☐ Drawing #1, 5 minutes
   ☐ Drawing #2, 5 minutes
   ☐ Drawing #3, 5 minutes

2. 5 minutes: Write a paragraph discussing one of the views and memories or meanings you associate with that window or view. Do use sentences, but you may use a stream of consciousness style.

   - For example: "For me it is a special pleasure to have a window over the sink in my kitchen. I know the tempo of my neighborhood, the couples, dogs, kids riding scooters, and the cars visiting houses from my many hours of looking through the window over my kitchen sink as I cook and clean. I think of a friend and his comment years ago, in the first home I lived in on my own, when he saw my curtains hanging in the window over the sink, of how his mother didn't approve of a woman without curtains in her kitchen windows. His mom traveled all over the country staying in RV parks and when she visited she boiled peanuts. I think of the oak outside my current kitchen sink window and how it was too short four years ago to keep the street light from flooding into my house at night, but now I know the time of the year by the way in which its canopy shelters my house with nighttime shade from March to November. When I first bought the house I saw my neighbor's beautiful irises in bloom through the chain link fence in her backyard and they so captivated me that I painted them on the wall in my daughter's room."

## Wednesday, May 6

20 minutes: Select one location from which to view the world outside. Sketch your view, including the edges of your window frames to show that you are "looking through".

- **As you draw**, notice birds or other animals passing by, people walking, jogging, biking or driving by, planes overhead, the effects of wind through the trees, and so forth.
- What do you hear? Birds singing or chirping, dogs barking, automobiles driving by or roaring down a highway, a train, children's voices, a jet overhead, wind rustling through leaves…?
- What do you smell? What do you feel? If the sun is shining through your window, it may be warming your skin. If the window is open perhaps a breeze is wafting in.
- **Write notes** on the page of your drawing or on the page facing it, about what you hear, see, smell and feel. You can write these notes as you draw or after you draw.
- Use the full 20 minutes to take in as much as you can, as you draw and make notations. The more you see, the more there is to see!

## Thursday, May 7

Today you will begin a full-page drawing of your view of the world outside as seen through the window (or door, porch or balcony). This drawing should be well-composed, so you will start with some quick compositional sketches before beginning the actual project. Then you will lightly begin the overall layout on a full fresh page of your sketchbook.

1. 10 minutes: Using one full page of your sketchbook, quickly draw 4 picture planes of the same proportions as your paper. Into each of these picture planes quickly sketch the window and view, varying placement, scale, and cropping to achieve a balanced, harmonious, and visually interesting composition.
    - Include the window frame and part of the interior wall in some of the compositions
    - If helpful you will be able to slightly "trim" your picture plane by lightly drawing a straight, even border along the bottom of the picture plane (and ultimately along the bottom of the page in your final drawing).

2. 10 minutes: Select your strongest composition, turn to a fresh sheet in your sketchbook, and lightly draw the overall compositional layout. Work from general to specific, being attentive to proportions and shape relationships.

- Your drawing will be at least 8x8 inches - in general a square is difficult to work with compositionally, so plan to use a rectangle.
- If cropping the picture plane along the bottom of the page, be sure to first use a ruler or crisply folded piece of paper to get a clean, straight line at right angles to the adjacent sides of the page.
- For today start with light contour lines and only lightly shade areas, as your emphasis today should be on the shape relationships and proportions.
- Use the approach we learned in drawing compositional studies, where we observe placement and proportion by looking at relationships to the edges of the picture plane and horizontal and vertical alignments.  Here your window is a picture plane within the picture plane of your drawing!

You will have the opportunity to use dry media of your choice in this project - so colored pencil or pen is also an option (and there are other possibilities depending on what you have at home).  This is a great project to incorporate your knowledge of color and levels of saturation!  Of course, every media requires a certain investment of time and craft, so keep that in mind next week when you return to this drawing.  As you consider what you would like to use, look back at the examples by the New York artists.

**Friday, May 8:** Use Friday to attend office hours or to catch up on the week's work.

Have a great weekend!

Reading for Monday, May 5, 2020: Excerpts from a NY Times Newspaper Article:

# 17 Artists Capture a Surreal New York From Their Windows

by Antonio de Luca, Sasha Portis and Adriana Ramic, April 16, 2020

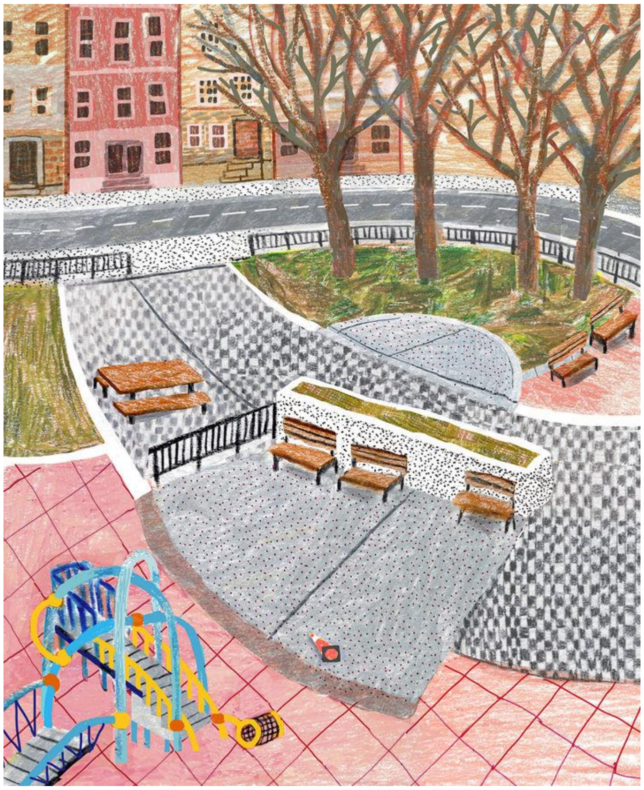
*Mark Pernice, Hauppauge, Long Island*

Windows are often described as the eyes of a building. They are a symbol of pondering, an aperture through which we can experience the world outside while remaining inside — an important feature now that millions of New Yorkers have had to move their lives indoors.

We reached out to 17 illustrators and artists currently sheltering in place in neighborhoods across the city and asked them to draw what they see out of their windows, and to show us what it feels like to be in New York at this rare moment in time.

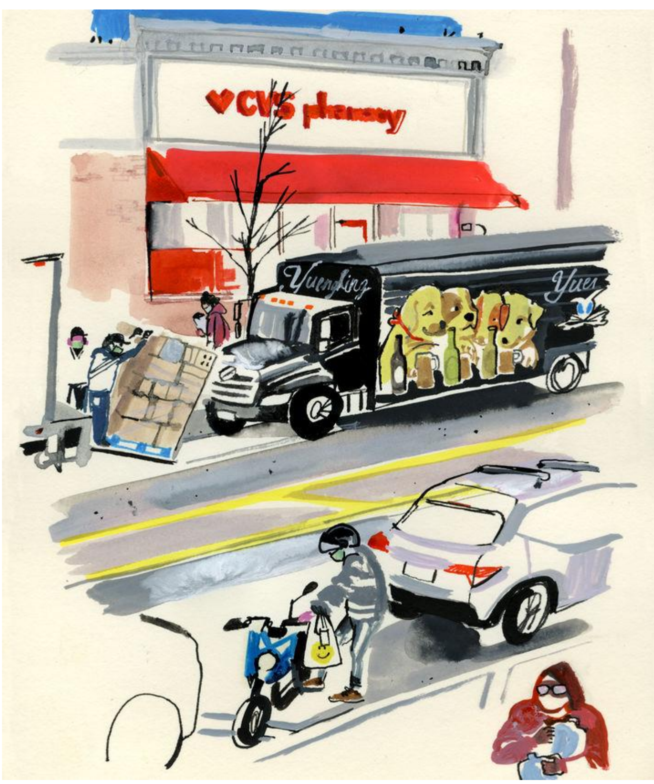
We received images full of conflicting and immediately recognizable emotions: images that communicate the eerie stillness of the city and make connections to history, odes to essential workers and the changing of the seasons.

The act of drawing offers a different kind of truth than photography can. It is an additive form where images are built up from a blank surface. Illustration can evoke empathy and bring shared experiences into view as millions of people around the world find themselves in a similar position: staring out their windows, wondering what’s ahead.



*JooHee Yoon, Prospect Heights, Brooklyn*


*Yuko Shimizu, Morningside Heights, Manhattan*




*Lauren Tamaki, Park Slope, Brooklyn*

"My sense of time seems to stretch and shrink in weird ways...."

—JooHee Yoon

For three weeks, I have not seen anything move.

—Yuko Shimizu

This drawing is my little ode to delivery people. They're putting themselves at great risk to keep this city running while medical staff are on the front lines. If you're able, tip very generously.

—Lauren Tamaki

*Katherine Lam, Ridgewood, Queens*

It's as if I'm in a place that looks like New York, but I don't recognize it at all.

—Katherine Lam

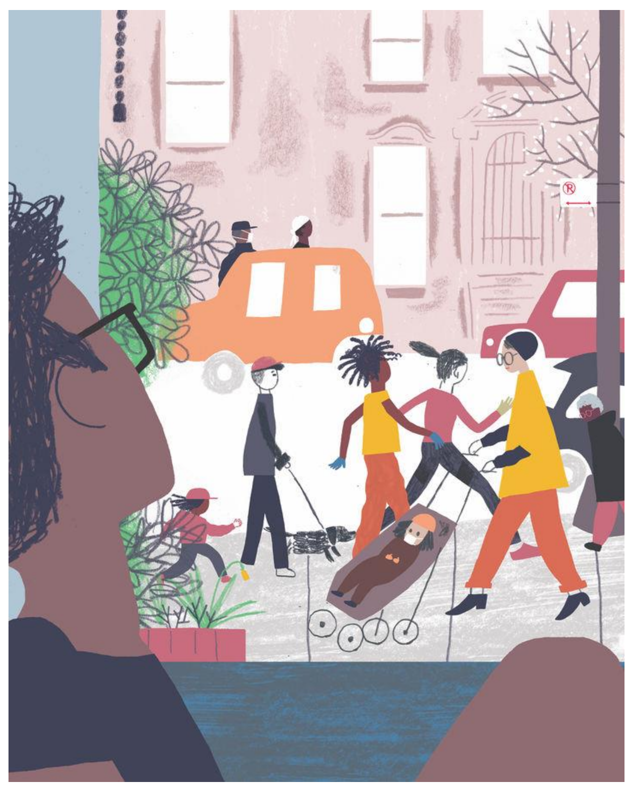
There's a tree outside our window that seems like it's in the apartment with us. Throughout the day I feel a bunch of different things: disconnected, disappointed, sad, angry.

—Daniel Salmieri

Looking out the window feels like we're at a human zoo watching the wild outdoors from the safety of our couch.

—Christopher Silas Neal

*Daniel Salmieri, Bed-Stuy, Brooklyn*



*Christopher Silas Neal, Bed-Stuy, Brooklyn*

My feelings go from a dull, low-level stress to a heightened sense of connection with all of my neighbors.

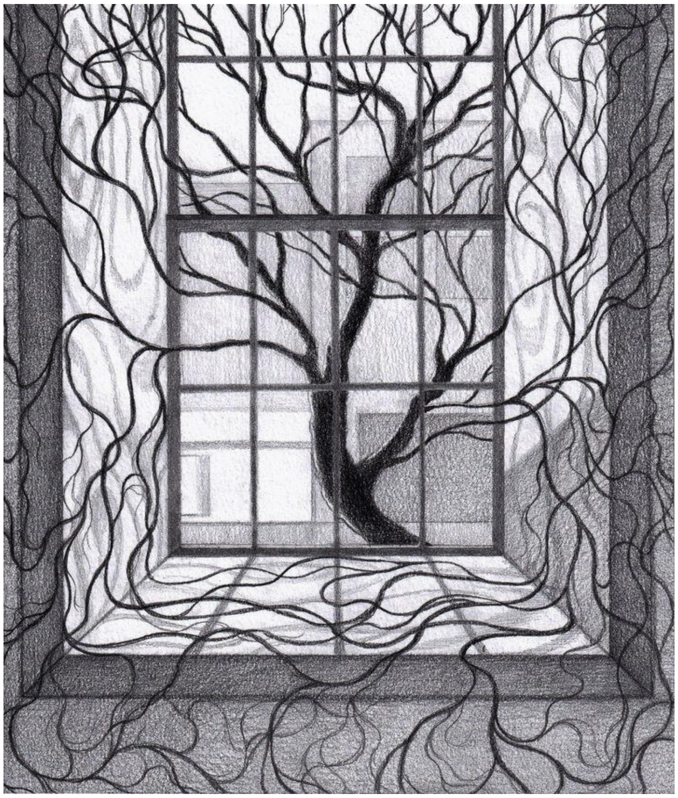
—Josh Cochran

I have been simultaneously enjoying and being disturbed by the silence at the moment.

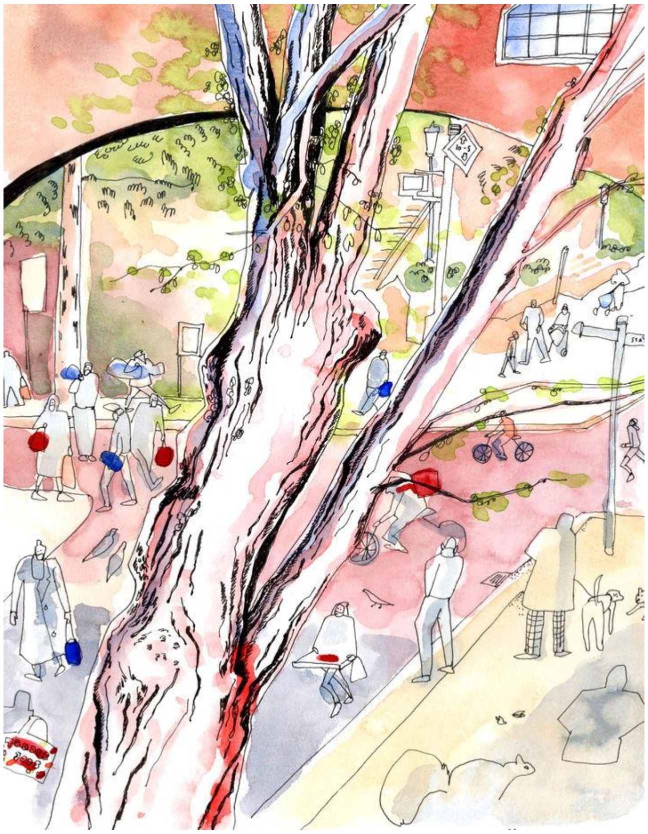
—Peter Arkle

“Everything feels ghostly, and every movement through the neighborhood seems unique and important.”

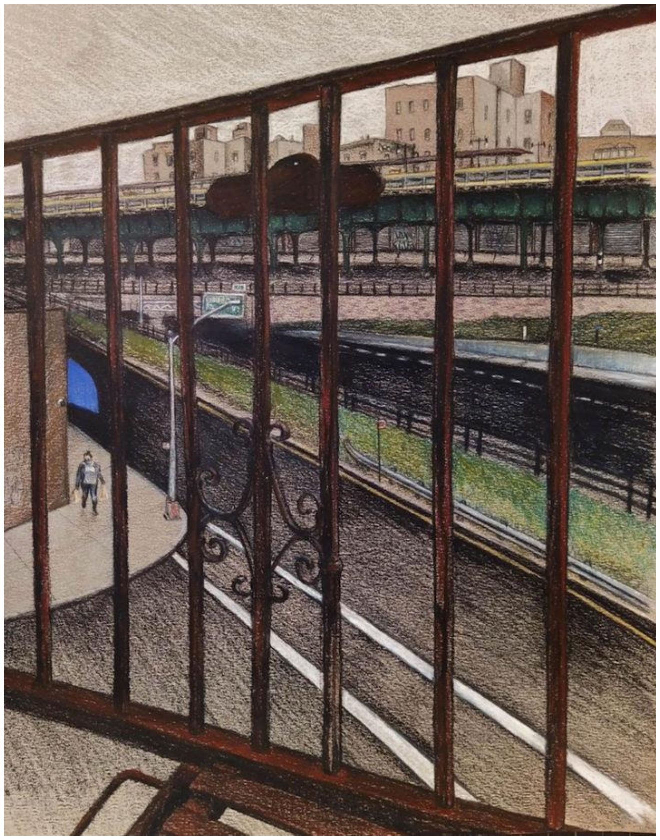
—Patrick Edell



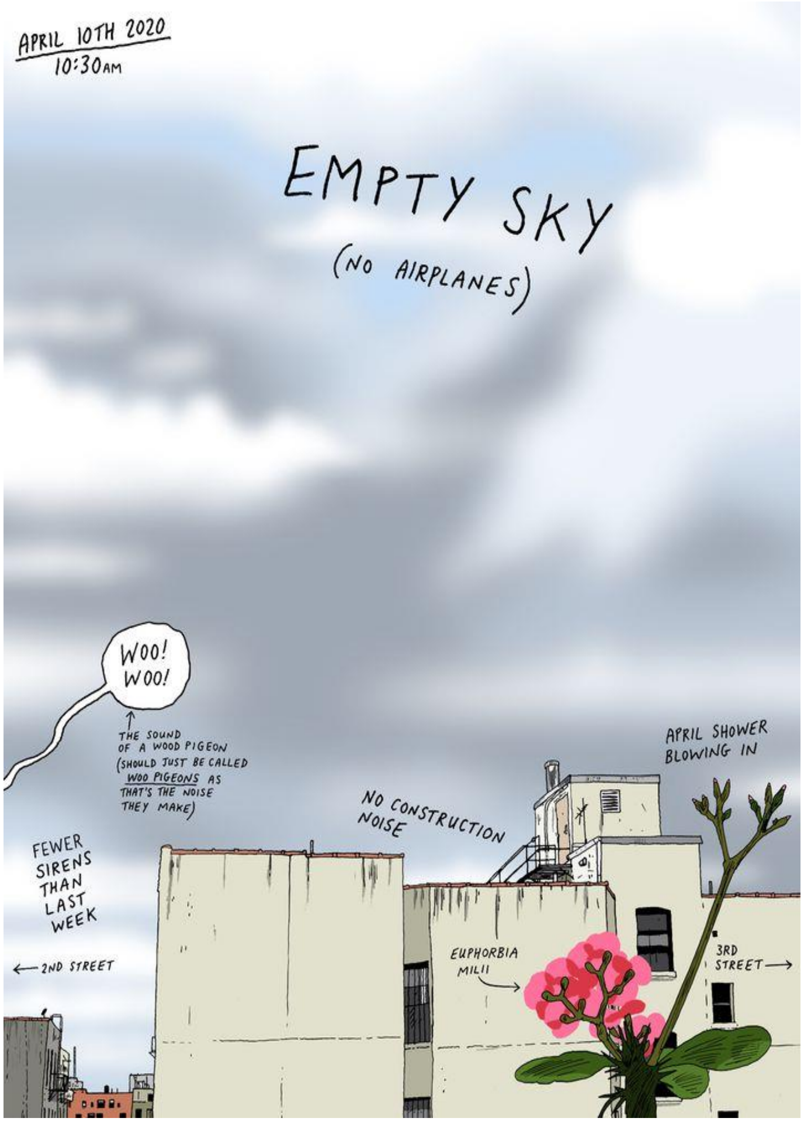
*Cindy Ji Hye Kim, Mott Haven, the Bronx*



*Normandie Syken, Forest Hills, Queens*



*Shellyne Rodriguez, Parkchester, the Bronx*




*Peter Arkle, East Village, Manhattan*

01. THE MORE YOU SEE THE MORE IS SEEN.
02. A MIRACLE—AN ANTIDOTE.
03. SMALL IS STILL BEAUTIFUL.

—Maziyar Pahlevan, Astoria, Queens

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: 11 Calculus I
**Teacher(s)**: Mr. Simmons

**Weekly Plan**:

Monday, May 4
☐ Read "The Big Picture"
☐ Read "Review Overview"
☐ Week 6: Practice Problems

Tuesday, May 5
☐ Week 6: Conceptual Questions 1 and 2

Wednesday, May 6
☐ Week 6: Conceptual Questions 3 and 4

Thursday, May 7
☐ Week 6: Conceptual Questions 5 and 6

Friday, May 8
☐ Attend office hours
☐ Catch up or review the week's work

## Monday, May 4

Dearest students of Calculus,

Thank you all for persevering during this unfortunate time. I'm sad that I can't be there with you to help us all understand better the mathematical truths we've been looking at.

This week begins our review of the whole year. We will be stepping back to get a big-picture look at things, and going back to solidify our knowledge of what we've learned. For today, please complete the following tasks:

1. Read "The Big Picture."
2. Read "Calculus I Review Overview."
3. Complete the practice problems for Week 6. (If you want to save completing these practice problems until after answering the conceptual problems, that's fine too.)

## Tuesday, May 5

1. Answer, in full, complete, grammatical sentences, the first and second conceptual questions for Week 6. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone who's never heard of them before. (If you wish, you may answer these two questions together, not in two separate answers.)

## Wednesday, May 6

1. Answer, in full, complete, grammatical sentences, the third and fourth conceptual questions for Week 6. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone who's never heard of them before. (If you wish, you may answer these two questions together, not in two separate answers.)

## Thursday, May 7

1. Answer, in full, complete, grammatical sentences, the fifth and sixth conceptual questions for Week 6. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone who's never heard of them before. (If you wish, you may answer these two questions together, not in two separate answers.)

# The Big Picture

*Mr. Simmons*
*11 Calculus I*

We have come to a point in our course where it is natural to turn our gaze backward and survey all that we've done. We're wrapping up our studies of derivatives, and next year you will begin with the study of integrals. Derivatives answer the first great question of Calculus: what is the slope of a tangent line? Integrals will answer the second: what is the area under a curve? The two are intimately related, but we will wait until next year to see how.

What we want to do now is threefold:

1. Solidify our understanding of the purpose and scope of Calculus.
2. Review the concepts of Calculus.
3. Practice the skills of Calculus.

This piece of writing is intended to achieve the first of these ends.

Let's step back and look at the big picture for a second. Calculus is the mathematical study of continuous change. What does that mean? Often, it means it's the mathematical study of physical change. The $x$-axis often represents time and the $y$-axis position. A curve, then, represents the motion of an object, and we can analyze that motion using derivatives. A derivative here represents instantaneous change. Under this interpretation, we can rephrase Fermat's theorem[1] to say that when you toss a ball up in the air, at its highest point, it is stationary for an instant, and knowing this is incredibly helpful in pinpointing exactly when it will reach that point. A bit of imagination let's us see how this theorem could be helpful to NASA for planning a rocket launch, for example. The theorems of Calculus are incredibly useful in physics, medicine, engineering, and many other fields.

But Calculus is the *mathematical* study of continuous change. We don't study it solely because it is useful in the sciences. You're not all going to be scientists. This is a math class, and we're learning to do math. That means we're learning to abstract ideas, form conjectures, and then prove those conjectures into theorems. What does all that mean?

To start with, I just used "abstract" as a verb. What does that mean? When you were young, you looked at a bunch of different apples, and you abstracted from your experience of all those different apples the general idea of *apple*. Then you started counting: one apple, two apples, three apples. Eventually you abstracted from all your counting the general ideas *one*, *two*, *three*, etc. Not "one apple," but just, *one*. Not "two apples," but just, *two*. We called these numbers. Then we started asking question about numbers, like "What's $5 + 2$?" or "What's $3928 \div 42$?" When we got bored answering simple questions like that, we started asking "What number would I multiply 36

[1] which, remember, says that if $f$ is defined on $(a, b)$ and has a local maximum (or minimum) at $x$, and $f$ is differentiable at $x$, then $f'(x) = 0$

by to get 2059?" We got tired of writing out long sentences like that, so we shortened it to "Say $36x = 2059$. What's $x$?" or even "Solve $36x = 2059$ for $x$."

(By the way, I didn't give you a definition of the verb "abstract." You're abstracting one from these sentences.)

Then you abstracted from the question, "What are $8 \times 2 + 10$, $8 \times 3 + 10$, and $8 \times 4 + 10$?," the general idea "$y = 8x + 10$." We called that a function. We noticed things were getting pretty darn abstract, so we came up with a whole new way to represent these ideas. Not just letters in place of numbers, but pictures in place of sentences. Graphs. It made it way easier to interpret functions.

Then you looked at a bunch of similar functions, like $y = 2x + 4$, $y = 5x - 3$, and $y = 39x + 9$, and you abstracted from them the general function equation

$$y = mx + b.$$

We called this a form. (Specifically, that one's called "slope–intercept form.") We abstracted some other forms too, like

$$y - y_1 = m\left(x - x_1\right),$$

$$y = ax^2 + bx + c,$$

$$y = \frac{a_p x^p + a_{p-1} x^{p-1} + \cdots + a_2 x^2 + a_1 x + a_0}{b_q x^q + b_{q-1} x^{q-1} + \cdots b_2 x^2 + b_1 x + b_0},$$

$$y = ab^x,$$

$$y = \sin\left(x\right),$$

and others. And we learned how to graph them all.

Now, in Calculus, we've come to a whole new level of abstraction. We've started saying interesting things about these functions. We started saying things like "A function whose graph is a smooth curve from here to here will have a highest and lowest point within that space." *Any* smooth function. We've gone from apples, to numbers, to variables, to functions, to classes of functions, to types of classes of functions, to statements about types of classes of functions. At first we thought the type we cared most about was the type called "continuous," because we thought those curves were always smooth, but then we discovered that some of them have ugly corners, like the absolute value function. So we got interested in a nicer type called "differentiable." Those functions were all very smooth. They reflected the real world, because things in the real world move around, speed up and slow down, in a smooth way. The statements we've been making about functions—at first functions in general, then continuous functions, then differentiable functions—are called theorems.

Theorems are at a very high level of abstraction. "If $f$ is defined on $(a, b)$ and has a local maximum (or minimum) at $x$, and $f$ is differentiable at $x$ then $f'\left(x\right) = 0$." What's $f$? Well, it's any differentiable function. So is it rational? Exponential? Trigonometric? If it's rational, is it linear? Quadratic? Cubic? Higher-order polynomial? Hyperbolic? If it's linear, is it $y = 2x$? $y = 31x + 5$? $y = -\frac{5}{6}x + \frac{2}{3}$? If it's, say, $y = 4x - 1$, then what number is $y$? Well, it depends on what number $x$ is. What number is $x$? It could be any number in the domain of the function. So, 4? 19485? $-0.009284$? $\pi$? What do $x$ and $y$ even represent? What are they the numbers of? Time versus position? Time versus money? Frequency versus amount? Are they just abstract variables, there simply to draw a beautiful graph?

There are so many layers of abstraction here. That's what makes Calculus so difficult. You might think theorems like this are too abstract to be useful. But they are *darn* useful. In fact, the more general the type of function that a theorem applies to (i.e., the more general the "if" part of the theorem), the more useful it is. And also the more specific the thing it says about that type of function (i. e., the more specific the "then" part of the theorem), the more useful it is. This theorem in particular helps us optimize a function, meaning find its maximum. That can be useful if you ever want to get the highest possible amount of something, which people often do (e. g., of money).[2]

So that's what a theorem is, a general, and therefore abstract, statement in math. What do I mean by saying that a math class teaches you to abstract ideas, form conjectures, and prove those conjectures into theorems? Well, forming conjectures is just the last step of abstraction as described above. A mathematician might notice a pattern: "All the differentiable functions I've ever seen have had $f'(x) = 0$ at their maxima. I wonder if that's true for *all* differentiable functions." That's a conjecture. That one in particular was proven by Pierre Fermat, causing it to be called a theorem (which just means a conjecture that's been proved). But there are still conjectures out there, yet to be proven. For example, the twin prime conjecture.[3] And there are an infinite number of conjectures yet to be made, maybe even an infinite number of interesting ones.

Once we've made a conjecture, a guess, we have to try to prove it. If it is proved, we call it a theorem. If it is disproved, we call it trash. (I'm kidding.) While we might use physical observations and intuitions to come up with conjectures, we can't depend on them to prove theorems. We prove theorems with deductive logic. Deductive logic doesn't rely on intuition, but on rigorous rules of inference.

For example, an intuitive understanding of a limit says that it's what the output approaches as the input approaches some value. If the input is interpreted as time and the output is interpreted as the position of an object, we would say that the limit is the place the object is going toward. In Calculus, it's essential to have this intuitive understanding, in order to be able to make guesses about which statements about limits might be true, and which ones might be false. But in math, if we're actually going to prove one of our guesses, we need a rigorous definition of a limit. "Going toward" isn't a rigorous mathematical concept. The formal, rigorous definition of a limit that we learned is that "The function $f$ approaches the limit $l$ near $a$ means: for every $\varepsilon > 0$ there is some $\delta > 0$ such that, for all $x$, if $0 < |x - a| < \delta$, then $|f(x) - l| < \varepsilon$." This is rigorous, because it relies not on any intuitions, but on terms that themselves have rigorous definitions, like "absolute value."

This is why math is so hard. It's so abstract. Just look at all those symbols. But that abstraction frees us from relying on physical intuitions, and it let's us prove things rigorously. That might seem silly, since we can just say, "But the informal definition *worked*." But rigor is what has allowed

[2] If you want to see a bunch of practical applications of calculus, look at page vi of your textbook. Finney loves practical applications.

[3] If you're interested, "twin primes" are any two primes that are only 2 apart. The first few twin prime pairs are 3 and 5, 5 and 7, 11 and 13, 17 and 19, 27 and 29, 29 and 31, 41 and 43. . . . You can already start to see they're getting rarer as we get higher. So do they eventually end? Is there a last twin prime pair? The twin prime conjecture states than there's no highest twin prime pair, that they just keep going, that no matter how high a number you pick, you'll be able to find a twin prime pair that's higher. "Beginning in 2007, two distributed computing projects, Twin Prime Search and PrimeGrid, have produced several record-largest twin primes. As of September 2018, the current largest twin prime pair known is $2996863034895 \cdot 2^{1290000} \pm 1$, with $388,342$ decimal digits. It was discovered in September 2016. There are $808,675,888,577,436$ twin prime pairs below $10^{18}$" (Wikipedia). But it's not enough just to show that they seem to keep going. Mathematicians want to prove they keep going. This conjecture has not been proven.

Calculus to advance far beyond what would have been possible otherwise. We need both intuition and rigor.

I want to make sure we're not missing the forest for the trees. Perhaps the theorems of Calculus seem to have been presented as abstract collections of symbols, with no real meaning. But of course, from what I've said above, that's not what they are. They do have meaning.

Rolle's theorem doesn't just say "If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, and $f(a) = f(b)$, then there is a number $x$ in $(a, b)$ such that $f'(x) = 0$"; it implies that if an object is in the same place now as it was a few minutes ago, then at some point between then and now it had to be stationary, even if only for an instant.

The Mean Value Theorem implies that if I'm here at one time and sixty miles away an hour later, then at some point, even if only for a moment, I had to be going 60 mph.

The theorems have meaning. The physical interpretations help us understand that meaning. They don't define it: the theorems are abstract mathematical truths. The variables $x$ and $y$ don't have to be time and position. That is often how the theorems are applied, and it can be useful to think of them that way, to see them as fitting into a larger picture. But always remember: in mathematics, we are studying eternal truths. The functions we're studying are abstract relations between quantities, independent from how those quantities are interpreted. The fact that they are true relations makes them incredibly useful in the sciences, but that's not primarily why we're studying them.

I hope this brief big-picture look at Calculus helps at least a little in conceptualizing and contextualizing the theorems of Calculus, as well as contextualizing Calculus among your other studies. As we come to the end of this academic year and look back on what we learned, we're not just going to review skills—how to apply the Chain Rule, for instance—but also make sure we have a firm understanding of each theorem and technique of Calculus as it relates to Calculus as a whole, the study of continuous change.

# Calculus I Review Overview

*Mr. Simmons*
*Calculus I*

## The Road Ahead

In these next three weeks (the three first full weeks of May), we're going to be reviewing Calculus I. (So that the numbers align with the week numbers for packets, we're going to start with "Week 6.") We will have an open-book assessment in Week 9 (the last week of the year).

This year, we've studied differential calculus (as opposed to integral calculus). Differential calculus answers the question of the slope of a tangent line; integral calculus answers the question of the area under a curve. The two will be connected in Calculus II, but for now let's take some time to make sure we understand the first of these.

The main idea of this year of Calculus has been to understand derivatives of functions. Derivatives are how we analyze continuous change. We learned years ago how to calculate the slope of a line. A line has a constant slope; a linear function has a constant rate of change. Whenever the independent variable changes by some amount, the dependent variable changes by a constant multiple of that amount.

But what about curves? Ever since ancient Greece, where $\pi$ was discovered, curves have been giving us trouble. That's where derivatives come in. If we look at a curve, we notice that we can draw a tangent line. Then we can measure the slope of that tangent line. Great! That tells us the rate of change of the curve right at the point where the tangent line touches it.

That's all that differential calculus is. It's just looking at curves and asking how sloped they are, how fast they change, and how fast that change is changing (and how fast *that* change is changing, and so on). But answering that question turned out not to be quite as easy as we might have guessed.

We ended up needing to base our definition of derivatives on a definition of limits. The derivative is the slope of the tangent line, but what's a tangent line? The best way we found to define the tangent line at a certain point (call it the tangent point) was for it to be the limit of secant lines (lines that go through the curve at two different points) as those two points get closer to the tangent point. So we defined the derivative at a point to be the limit of the secant line's slope as the secant line approaches the tangent line at that point. And there you have it: the slope of a curve, the instantaneous rate of change of a curvy function.

These are the main ideas behind differential calculus. In the next three weeks, we'll review the major concepts that help us understand these main ideas, and we'll practice again some of the skills we learned. Each week, I will give you two types of questions to answer: conceptual questions and practice questions. As an aid to answering these questions, I encourage you to look at the relevant chapter in your textbook.[1] As always, if you have questions, let me know. Below is a rough outline

[1] While Spivak's text is more rigorous and precise than Finney's, Finney can be more intuitive and easier to read for the purposes of a general overview, and you have easy access to it already. If you would like access to the relevant

of each week's main questions.

## Week 6: Functions, Limits, and Continuity

When we say we want to analyze continuous change, we're talking about continuous change of a function. So what's a function? We want to be able to define a tangent line, and I've hinted that we're going to use limits to do that. What's a limit? Finally, which kind of functions do we most want to look at? The answer is differentiable functions, which are a subclass of continuous functions... but what does "continuous" mean?

## Week 7: Derivatives

After understanding functions, limits, and continuity, we can finally give a little bit more form to our definition of derivatives. We know that the derivative is the slope of a tangent line, but how do we find that? Once we've found a derivative function, what can we do with it?

## Week 8: Derivatives and Graphs

Knowing the derivative of a function can be useful for graphing it. Let's understand how, and let's practice doing it.

excerpts from Spivak, please let me know by email.

# Week 6: Functions, Limits, and Continuity

*Mr. Simmons*
*Calculus I*

## Practice Problems

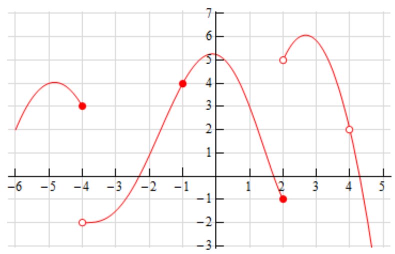
1. Given $f(x) = 3 - 5x - 2x^2$, evaluate
   (a) $f(4)$.
   (b) $f(0)$.
   (c) $f(-3)$.
   (d) $f(6-t)$.
   (e) $f(7-4x)$.
   (f) $f(x+h)$.
2. Evaluate $\frac{f(x+h)-f(x)}{h}$ for
   (a) $f(x) = 4x - 9$.
   (b) $f(x) = \frac{2x}{3-x}$.
3. Determine the domain of each function:
   (a) $f(x) = 3x^2 - 2x + 1$
   (b) $f(x) = -x^2 - 4x + 7$
   (c) $f(x) = 2 + \sqrt{x^2+1}$
   (d) $f(x) = 5 - |x+8|$
4. Find the following limits.
   (a) $\lim_{x\to -5} \frac{x^2-25}{x^2+2x-15}$
   (b) $\lim_{x\to 1} \frac{x^2-1}{x+1}$
   (c) $\lim_{x\to 2} \frac{x^2-4}{x-2}$
   (d) $\lim_{x\to 3} \frac{x^2-4}{x-2}$
   (e) $\lim_{h\to 0} \frac{(a+h)^2-a^2}{h}$

5. Let $f(x) = \begin{cases} 7 - 4x & \text{if } x < 1 \\ x^2 + 2 & \text{if } x \geq 1 \end{cases}$. Find the following limits:

   (a) $\lim_{x \to -6} f(x)$

   (b) $\lim_{x \to 1} f(x)$

6. In the function graph below, determine where the function is discontinuous.




## Conceptual Questions

Answer the following questions in your own words. Try to avoid using symbols to the extent possible. Instead, write in full, complete, grammatical sentences. Answer these questions as if you're teaching these concepts to someone who's never heard of them before. That might mean giving examples, counterexamples, or analogies, for example. If you use any notation, it means explaining that notation. (This is the most important part of the review.)

1. In your own words, what is a function?
2. In your own words, what is a domain?
3. Exactly how are functions represented by equations?
4. Exactly how are functions represented by graphs?
5. In your own words, what is a limit?
6. In your own words, what is a continuity?

GreatHearts Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: 11th Drama
**Teacher(s)**: Mrs. Jimenez (margaret.cousino@greatheartsirving.org)

**Weekly Plan**:

Monday, May 4
- [ ] Plan your private performance - complete sheet
- [ ] Locate costumes and props

Tuesday, May 5
- [ ] Act 2 Zoom Rehearsal OR
- [ ] Practice lines for 20 minutes

Wednesday, May 6
- [ ] Act 3 Zoom Rehearsal OR
- [ ] Practice lines for 20 minutes

Thursday, May 7
- [ ] Practice lines for 20 minutes

Friday, May 8
- [ ] attend office hours
- [ ] catch-up or review the week's work
- [ ] scan and submit work by Sunday, May 10

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

________________________________
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

________________________________
Parent Signature

## Monday, May 4

1. As I've communicated to y'all over email or Zoom, we will be doing two "performances" which will make up most of your semester grade, as Drama is a performance-based class.

    a. The first one is a whole cast performance over Zoom, which is what we are preparing for during our Zoom rehearsals. We will record this during the week of May 18-22; day and time TBD. Look out for another poll on Google Classroom and respond ASAP.

    b. The second one is your own privately recorded performance of only some of your lines. **These will be due Wednesday, May 27**, but you can record and submit them via Google Classroom at any point before then once you receive the OK from me. Look for the assignment "Twelfth Night: Private Performance." *You can also find your individual line assignments there.*

    c. For those of you with small acting roles, I will have you record your lines *and* a new version of the monologue you performed Quarter 3. I know you were supposed to have larger backstage roles for the original performance, but we will have to accomodate and supplement with more acting due to the circumstances.

    d. Finally, you only need to fully memorize and master the lines I'm assigning you for your private performance. You will record yourself off-script, so you should spend your line memorizing time on these lines. You should be very familiar with your other lines at this point and not dependent on your script, but it is acceptable to read from your script during the Zoom performance so long as it is not in view of the camera, your eyes are not glued to it, and you can turn the pages absolutely silently. **You still need to be acting!**

2. Your assignment for today is to prepare for your private performance. Please do the following:

    a. Look up which lines will be included in your private performance on Google Classroom.

    b. Find a **costume** at home. We do not need to dress in period clothing. Look through your closet and find an outfit (or outfits) that suit your character(s). Is your character rich or poor? A servant? In mourning? Wearing yellow stockings? A priest? A girl pretending to be a boy? Etc. Whoever your character is, find a costume that represents him/her.

    c. Find **props** for your character around the house. Not all the line assignments require props but read through yours to see if the script itself calls for a prop or if you think a certain prop would supplement your interpretation. Is your character reading a letter? Holding a ring? A drunkard and so needs a flask or bottle? Using a sword? A wallet? Money? Etc. You can be creative with this--make a prop or re-purpose a household item for prop if you don't have it.

    d. Decide on the **location(s)** for your private performance. Does the scene take place indoors or outdoors? Where exactly? Or how do you interpret the scene? I know we are all stuck at home, but I do *not* want to see your unmade bed in the background for your private performance! Be intentional about finding/making a place to record. Use one corner of your bedroom or another room in the house and set it up for your scene. It does not need to be elaborate, just decent :)

    e. Fill out the "Private Performance Preparation" sheet in this packet and submit it via Google Classroom by Sunday, May 10.

    f. The costumes and props you find for your private performance you will also use for our Zoom performance. For the Zoom performance, everyone needs to sit/stand in front of a blank, plain white/beige/light-colored wall. I will post a props list to Google for the Zoom performance.

## Tuesday, May 5

1. Actors in Act 2 need to attend today's Zoom rehearsal. Zoom link can be found on Google Classroom.
2. Characters in Act 2: Student director, Antonio, Sebastian, Sir Toby, Sir Andrew, Clown, Maria, Malvolio, Orsino, Curio, Viola, Fabian
3. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to. Everyone still has to submit a line memorizing sheet, however.

## Wednesday, May 6

1. Actors in Act 3 need to attend today's Zoom rehearsal. Zoom link can be found on Google Classroom.
2. Characters in Act 3: Student director, Viola, First Officer, Second Officer, Clown, Sir Toby, Sir Andrew, Olivia, Fabian, Maria, Sebastian, Antonio, Malvolio
3. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to.

## Thursday, May 7

1. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to.

## Friday, May 8

1. Finish any work from this week that you need to.
2. Upload your line memorizing sheet and Private Performance Preparation sheet to Google Classroom as a single PDF by Sunday, May 11.
3. Attend office hours at 11am if you have questions.

## Drama Weekly Line Memorization

*NB: If you are attending Zoom rehearsals this week and so are exempt from the 20 mins a day, please write "Zoom rehearsal" by the day(s) you attended and still submit this through Google Classroom so I have a record. Thanks!*

**Name:** **Week: 5/4-5/10**

| Day: | Minutes practiced: |
|---|---|
| Monday | |
| Tuesday | |
| Wednesday | |
| Thursday | |
| Friday | |
| Saturday | |
| Sunday | |

***Minimum time: 20 mins/day, 5 days/week***

**I verify that this is a true and accurate account of the time I have spent memorizing my lines this past week.**

**Signature:** **Date:**

## Private Performance Preparation

| |
|---|
| Name: |
| Character(s): |
| Scene(s): |

| |
|---|
| Costume(s): |
| Prop(s): |
| Location(s): |

GreatHearts Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: 11 Greek
**Teacher(s)**: Miss Salinas annie.salinas@greatheartsirving.org

**Weekly Plan**:

Monday, May 4
☐ Worksheet: People-watching, pt. 1

Tuesday, May 5
☐ Worksheet: People-watching, pt. 2

Wednesday, May 6
☐ Watch the dictation video on Google Classroom
☐ Worksheet: Dictation, pt. 1

Thursday, May 7
☐ Watch the dictation video on Google Classroom
☐ Worksheet: Dictation, pt. 2

Friday, May 8
☐ attend office hours
☐ catch-up or review the week's work

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

χαιρετε φιλοι! We’re back! You may have noticed that this week, all your classes are back working on packets rather than on daily Google Classroom assignments. The idea with transitioning back to packets is that it’ll be simpler for students and parents to not have to figure out which classes want assignments submitted in different ways...like a Starbucks coffee order, the more options there are, the more likely there are to be mistakes. (Don’t hate me, Starbucks, I love and miss you.) While it’ll make getting feedback to you guys a little slower, I must confess I am extremely relieved to be going back to the packet method - it was taking me hours and hours and hours to digitize, organize, troubleshoot, and post digital assignments for my five classes. (~~yikes~~) And since we’re already working ‘round the clock anyway...you get the idea. (~~yikes again~~) Plus, now we get videos, which I’m excited about! We’ll have two for Greek class this week: one on each Wednesday and Thursday that you’ll need to watch in order to complete the assignment.

## Monday, May 4

Today, you’ll try your hand at writing simple questions and responses in Greek! Complete the worksheet for Monday, “People-watching pt. 1”.

## Tuesday, May 5

Today we’re continuing to create sentences in Greek. Complete Tuesday’s worksheet, “People-watching pt. 2”.

## Wednesday, May 6

Today and tomorrow, we’ll be working on dictation: writing down Greek words that you hear. Watch the video on Google Classroom, and follow the instructions on the worksheet for Wednesday.

## Thursday, May 7

Dictation, pt. 2: Watch the video on Google Classroom, and follow the instructions on the worksheet for Wednesday.

## Friday, May 8

Have questions about the week’s work? Want to go over something to make sure you got it? Just simply desperate to converse with someone you’re not related to? Come to Greek Office Hours at 10:30am - link available in the stream of our Google Classroom! See you there.

**Monday**

# People-watching (pt. 1)

This week, we're practicing writing short sentences in Greek. The first sentences you'll write practice the middle-voice participles we learned in Chapter 8α. All of them will answer the question, "Who stops _______ing?" For example:

| | |
|---|---|
| τὶς παύεται ἐργαζόμεν**ος**; | Who stops working? |
| ἡ Μυρρίνη παύεται ἐργαζομέν**η**. | Myrrhine stops working. |

Notice what I've put in bold: in the question sentence, we didn't know who would be doing the action, so we defaulted to the masculine singular participle ending.

- But when we figured out it was Myrrhine, we had to change it to the ________________ gender.
- All of these participles will simply be in the ______________ case, because they're predicates to the subject and don't become direct objects or anything complicated.
- However, if Myrrhine had been plural, we would have had to change the verb and the participle to become ____________ as well.

Watch, here's another example:

| | |
|---|---|
| τις παυ**εται** διαλεγομεν**ος**; | Who stops chatting? |
| αἱ κοραι και οἱ νεανιαι παυ**ονται** διαλεγομεν**οι**. | The young men and young women stop chatting. |

Notice how in that example, the verb and the participle both became nominative plural because we have a plural subject.

Possible people:

- ὁ Δικαιόπολις
- ἡ Μυρρίνη
- ὁ Φίλιππος
- ἡ Μέλιττα
- ὁ Ξανθίας
- ὁ πάππος
- αἱ γυναῖκες
- οἱ ἄνδρες
- αἱ κόραι
- οἱ ἑταιροι

Possible actions:

- διαλέγομαι (+dat)
- εὔχομαι
- θεάομαι
- ἡγέομαι (+dat)
- ἕπομαι (+dat)
- μάχομαι
- ἔρχομαι

Now it's your turn. Using the vocab on the previous page, create five sentence pairs of your own. Don't forget to turn the verbs into participles; if you get stuck or want to see a declension chart with the endings all written out, page 134 of your textbook will help. Try to use different verbs in each sentence.

1. Q: ____________________

   A: ____________________

Translation:

   Q: ____________________

   A: ____________________

2. Q: ____________________

   A: ____________________

Translation:

   Q: ____________________

   A: ____________________

3. Q: ____________________

   A: ____________________

Translation:

   Q: ____________________

   A: ____________________

4. Q: ____________________

   A: ____________________

Translation:

   Q: ____________________

   A: ____________________

5. Q: ____________________

   A: ____________________

Translation:

   Q: ____________________

   A: ____________________

**Tuesday**

# People-watching (pt. 2)

*Today, we'll be writing more sentences in Greek using the participles we learned in Chapter 8.*

"Whom do you see? What is s/he doing?" Answer by writing a sentence that uses a participle that describes the direct object.

For example:

| | |
|---|---|
| τίνα ὁρ**ᾷς**; τὶ ποιεῖ; | Whom do you see? What is s/he doing? |
| ὁρ**ῶ** **τὸν** δοῦλ**ον** ἐργαζόμεν**ον**. | I see the slave working. |

Notice the verb also changed endings, from 2nd person ("you") to 1st person ("I"). Notice also that this time, we've introduced a new case: the _______________ case. Here, the direct object and the participle describing their action will both get these endings.

Here's a plural example, using the numbers we've been working on:

| | |
|---|---|
| τίνα ὁρᾷς; τὶ ποιεῖ; | Whom do you see? What is s/he doing? |
| ὁρῶ τετταρ**ας** ἑταιρ**ους** ἐπανερχομεν**ους**. | I see four companions returning. |

Notice the accusative ending on the noun, adjective, and participle.

Possible people:

- ὁ Δικαιόπολις
- ἡ Μυρρίνη
- ὁ Φίλιππος
- ἡ Μέλιττα
- ὁ Ξανθίας
- ὁ πάππος
- αἱ γυναῖκες
- οἱ ἄνδρες
- αἱ κόραι
- οἱ ἑταιροι

Possible actions:

- πορεύομαι
- ἀφικνέομαι (εἰς τὴν αὐλήν)
- εἰσέρχομαι (εἰς τὸ ἄντρον)
- φοβέομαι (τὸν δεσπότην)
- πείθομαι (τῷ δεσπότῃ)
- ἡγέομαι (+ dat.)
- ἕπομαι (+ dat.)
- διαλέγομαι ἀλλήλοις/ἀλλήλαις
- θεάομαι (τᾰ δράματα)

Using these possible nouns and verbs, write your sentences on the next page! Since all the sentences will answer the same question today, unlike yesterday, you do not have to write questions. **However, I would like you to include cardinal or ordinal numbers in three of your five sentences.**

As yesterday, try to use different people and actions in each sentence. If you need help turning verbs into participles with the correct case endings, page 134 of your textbook will help.

τίνα ὁρᾷς; τὶ ποιεῖ; Whom do you see? What is s/he doing?*

1. :________________________________________

Transl.:________________________________________

2. :________________________________________

Transl.:________________________________________

3. :________________________________________

Transl.:________________________________________

4. :________________________________________

Transl.:________________________________________

5. :________________________________________

Transl.:________________________________________

*Don’t forget to include cardinal or ordinal numbers in at least 3 of your 5 sentences.

**Wednesday**

# Dictation: nouns and articles pt. 1

On Google Classroom, you will find a video of me reading this passage aloud. Log on and listen to the passage. As you listen, write in the missing words. Then, below, give me the declension (1st, 2nd, or 3rd), gender (masc, fem, or neut) and case (nom, gen, dat, acc, or voc) of each missing word. (HINT: the articles will help you.) You do not have to translate the passage.

προχωροῦσιν οὖν ἀνὰ τὸ ________[1] καὶ, ἐπεὶ εἰς ἄκρον ἀφικνοῦνται, τὰς ________[2] ὁρῶσι κάτω κειμένας. ὁ δὲ Φίλιππος τὴν ____________[3] θεώμενος, ”ἰδού,”φησίν, “ὡς καλὴ ἐστιν ἡ __________.[4] ἆρ’ ὁρᾶτε τὴν Ἀκρόπολιν;” ἡ δὲ Μέλιττα, “ὁρῶ δή. ἆρ’ὁρᾶτε καὶ τὸν ____________;[5] ὡς καλός ἐστι καὶ μέγας.” ὁ δὲ Φίλιππος, “ἀλλὰ, ὦ πάππα· καταβαίνομεν γὰρ πρὸς τὴν ________.”[6]

1. declension: ____________ gender: __________ case: ___________
2. declension: ____________ gender: __________ case: ___________
3. declension: ____________ gender: __________ case: ___________
4. declension: ____________ gender: __________ case: ___________
5. declension: ____________ gender: __________ case: ___________
6. declension: ____________ gender: __________ case: ___________

**Thursday**

# Dictation: nouns and articles pt. 2

On Google Classroom, you will find a video of me reading this passage aloud. Log on and listen to the passage. As you listen, write in the missing words. Then, below, give me the declension (1st, 2nd, or 3rd), gender (masc, fem, or neut) and case (nom, gen, dat, acc, or voc) of each missing word. (HINT: the articles will help you.) You do not have to translate the passage.

ταχέως οὖν καταβαίνουσι καὶ εἰς τὰς __________[1] ἀφικόμενοι, τὸν ἡμίονον προσάπτουσι __________[2] τινὶ καὶ εἰσέρχονται. ἐν δὲ τῷ __________[3] πολλοὺς ἀνθρώπους ὁρῶσιν ἐν ταῖς __________[4] βαδίζοντας· ἄνδρες γάρ, ____________,[5] νεανίαι, παῖδες, ___________[6] τε καὶ ξένοι, σπεύδουσι πρὸς τὴν __________.[7] ἡ οὖν Μυρρίνη φοβουμένη ὑπὲρ τῶν ___________,[8] "ἔλθε δεῦρο, ὦ ___________,"[9] φησίν, "καὶ λαμβάνου τῆς ___________.[10] μὴ λεῖπέ με ἀλλ' ἕπου ἅμα ἐμοί."

1. declension: ____________ gender: __________ case: ___________
2. declension: ____________ gender: __________ case: ___________
3. declension: ____________ gender: __________ case: ___________
4. declension: ____________ gender: __________ case: ___________
5. declension: ____________ gender: __________ case: ___________
6. declension: ____________ gender: __________ case: ___________
7. declension: ____________ gender: __________ case: ___________
8. declension: ____________ gender: __________ case: ___________
9. declension: ____________ gender: __________ case: ___________
10. declension: ____________ gender: __________ case: ___________

GreatHearts Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**

**Course**: 11 Humane Letters

**Teacher(s)**: Mr. Brandolini david.brandolini@greatheartsirving.org
Mr. Mercer andrew.mercer@greatheartsirving.org

**Weekly Plan**:

Monday, May 4

☐ Written reflection on *Hamlet*

☐ Read Aristotle, *Nichomachean Ethics* I.1-7 (p. 1-10 of the Hackett edition)

Tuesday, May 5

☐ Read Aristotle, *Nichomachean Ethics* I.8-II.6 (p. 10-25)

Wednesday, May 6

☐ Read Aristotle, *Nichomachean Ethics* II.7-III.5 (p. 25-40)

☐ Prepare for seminar by thinking through questions posted on Google Classroom

Thursday, May 7

☐ Attend Zoom seminar on Aristotle readings

Friday, May 8

☐ Attend office hours

☐ Catch-up or review the week's work

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

________________________________
Student Signature

________________________________
Parent Signature

## Monday, May 4

1. *Hamlet* reflection. (30-40 minutes)
Today, in a couple of paragraphs, write an informal* reflection on *Hamlet* in light of our understanding of the tragic flaw, or *hamartia*, in the Greek tragedies we have read. What is Hamlet's tragic flaw? Does he have a moment of recognition to see this in himself? Where might we see him reconcile with his flaw and find hope of redemption?

2. Read Aristotle, *Nichomachean Ethics* I.1-7 (p. 1-10 of the Hackett edition) (rest of class)
We leave the throes of tragedy behind us and move towards something a little more direct as we begin Aristotle's *Nichomachean Ethics*. Many ideas and themes in this work will be familiar to us now that we've put a respectable number of Platonic Dialogues under our belts; that being said, Aristotle's work is non-narrative, so hopefully many of his ideas will be more clear. We will not be having any lecture material on this reading in this packet; instead, we ask that you read an annotate with care, noting down ideas or questions you have that you think may prove fruitful in our seminar this week.

Note: some of today's reading can spill into Tuesday's relatively easily if you find you would like more time to work on *Hamlet*, but try to read the majority of it today.

**Note that informal simply means that you are not strictly required to use quotations or perform extensive analysis; keep your language and thoughts elevated nonetheless!*

## Tuesday, May 5

Read Aristotle, *Nichomachean Ethics* I.8-II.6 (p. 10-25)

## Wednesday, May 6

Read Aristotle, *Nichomachean Ethics* II.7-III.5 (p. 25-40)
Ahead of the seminar, we will be sending communication via google classroom with official seminar questions for you to look over in preparation for our discussion. At the same time, work to refine any thoughts or questions you saw while reading over the text!

## Thursday, May 7

We will be hosting our first video seminar of the year today! Check your email and/or google classroom for information on accessing the meeting.

## Friday, May 8

Please attend the optional Zoom office hours if there is anything you need to discuss with your teacher. The rest of this day is to be spent catching up on or reviewing material from this week for all your classes.

**Monday: 11th Grade Humane Letters *Hamlet* reflection**

*Answer in at least two thoughtful paragraphs. See the lesson plan above for the prompt.*

GreatHearts Irving

# Physics Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

## May 4-8, 2020

**Course**: 11 Physics
**Teacher**: Miss Weisse natalie.weisse@greatheartsirving.org
**Resource:** *Miss Weisse's Own Physics Textbook* — new pages found at the end of this packet

**Weekly Plan**:

Monday, May 4

- [ ] Review *Unit 8 Part 4 – Hooke's Law and Elastic Energy*
- [ ] Complete Unit 8 Worksheet 2 - Extra Problems
- [ ] Email Miss Weisse with Questions and to Request Solutions

Tuesday, May 5

- [ ] Read *Unit 8 Part 5 – Gravitational (Potential) and Kinetic Energy*
- [ ] Watch *"Introduction to Kinetic Energy Lab – Tuesday May 4"* Video
- [ ] Complete Pre-Lab Work
- [ ] Linearizing Graphs Review

Wednesday & Thursday, May 6-7

- [ ] Collect Data
- [ ] Graph & Linearize Data
- [ ] Analysis of Linearized Data
- [ ] Determine the Meaning of the Slope
- [ ] Write Conclusion
- [ ] Email Miss Weisse with Questions

Friday, May 8

- [ ] Attend Our Post Lab Discussion During Office Hours at 9:30 AM!
- [ ] Submit This Week's Work on Google Classroom by The End of the Day Sunday, May 10

## Statement of Academic Honesty

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I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

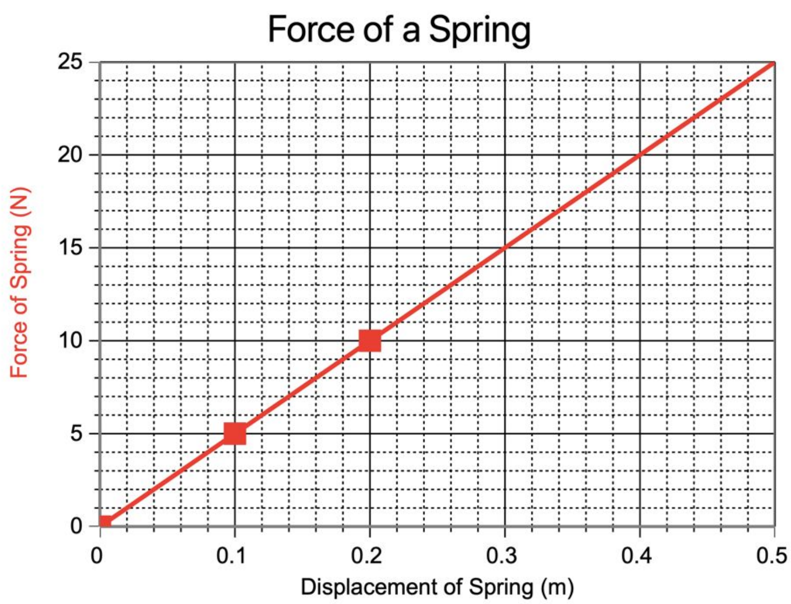
## Monday, May 4

- ➔ Review *Unit 8 Part 4 – Hooke's Law and Elastic Energy*
- ➔ Complete Unit 8 Worksheet 2 - Extra Problems
- ➔ Email Miss Weisse with Questions and to Request Solutions

# Unit 8 Energy Storage and Transfer Model
# Worksheet 2 – Extra Practice

1) What does the spring constant number tell you about a spring? In other words, if two springs are given to you, and one has a spring constant of 100 N/m and the other has a spring constant of 125 N/m, what do these numbers tell you about the springs?

2) Using the graph below, determine the following:




a. The spring constant of the spring

b. The equation that best represents the graph.

c. The amount of force if the spring were stretched 0.22 m.

d. Graphically determine the amount of energy stored in the spring from stretching the spring from x = 0 to x = 0.2 m

e. Graphically determine the amount of energy stored in the spring from stretching the spring from x = 0.3 to x = 0.5 m

3) You are given a spring and you have a 0.50 kg mass and a 1.0 kg mass and a meter stick. Explain how you could use these devices to determine the spring constant in the spring.

4) Upon hanging the 1.0 kg mass on the spring it stretches 4 cm. When you attach the 0.50 kg mass to the bottom of the 1.0 kg mass, the spring stretches 2 more cm. What is the spring constant? (Hint: what force is being applied to the spring? Can you make a graph of the Force and the distance stretched to help?)

5) Conceptual Question: Two springs with the same spring constant are connected together to make a longer spring. If you hang a 1 kg mass to the end, does each spring stretch the same as, half as much, or twice as much as if the 1 kg mass was hung from just 1 of the springs. Consider a force diagram for each spring to think about how much force is applied to each spring.

## Tuesday, May 5

- ➔ Read *Unit 8 Part 5 – Gravitational and Kinetic Energy*
- ➔ Watch *"Introduction to Kinetic Energy Lab – Tuesday May 4"* Video found in the "Materials" section in Google Classroom
- ➔ Complete Pre-Lab Work (spelled out in *Unit 8 – Part 5*)
    - ◆ Purpose, Variables, Constants, Description of Procedure
    - ◆ Create your data table as shown below. You will be graphing the data from columns in **green**.
- ➔ Linearizing Graphs Review (at the end of *Unit 8 – Part 5*)

**Data Table**

| Height (cm) | Height (m) | **$E_k$** ($= E_g = mgh$) | Velocity (m/s) | | | |
|---|---|---|---|---|---|---|
| | | | Trial 1 | Trial 2 | Trial 3 | **Average Velocity** |
| | | | | | | |
| | | | | | | |
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## Wednesday *and* Thursday, May 6-7

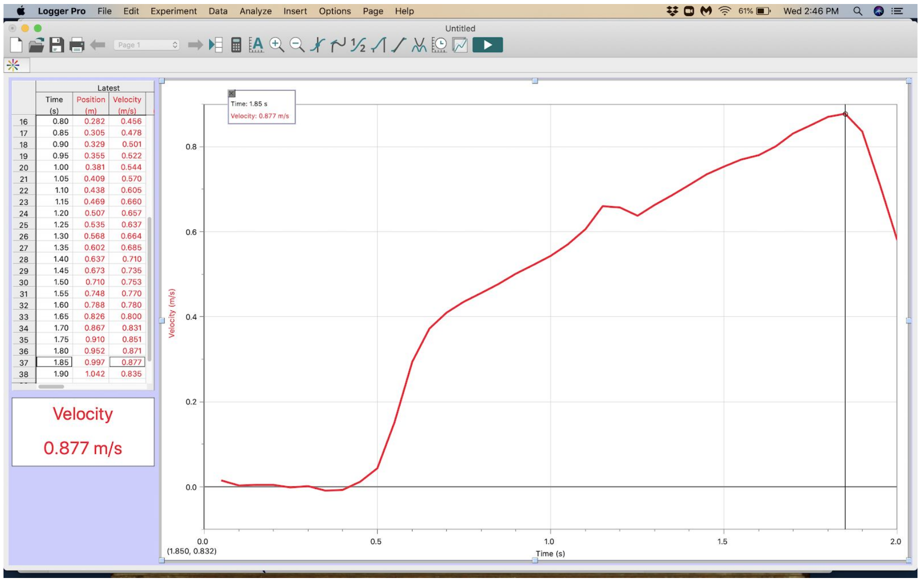
- ➔ Collect Data from Screen Shots of *LoggerPro* found on the following pages before *Miss Weisse's Own Physics Textbook*
    - ◆ You will be collecting data for velocities at 10 different ramp heights, and three trials for each height.
    - ◆ After you've collected all the data,
        - Convert heights to meters
        - Complete the $E_g$ (and therefore $E_k$) calculations
        - Average the three trials of velocity data per height
- ➔ Graph & Linearize Data
    - ◆ Even though Kinetic Energy is technically our Independent Variable, we want an equation for $E_k$ so we are going to use it as our y-axis, dependent variable.
    - ◆ Linearize your graph – compare the shape to the common graph shapes in the notes you read yesterday
        - determine the relationship between $E_k$ and velocity.
        - Regraph the data after manipulating your velocity values
        - Draw your best fit line, find it's slope, and write your equation
- ➔ Analysis of Linearized Data
    - ◆ What does the slope tell us?
    - ◆ What does the y-intercept tell us?
- ➔ Determine the *Meaning* of the Slope
    - ◆ *Think about the units! Rewrite Joule as kg $m^2$/ $s^2$ and see what happens! It has to do with specific physical properties of our set-up.*
- ➔ Write Conclusion (refer to guidelines given last week)
- ➔ Email Miss Weisse with Questions

## Friday, May 8

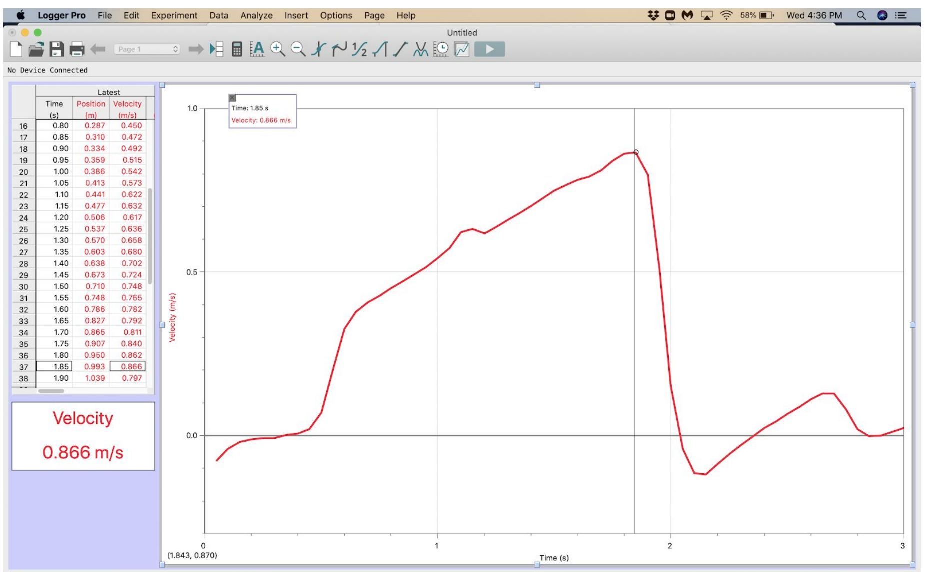
- ➔ Attend Our Post-Lab Discussion During Office Hours at 9:30 AM!

****************************** **LAB DATA** ******************************

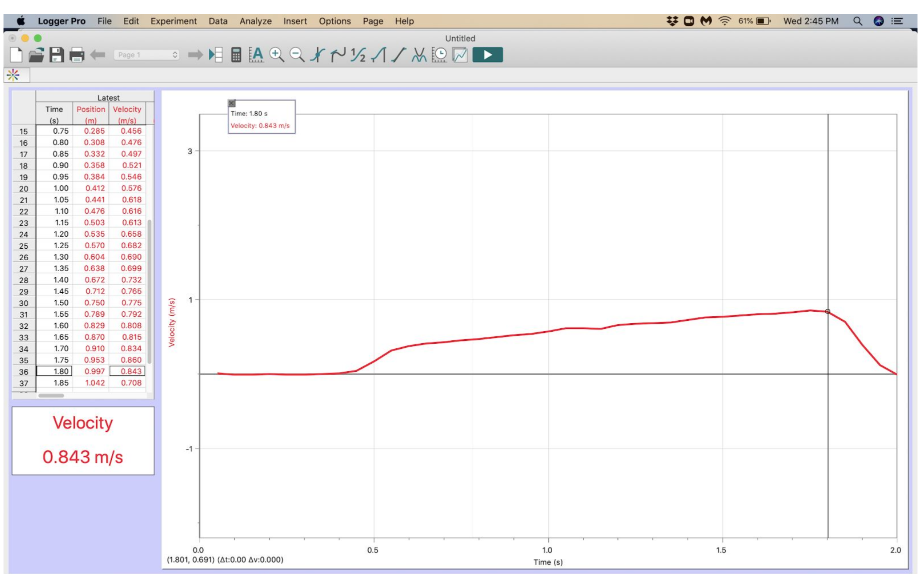
## Height 9 cm




| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 16 | 0.80 | 0.282 | 0.456 |
| 17 | 0.85 | 0.305 | 0.478 |
| 18 | 0.90 | 0.329 | 0.501 |
| 19 | 0.95 | 0.355 | 0.522 |
| 20 | 1.00 | 0.381 | 0.544 |
| 21 | 1.05 | 0.409 | 0.570 |
| 22 | 1.10 | 0.438 | 0.605 |
| 23 | 1.15 | 0.469 | 0.660 |
| 24 | 1.20 | 0.507 | 0.657 |
| 25 | 1.25 | 0.535 | 0.637 |
| 26 | 1.30 | 0.568 | 0.664 |
| 27 | 1.35 | 0.602 | 0.685 |
| 28 | 1.40 | 0.637 | 0.710 |
| 29 | 1.45 | 0.673 | 0.735 |
| 30 | 1.50 | 0.710 | 0.753 |
| 31 | 1.55 | 0.748 | 0.770 |
| 32 | 1.60 | 0.788 | 0.780 |
| 33 | 1.65 | 0.826 | 0.800 |
| 34 | 1.70 | 0.867 | 0.831 |
| 35 | 1.75 | 0.910 | 0.851 |
| 36 | 1.80 | 0.952 | 0.871 |
| 37 | 1.85 | 0.997 | 0.877 |
| 38 | 1.90 | 1.042 | 0.835 |

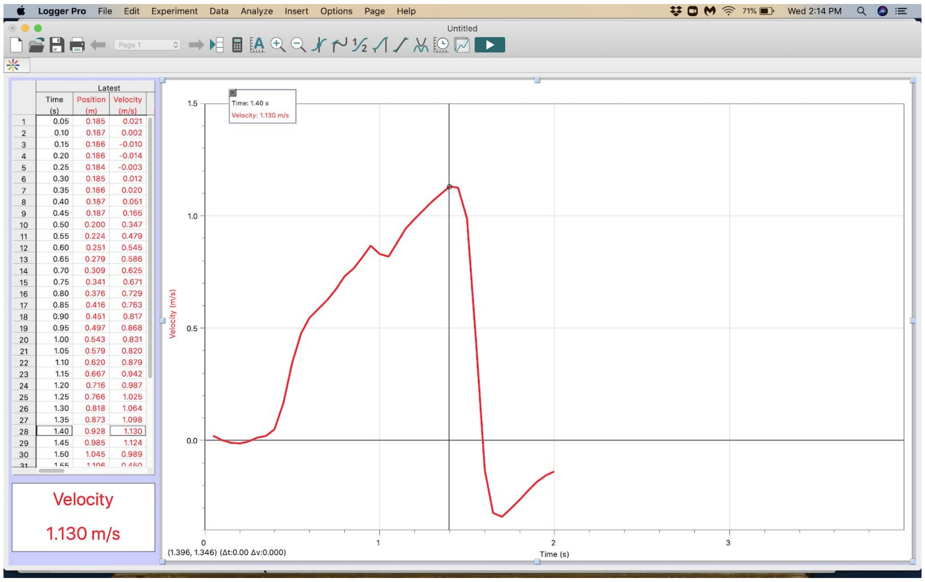


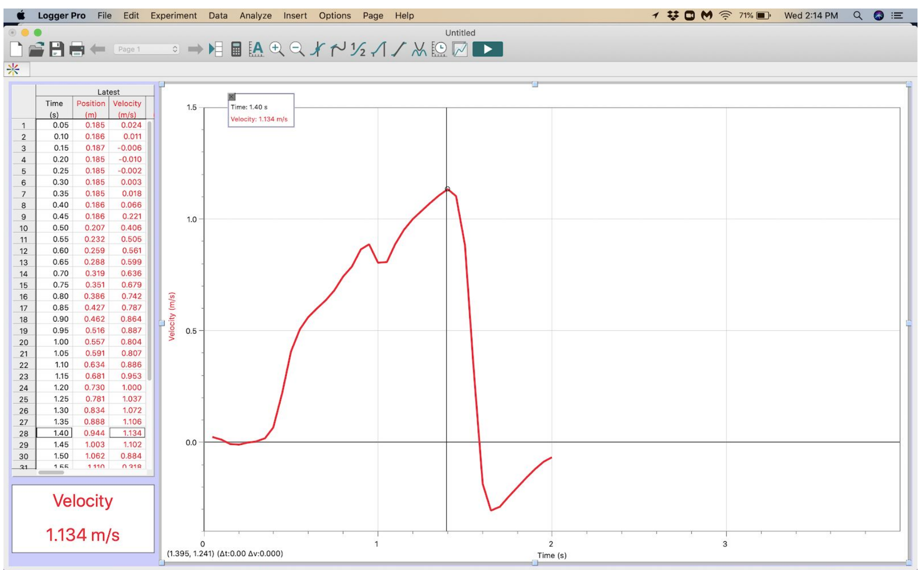

| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 16 | 0.80 | 0.287 | 0.450 |
| 17 | 0.85 | 0.310 | 0.472 |
| 18 | 0.90 | 0.334 | 0.492 |
| 19 | 0.95 | 0.359 | 0.515 |
| 20 | 1.00 | 0.386 | 0.542 |
| 21 | 1.05 | 0.413 | 0.573 |
| 22 | 1.10 | 0.441 | 0.622 |
| 23 | 1.15 | 0.477 | 0.632 |
| 24 | 1.20 | 0.506 | 0.617 |
| 25 | 1.25 | 0.537 | 0.636 |
| 26 | 1.30 | 0.570 | 0.658 |
| 27 | 1.35 | 0.603 | 0.680 |
| 28 | 1.40 | 0.638 | 0.702 |
| 29 | 1.45 | 0.673 | 0.724 |
| 30 | 1.50 | 0.710 | 0.748 |
| 31 | 1.55 | 0.748 | 0.765 |
| 32 | 1.60 | 0.786 | 0.782 |
| 33 | 1.65 | 0.827 | 0.792 |
| 34 | 1.70 | 0.865 | 0.811 |
| 35 | 1.75 | 0.907 | 0.840 |
| 36 | 1.80 | 0.950 | 0.862 |
| 37 | 1.85 | 0.993 | 0.866 |
| 38 | 1.90 | 1.039 | 0.797 |

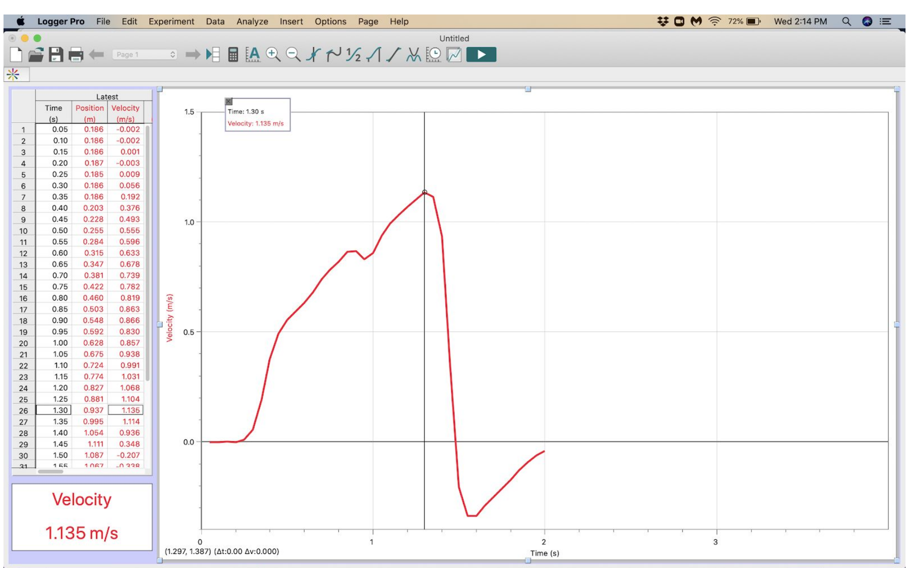



| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 15 | 0.75 | 0.285 | 0.456 |
| 16 | 0.80 | 0.308 | 0.476 |
| 17 | 0.85 | 0.332 | 0.497 |
| 18 | 0.90 | 0.358 | 0.521 |
| 19 | 0.95 | 0.384 | 0.546 |
| 20 | 1.00 | 0.412 | 0.576 |
| 21 | 1.05 | 0.441 | 0.618 |
| 22 | 1.10 | 0.476 | 0.616 |
| 23 | 1.15 | 0.503 | 0.613 |
| 24 | 1.20 | 0.535 | 0.658 |
| 25 | 1.25 | 0.570 | 0.682 |
| 26 | 1.30 | 0.604 | 0.690 |
| 27 | 1.35 | 0.638 | 0.699 |
| 28 | 1.40 | 0.672 | 0.732 |
| 29 | 1.45 | 0.712 | 0.765 |
| 30 | 1.50 | 0.750 | 0.775 |
| 31 | 1.55 | 0.789 | 0.792 |
| 32 | 1.60 | 0.829 | 0.808 |
| 33 | 1.65 | 0.870 | 0.815 |
| 34 | 1.70 | 0.910 | 0.834 |
| 35 | 1.75 | 0.953 | 0.860 |
| 36 | 1.80 | 0.997 | 0.843 |
| 37 | 1.85 | 1.042 | 0.708 |

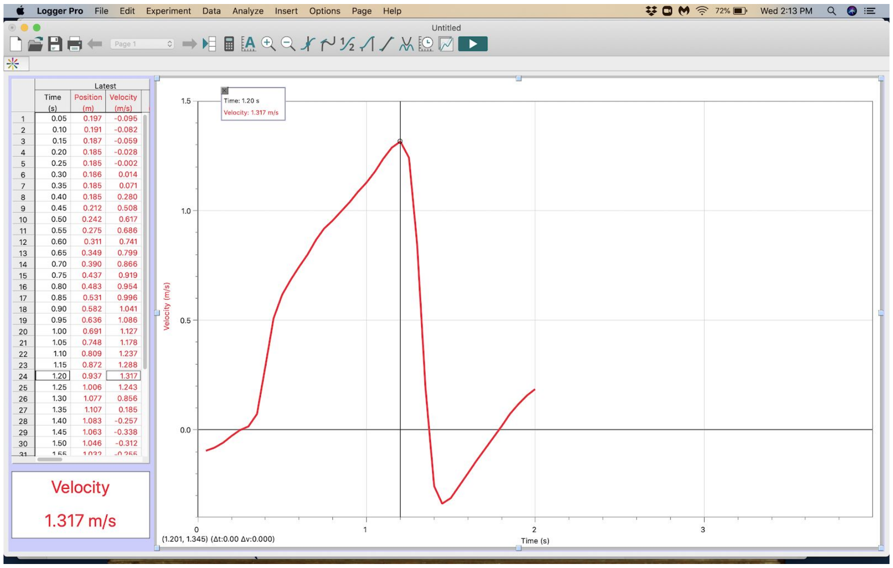
# Height 13 cm

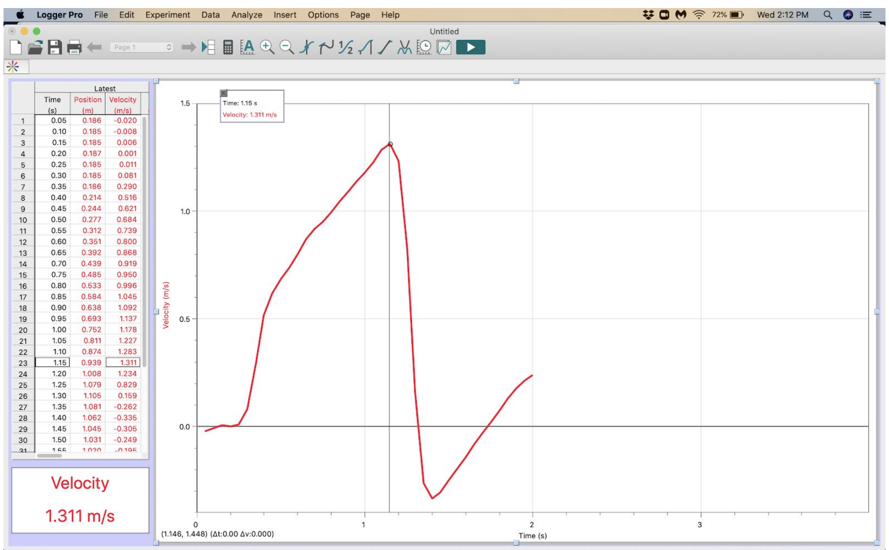

Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:14 PM
Untitled
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.185 0.021
2 0.10 0.187 0.002
3 0.15 0.186 -0.010
4 0.20 0.186 -0.014
5 0.25 0.184 -0.003
6 0.30 0.185 0.012
7 0.35 0.186 0.020
8 0.40 0.187 0.051
9 0.45 0.187 0.165
10 0.50 0.200 0.347
11 0.55 0.224 0.479
12 0.60 0.251 0.545
13 0.65 0.279 0.586
14 0.70 0.309 0.625
15 0.75 0.341 0.671
16 0.80 0.376 0.729
17 0.85 0.416 0.763
18 0.90 0.451 0.817
19 0.95 0.497 0.868
20 1.00 0.543 0.831
21 1.05 0.579 0.820
22 1.10 0.620 0.879
23 1.15 0.667 0.942
24 1.20 0.716 0.987
25 1.25 0.766 1.025
26 1.30 0.818 1.064
27 1.35 0.873 1.098
28 1.40 0.928 1.130
29 1.45 0.985 1.124
30 1.50 1.045 0.989
Velocity
1.130 m/s
Time: 1.40 s
Velocity: 1.130 m/s
Velocity (m/s)
Time (s)
(1.396, 1.346) (Δt:0.00 Δv:0.000)

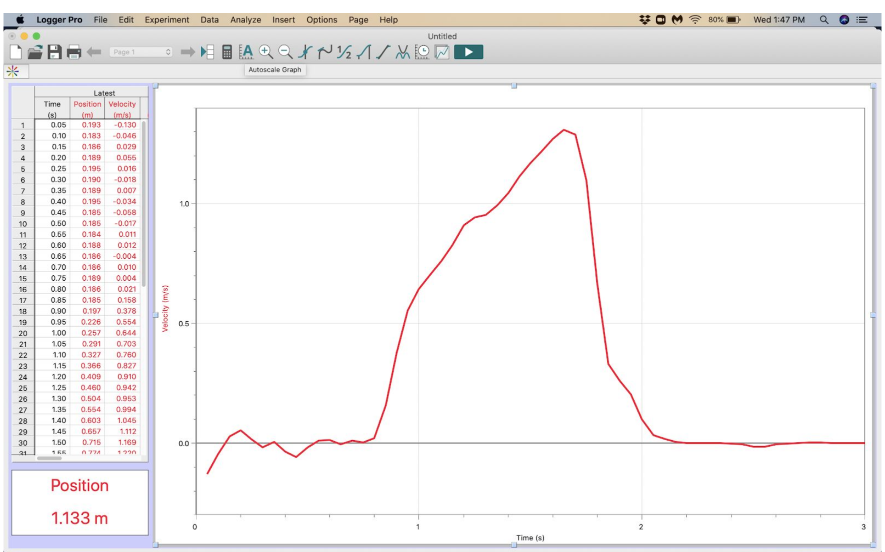

Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:14 PM
Untitled
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.185 0.024
2 0.10 0.186 0.011
3 0.15 0.187 -0.006
4 0.20 0.185 -0.010
5 0.25 0.185 -0.002
6 0.30 0.185 0.003
7 0.35 0.185 0.018
8 0.40 0.186 0.066
9 0.45 0.186 0.221
10 0.50 0.207 0.406
11 0.55 0.232 0.505
12 0.60 0.259 0.561
13 0.65 0.288 0.599
14 0.70 0.319 0.636
15 0.75 0.351 0.679
16 0.80 0.386 0.742
17 0.85 0.427 0.787
18 0.90 0.462 0.864
19 0.95 0.516 0.887
20 1.00 0.557 0.804
21 1.05 0.591 0.807
22 1.10 0.634 0.886
23 1.15 0.681 0.953
24 1.20 0.730 1.000
25 1.25 0.781 1.037
26 1.30 0.834 1.072
27 1.35 0.888 1.106
28 1.40 0.944 1.134
29 1.45 1.003 1.102
30 1.50 1.062 0.884
Velocity
1.134 m/s
Time: 1.40 s
Velocity: 1.134 m/s
Velocity (m/s)
Time (s)
(1.395, 1.241) (Δt:0.00 Δv:0.000)


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:14 PM
Untitled
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.186 -0.002
2 0.10 0.186 -0.002
3 0.15 0.186 0.001
4 0.20 0.187 -0.003
5 0.25 0.185 0.009
6 0.30 0.186 0.056
7 0.35 0.186 0.192
8 0.40 0.203 0.376
9 0.45 0.228 0.493
10 0.50 0.255 0.555
11 0.55 0.284 0.596
12 0.60 0.315 0.633
13 0.65 0.347 0.678
14 0.70 0.381 0.739
15 0.75 0.422 0.782
16 0.80 0.460 0.819
17 0.85 0.503 0.863
18 0.90 0.548 0.866
19 0.95 0.592 0.830
20 1.00 0.628 0.857
21 1.05 0.675 0.938
22 1.10 0.724 0.991
23 1.15 0.774 1.031
24 1.20 0.827 1.068
25 1.25 0.881 1.104
26 1.30 0.937 1.135
27 1.35 0.995 1.114
28 1.40 1.054 0.936
29 1.45 1.111 0.348
30 1.50 1.087 -0.207
Velocity
1.135 m/s
Time: 1.30 s
Velocity: 1.135 m/s
Velocity (m/s)
Time (s)
(1.297, 1.387) (Δt:0.00 Δv:0.000)

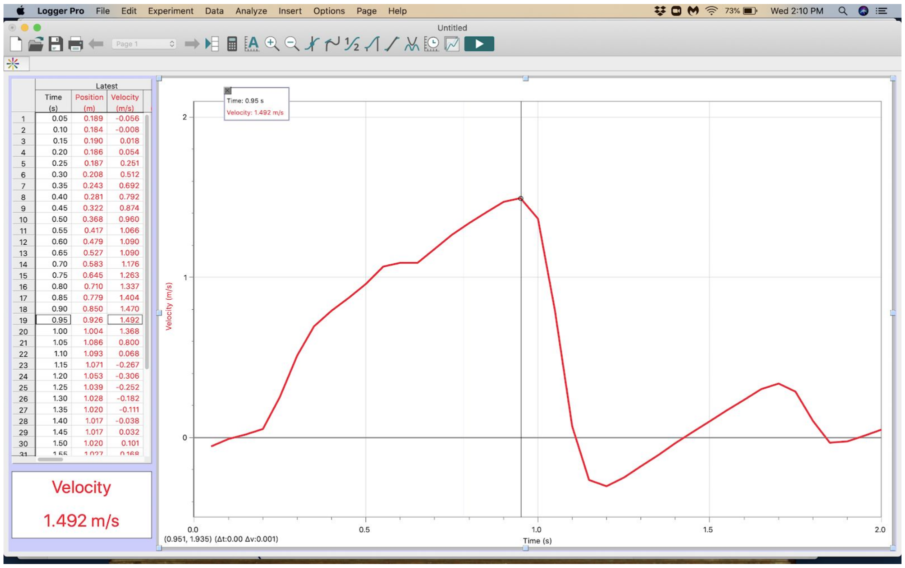
# Height 17 cm

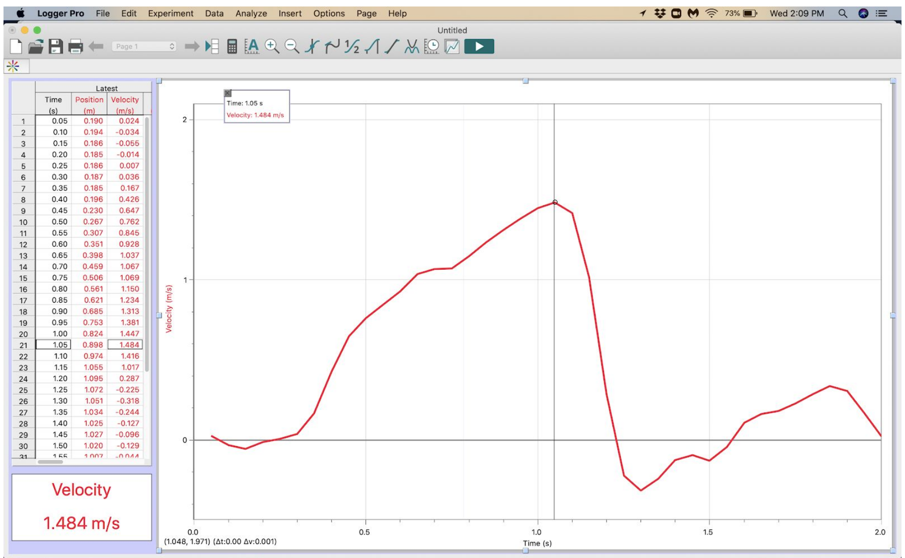

Logger Pro
Untitled
Latest
Time (s)
Position (m)
Velocity (m/s)
Time: 1.20 s
Velocity: 1.317 m/s
Velocity (m/s)
Time (s)
(1.201, 1.345) (Δt:0.00 Δv:0.000)
Velocity
1.317 m/s

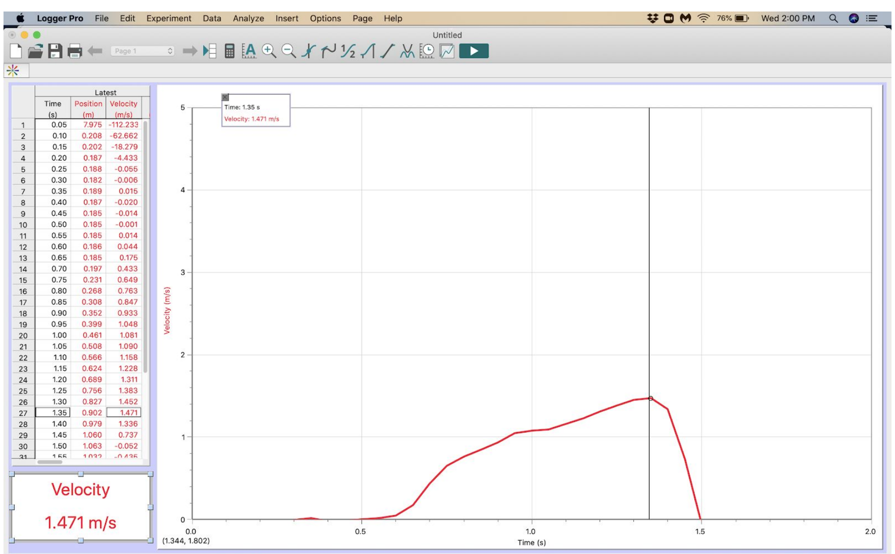

Logger Pro
Untitled
Latest
Time (s)
Position (m)
Velocity (m/s)
Time: 1.15 s
Velocity: 1.311 m/s
Velocity (m/s)
Time (s)
(1.146, 1.448) (Δt:0.00 Δv:0.000)
Velocity
1.311 m/s


Logger Pro
Untitled
Autoscale Graph
Latest
Time (s)
Position (m)
Velocity (m/s)
Velocity (m/s)
Time (s)
Position
1.133 m

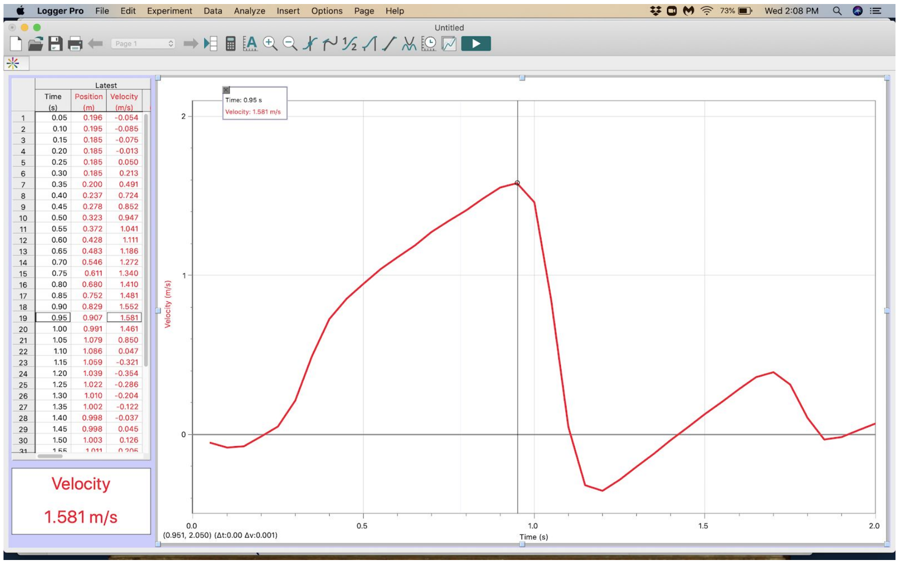
# Height 21 cm

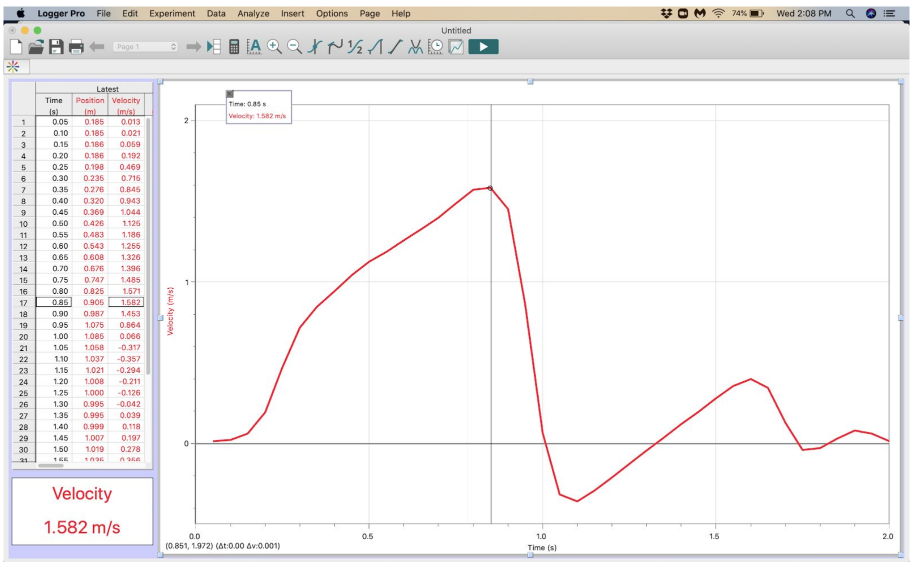

Time: 0.95 s
Velocity: 1.492 m/s
Velocity (m/s)
Time (s)
(0.951, 1.935) (Δt:0.00 Δv:0.001)
Velocity
1.492 m/s

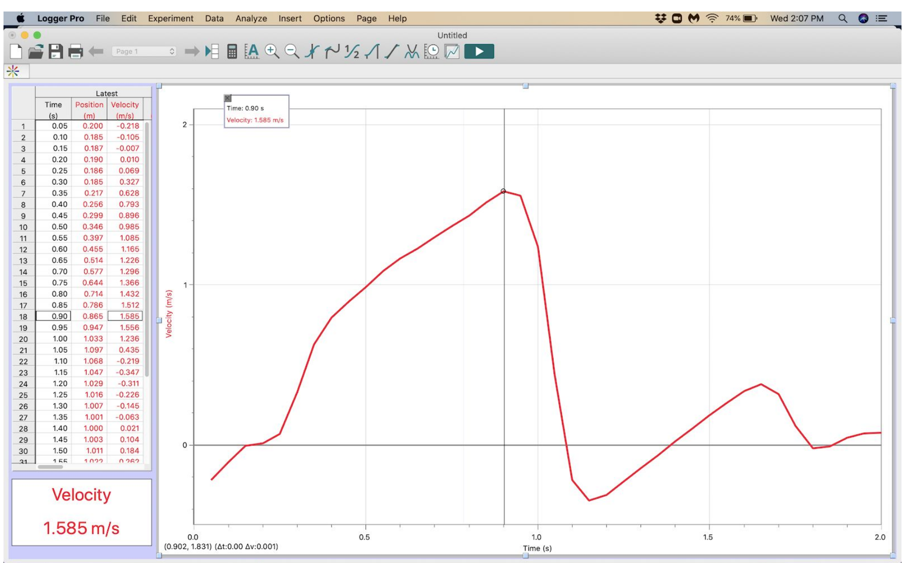

Time: 1.05 s
Velocity: 1.484 m/s
Velocity (m/s)
Time (s)
(1.048, 1.971) (Δt:0.00 Δv:0.001)
Velocity
1.484 m/s


Time: 1.35 s
Velocity: 1.471 m/s
Velocity (m/s)
Time (s)
(1.344, 1.802)
Velocity
1.471 m/s

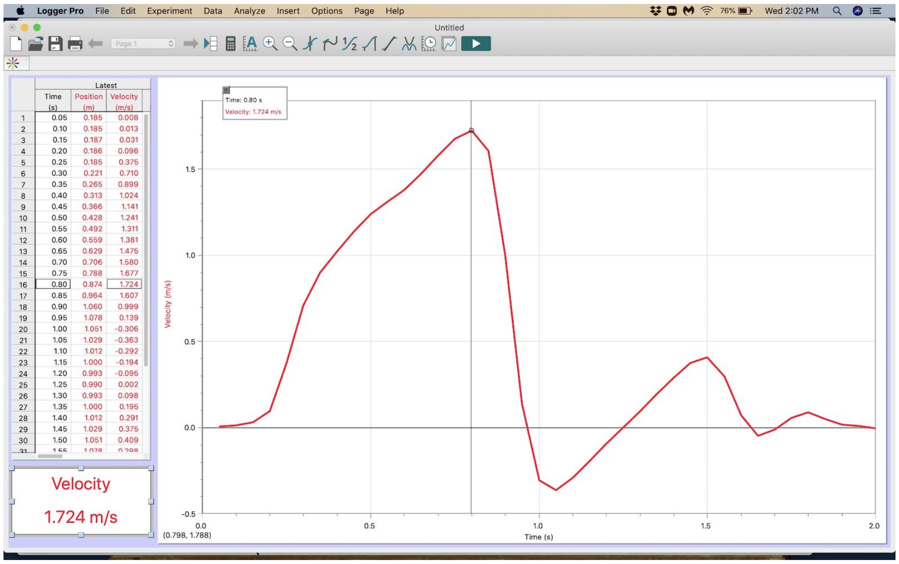
# Height 24 cm

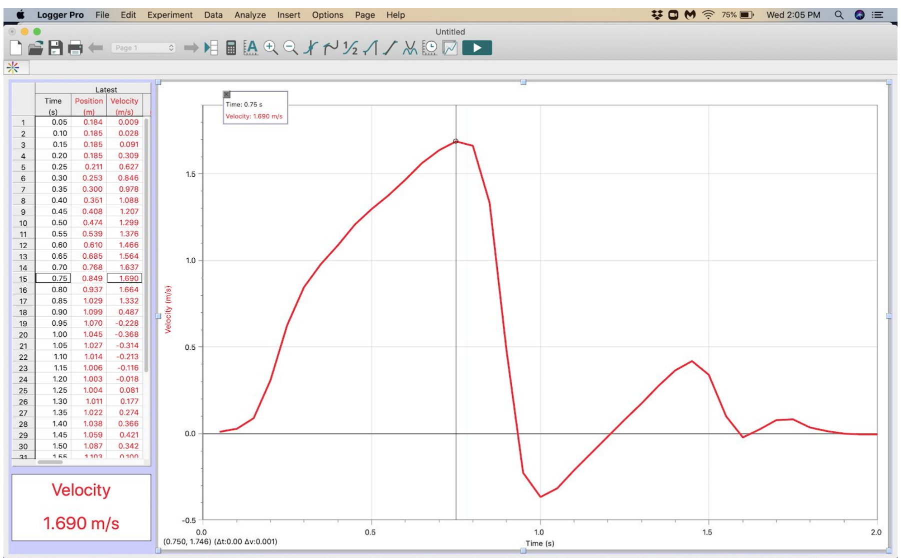

Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:08 PM
Untitled
Latest
Time (s) | Position (m) | Velocity (m/s)
1 0.05 0.196 -0.054
2 0.10 0.195 -0.085
3 0.15 0.185 -0.075
4 0.20 0.185 -0.013
5 0.25 0.185 0.050
6 0.30 0.185 0.213
7 0.35 0.200 0.491
8 0.40 0.237 0.724
9 0.45 0.278 0.852
10 0.50 0.323 0.947
11 0.55 0.372 1.041
12 0.60 0.428 1.111
13 0.65 0.483 1.186
14 0.70 0.546 1.272
15 0.75 0.611 1.340
16 0.80 0.680 1.410
17 0.85 0.752 1.481
18 0.90 0.829 1.552
19 0.95 0.907 1.581
20 1.00 0.991 1.461
21 1.05 1.079 0.850
22 1.10 1.086 0.047
23 1.15 1.059 -0.321
24 1.20 1.039 -0.354
25 1.25 1.022 -0.286
26 1.30 1.010 -0.204
27 1.35 1.002 -0.122
28 1.40 0.998 -0.037
29 1.45 0.998 0.045
30 1.50 1.003 0.126
Velocity
1.581 m/s
Time: 0.95 s
Velocity: 1.581 m/s
Velocity (m/s)
Time (s)
(0.951, 2.050) (Δt:0.00 Δv:0.001)

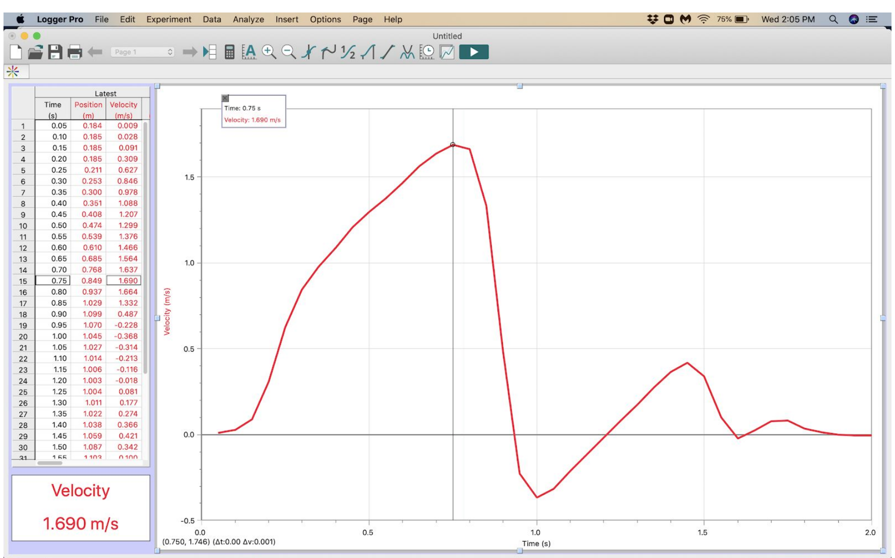

Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:08 PM
Untitled
Latest
Time (s) | Position (m) | Velocity (m/s)
1 0.05 0.185 0.013
2 0.10 0.185 0.021
3 0.15 0.186 0.059
4 0.20 0.186 0.192
5 0.25 0.198 0.469
6 0.30 0.235 0.715
7 0.35 0.276 0.845
8 0.40 0.320 0.943
9 0.45 0.369 1.044
10 0.50 0.426 1.125
11 0.55 0.483 1.186
12 0.60 0.543 1.255
13 0.65 0.608 1.326
14 0.70 0.676 1.396
15 0.75 0.747 1.485
16 0.80 0.825 1.571
17 0.85 0.905 1.582
18 0.90 0.987 1.453
19 0.95 1.075 0.864
20 1.00 1.085 0.066
21 1.05 1.058 -0.317
22 1.10 1.037 -0.357
23 1.15 1.021 -0.294
24 1.20 1.008 -0.211
25 1.25 1.000 -0.126
26 1.30 0.995 -0.042
27 1.35 0.995 0.039
28 1.40 0.999 0.118
29 1.45 1.007 0.197
30 1.50 1.019 0.278
Velocity
1.582 m/s
Time: 0.85 s
Velocity: 1.582 m/s
Velocity (m/s)
Time (s)
(0.851, 1.972) (Δt:0.00 Δv:0.001)


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
Wed 2:07 PM
Untitled
Latest
Time (s) | Position (m) | Velocity (m/s)
1 0.05 0.200 -0.218
2 0.10 0.185 -0.105
3 0.15 0.187 -0.007
4 0.20 0.190 0.010
5 0.25 0.186 0.069
6 0.30 0.185 0.327
7 0.35 0.217 0.628
8 0.40 0.256 0.793
9 0.45 0.299 0.896
10 0.50 0.346 0.985
11 0.55 0.397 1.085
12 0.60 0.455 1.165
13 0.65 0.514 1.226
14 0.70 0.577 1.296
15 0.75 0.644 1.366
16 0.80 0.714 1.432
17 0.85 0.786 1.512
18 0.90 0.865 1.585
19 0.95 0.947 1.556
20 1.00 1.033 1.236
21 1.05 1.097 0.435
22 1.10 1.068 -0.219
23 1.15 1.047 -0.347
24 1.20 1.029 -0.311
25 1.25 1.016 -0.226
26 1.30 1.007 -0.145
27 1.35 1.001 -0.063
28 1.40 1.000 0.021
29 1.45 1.003 0.104
30 1.50 1.011 0.184
Velocity
1.585 m/s
Time: 0.90 s
Velocity: 1.585 m/s
Velocity (m/s)
Time (s)
(0.902, 1.831) (Δt:0.00 Δv:0.001)

# Height 28 cm


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
76%
Wed 2:02 PM
Untitled
Page 1
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.185 0.008
2 0.10 0.185 0.013
3 0.15 0.187 0.031
4 0.20 0.186 0.096
5 0.25 0.185 0.375
6 0.30 0.221 0.710
7 0.35 0.265 0.899
8 0.40 0.313 1.024
9 0.45 0.366 1.141
10 0.50 0.428 1.241
11 0.55 0.492 1.311
12 0.60 0.559 1.381
13 0.65 0.629 1.475
14 0.70 0.706 1.580
15 0.75 0.788 1.677
16 0.80 0.874 1.724
17 0.85 0.964 1.607
18 0.90 1.060 0.999
19 0.95 1.078 0.139
20 1.00 1.051 -0.306
21 1.05 1.029 -0.363
22 1.10 1.012 -0.292
23 1.15 1.000 -0.194
24 1.20 0.993 -0.095
25 1.25 0.990 0.002
26 1.30 0.993 0.098
27 1.35 1.000 0.195
28 1.40 1.012 0.291
29 1.45 1.029 0.375
30 1.50 1.051 0.409
Velocity
1.724 m/s
Time: 0.80 s
Velocity: 1.724 m/s
Velocity (m/s)
1.5
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
Time (s)
(0.798, 1.788)


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
75%
Wed 2:05 PM
Untitled
Page 1
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.184 0.009
2 0.10 0.185 0.028
3 0.15 0.185 0.091
4 0.20 0.185 0.309
5 0.25 0.211 0.627
6 0.30 0.253 0.846
7 0.35 0.300 0.978
8 0.40 0.351 1.088
9 0.45 0.408 1.207
10 0.50 0.474 1.299
11 0.55 0.539 1.376
12 0.60 0.610 1.466
13 0.65 0.685 1.564
14 0.70 0.768 1.637
15 0.75 0.849 1.690
16 0.80 0.937 1.664
17 0.85 1.029 1.332
18 0.90 1.099 0.487
19 0.95 1.070 -0.228
20 1.00 1.045 -0.368
21 1.05 1.027 -0.314
22 1.10 1.014 -0.213
23 1.15 1.006 -0.116
24 1.20 1.003 -0.018
25 1.25 1.004 0.081
26 1.30 1.011 0.177
27 1.35 1.022 0.274
28 1.40 1.038 0.366
29 1.45 1.059 0.421
30 1.50 1.087 0.342
Velocity
1.690 m/s
Time: 0.75 s
Velocity: 1.690 m/s
Velocity (m/s)
1.5
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
Time (s)
(0.750, 1.746) (Δt:0.00 Δv:0.001)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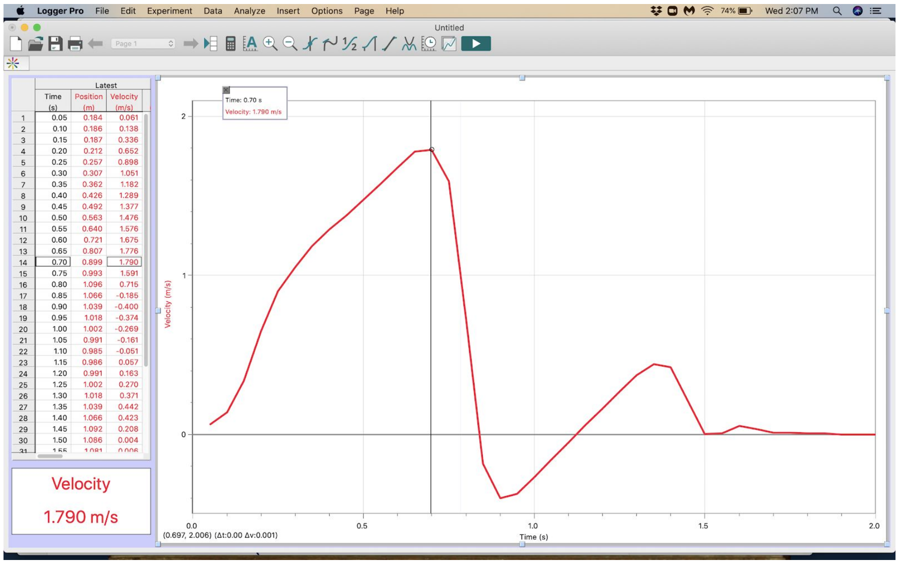
75%
Wed 2:05 PM
Untitled
Page 1
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.184 0.009
2 0.10 0.185 0.028
3 0.15 0.185 0.091
4 0.20 0.185 0.309
5 0.25 0.211 0.627
6 0.30 0.253 0.846
7 0.35 0.300 0.978
8 0.40 0.351 1.088
9 0.45 0.408 1.207
10 0.50 0.474 1.299
11 0.55 0.539 1.376
12 0.60 0.610 1.466
13 0.65 0.685 1.564
14 0.70 0.768 1.637
15 0.75 0.849 1.690
16 0.80 0.937 1.664
17 0.85 1.029 1.332
18 0.90 1.099 0.487
19 0.95 1.070 -0.228
20 1.00 1.045 -0.368
21 1.05 1.027 -0.314
22 1.10 1.014 -0.213
23 1.15 1.006 -0.116
24 1.20 1.003 -0.018
25 1.25 1.004 0.081
26 1.30 1.011 0.177
27 1.35 1.022 0.274
28 1.40 1.038 0.366
29 1.45 1.059 0.421
30 1.50 1.087 0.342
Velocity
1.690 m/s
Time: 0.75 s
Velocity: 1.690 m/s
Velocity (m/s)
1.5
1.0
0.5
0.0
-0.5
0.0
0.5
1.0
1.5
2.0
Time (s)
(0.750, 1.746) (Δt:0.00 Δv:0.001)

# Height 31 cm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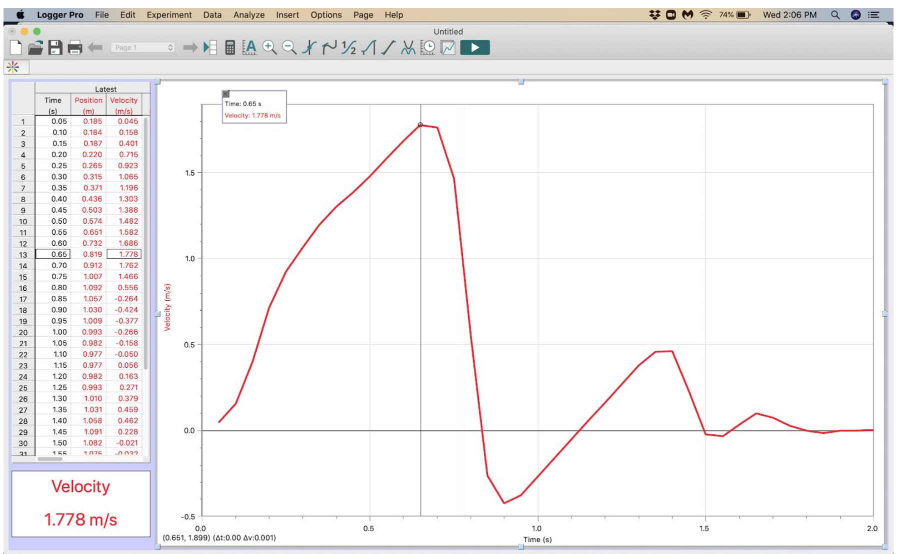
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.184 0.061
2 0.10 0.186 0.138
3 0.15 0.187 0.336
4 0.20 0.212 0.652
5 0.25 0.257 0.898
6 0.30 0.307 1.051
7 0.35 0.362 1.182
8 0.40 0.426 1.289
9 0.45 0.492 1.377
10 0.50 0.563 1.476
11 0.55 0.640 1.576
12 0.60 0.721 1.675
13 0.65 0.807 1.776
14 0.70 0.899 1.790
15 0.75 0.993 1.591
16 0.80 1.096 0.715
17 0.85 1.066 -0.185
18 0.90 1.039 -0.400
19 0.95 1.018 -0.374
20 1.00 1.002 -0.269
21 1.05 0.991 -0.161
22 1.10 0.985 -0.051
23 1.15 0.986 0.057
24 1.20 0.991 0.163
25 1.25 1.002 0.270
26 1.30 1.018 0.371
27 1.35 1.039 0.442
28 1.40 1.066 0.423
29 1.45 1.092 0.208
30 1.50 1.086 0.004
Velocity
1.790 m/s
Time: 0.70 s
Velocity: 1.790 m/s
Velocity (m/s)
Time (s)
(0.697, 2.006) (Δt:0.00 Δv:0.001)


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
74% Wed 2:06 PM
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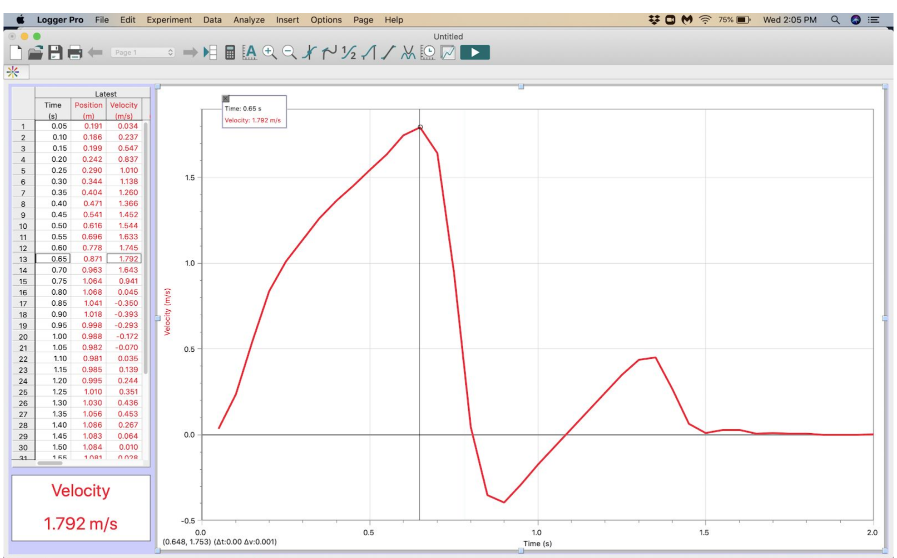
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.185 0.045
2 0.10 0.184 0.158
3 0.15 0.187 0.401
4 0.20 0.220 0.715
5 0.25 0.265 0.923
6 0.30 0.315 1.065
7 0.35 0.371 1.196
8 0.40 0.436 1.303
9 0.45 0.503 1.388
10 0.50 0.574 1.482
11 0.55 0.651 1.582
12 0.60 0.732 1.686
13 0.65 0.819 1.778
14 0.70 0.912 1.762
15 0.75 1.007 1.466
16 0.80 1.092 0.556
17 0.85 1.057 -0.264
18 0.90 1.030 -0.424
19 0.95 1.009 -0.377
20 1.00 0.993 -0.266
21 1.05 0.982 -0.158
22 1.10 0.977 -0.050
23 1.15 0.977 0.056
24 1.20 0.982 0.163
25 1.25 0.993 0.271
26 1.30 1.010 0.379
27 1.35 1.031 0.459
28 1.40 1.058 0.462
29 1.45 1.091 0.228
30 1.50 1.082 -0.021
Velocity
1.778 m/s
Time: 0.65 s
Velocity: 1.778 m/s
Velocity (m/s)
Time (s)
(0.651, 1.899) (Δt:0.00 Δv:0.001)


Logger Pro File Edit Experiment Data Analyze Insert Options Page Help
75% Wed 2:05 PM
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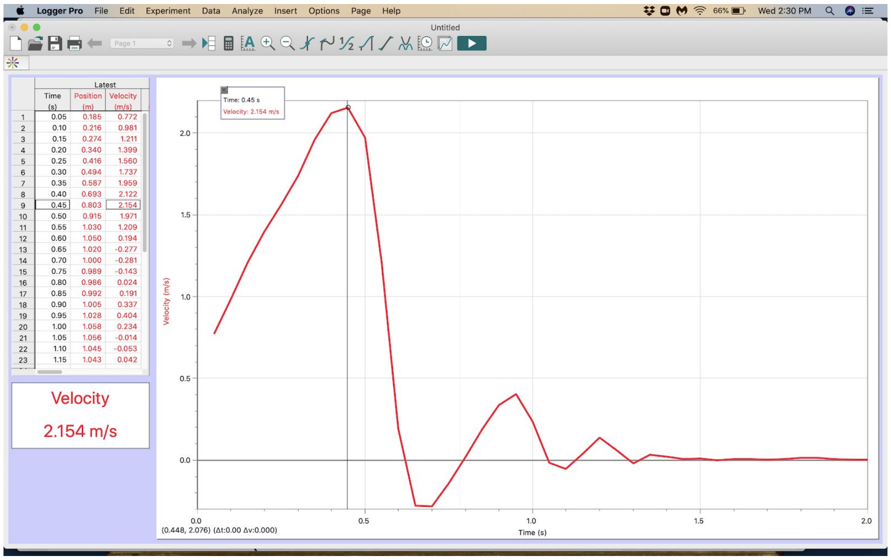
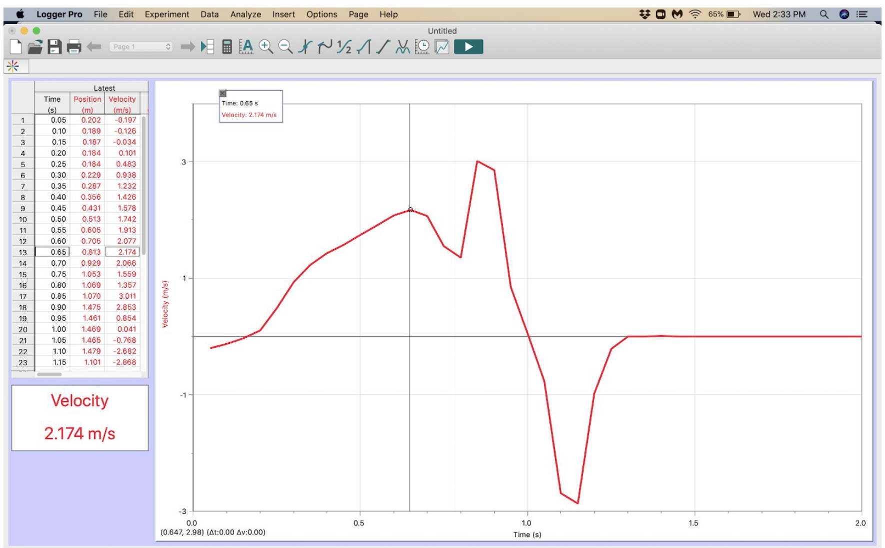
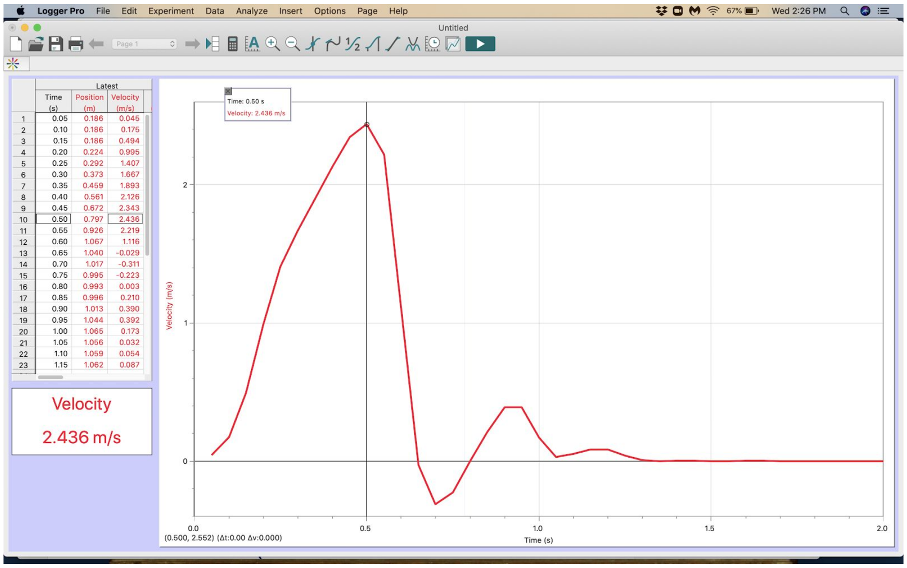
Latest
Time (s) Position (m) Velocity (m/s)
1 0.05 0.191 0.034
2 0.10 0.186 0.237
3 0.15 0.199 0.547
4 0.20 0.242 0.837
5 0.25 0.290 1.010
6 0.30 0.344 1.138
7 0.35 0.404 1.260
8 0.40 0.471 1.366
9 0.45 0.541 1.452
10 0.50 0.616 1.544
11 0.55 0.696 1.633
12 0.60 0.778 1.745
13 0.65 0.871 1.792
14 0.70 0.963 1.643
15 0.75 1.064 0.941
16 0.80 1.068 0.045
17 0.85 1.041 -0.350
18 0.90 1.018 -0.393
19 0.95 0.998 -0.293
20 1.00 0.988 -0.172
21 1.05 0.982 -0.070
22 1.10 0.981 0.035
23 1.15 0.985 0.139
24 1.20 0.995 0.244
25 1.25 1.010 0.351
26 1.30 1.030 0.436
27 1.35 1.056 0.453
28 1.40 1.086 0.267
29 1.45 1.083 0.064
30 1.50 1.084 0.010
Velocity
1.792 m/s
Time: 0.65 s
Velocity: 1.792 m/s
Velocity (m/s)
Time (s)
(0.648, 1.753) (Δt:0.00 Δv:0.001)

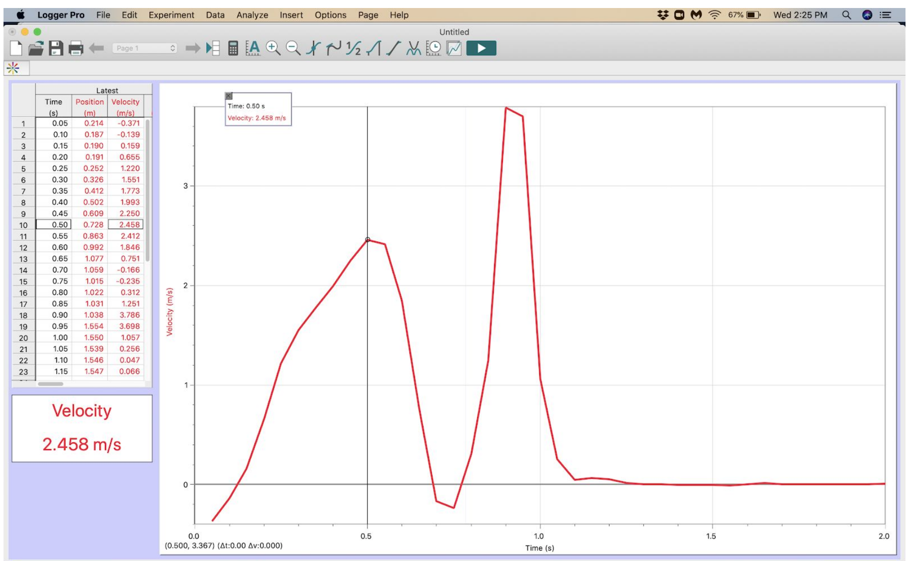
# Height 48 cm (there are only 2 trials for this height)

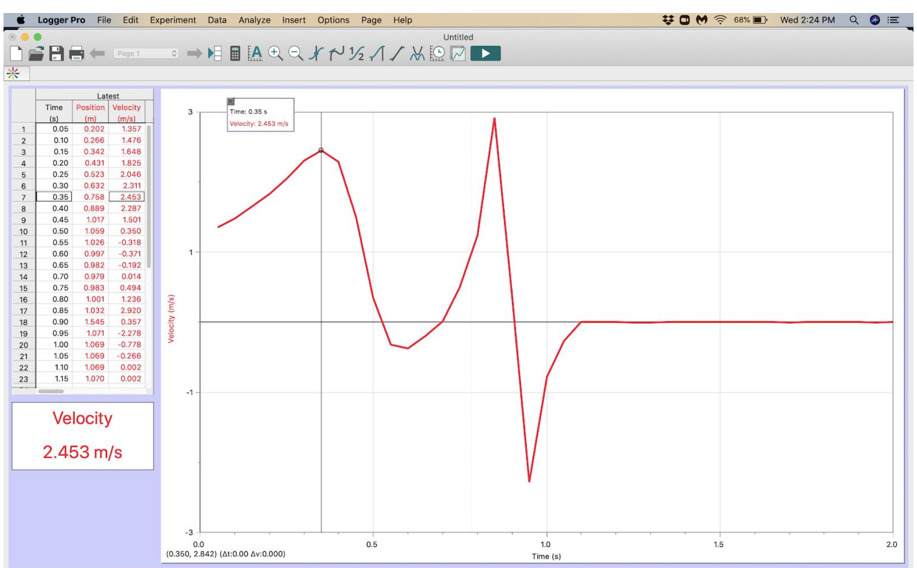


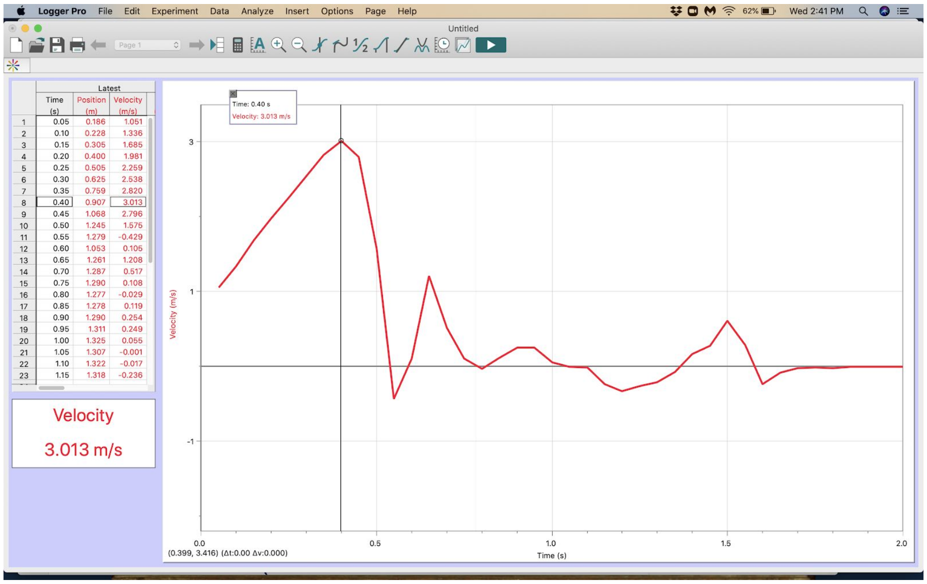
# Height 58 cm


Velocity
2.436 m/s
Velocity (m/s)
Time (s)

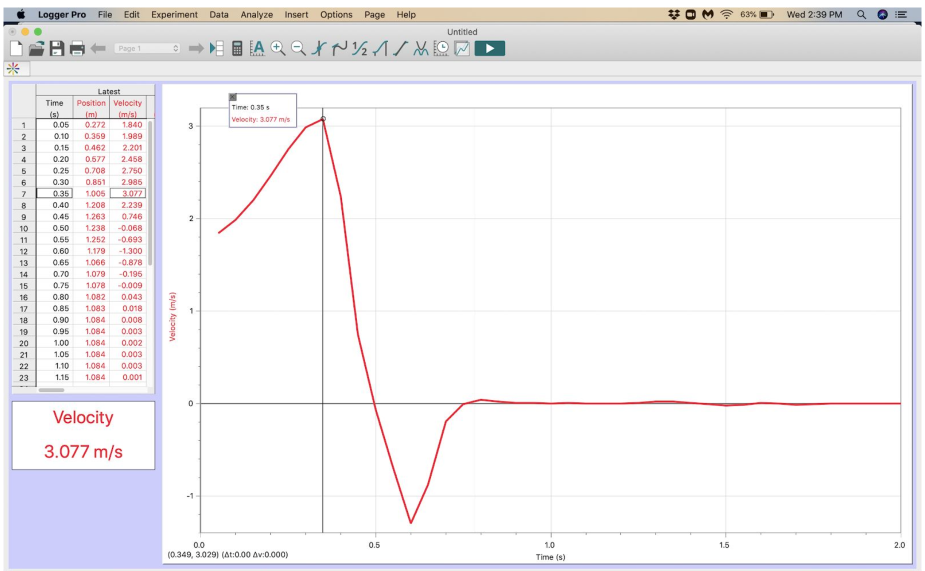

Velocity
2.458 m/s
Velocity (m/s)
Time (s)


Velocity
2.453 m/s
Velocity (m/s)
Time (s)

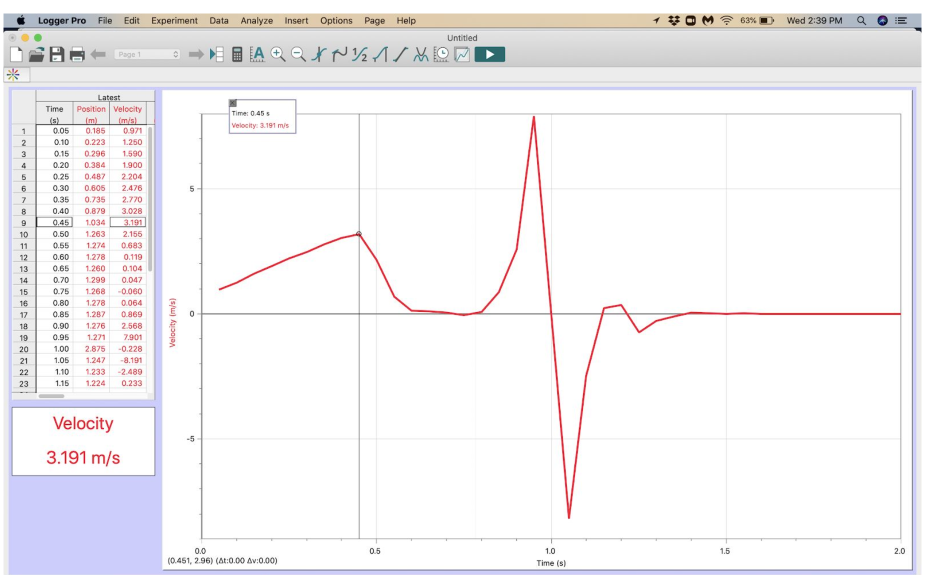
# Height 77 cm




| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 1 | 0.05 | 0.186 | 1.051 |
| 2 | 0.10 | 0.228 | 1.336 |
| 3 | 0.15 | 0.305 | 1.685 |
| 4 | 0.20 | 0.400 | 1.981 |
| 5 | 0.25 | 0.505 | 2.259 |
| 6 | 0.30 | 0.625 | 2.538 |
| 7 | 0.35 | 0.759 | 2.820 |
| 8 | 0.40 | 0.907 | 3.013 |
| 9 | 0.45 | 1.068 | 2.796 |
| 10 | 0.50 | 1.245 | 1.575 |
| 11 | 0.55 | 1.279 | -0.429 |
| 12 | 0.60 | 1.053 | 0.105 |
| 13 | 0.65 | 1.261 | 1.208 |
| 14 | 0.70 | 1.287 | 0.517 |
| 15 | 0.75 | 1.290 | 0.108 |
| 16 | 0.80 | 1.277 | -0.029 |
| 17 | 0.85 | 1.278 | 0.119 |
| 18 | 0.90 | 1.290 | 0.254 |
| 19 | 0.95 | 1.311 | 0.249 |
| 20 | 1.00 | 1.325 | 0.055 |
| 21 | 1.05 | 1.307 | -0.001 |
| 22 | 1.10 | 1.322 | -0.017 |
| 23 | 1.15 | 1.318 | -0.236 |




| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 1 | 0.05 | 0.272 | 1.840 |
| 2 | 0.10 | 0.359 | 1.989 |
| 3 | 0.15 | 0.462 | 2.201 |
| 4 | 0.20 | 0.577 | 2.458 |
| 5 | 0.25 | 0.708 | 2.750 |
| 6 | 0.30 | 0.851 | 2.985 |
| 7 | 0.35 | 1.005 | 3.077 |
| 8 | 0.40 | 1.208 | 2.239 |
| 9 | 0.45 | 1.263 | 0.746 |
| 10 | 0.50 | 1.238 | -0.068 |
| 11 | 0.55 | 1.252 | -0.693 |
| 12 | 0.60 | 1.179 | -1.300 |
| 13 | 0.65 | 1.066 | -0.878 |
| 14 | 0.70 | 1.079 | -0.195 |
| 15 | 0.75 | 1.078 | -0.009 |
| 16 | 0.80 | 1.082 | 0.043 |
| 17 | 0.85 | 1.083 | 0.018 |
| 18 | 0.90 | 1.084 | 0.008 |
| 19 | 0.95 | 1.084 | 0.003 |
| 20 | 1.00 | 1.084 | 0.002 |
| 21 | 1.05 | 1.084 | 0.003 |
| 22 | 1.10 | 1.084 | 0.003 |
| 23 | 1.15 | 1.084 | 0.001 |




| | Time (s) | Position (m) | Velocity (m/s) |
|---|---|---|---|
| 1 | 0.05 | 0.185 | 0.971 |
| 2 | 0.10 | 0.223 | 1.250 |
| 3 | 0.15 | 0.296 | 1.590 |
| 4 | 0.20 | 0.384 | 1.900 |
| 5 | 0.25 | 0.487 | 2.204 |
| 6 | 0.30 | 0.605 | 2.476 |
| 7 | 0.35 | 0.735 | 2.770 |
| 8 | 0.40 | 0.879 | 3.028 |
| 9 | 0.45 | 1.034 | 3.191 |
| 10 | 0.50 | 1.263 | 2.155 |
| 11 | 0.55 | 1.274 | 0.683 |
| 12 | 0.60 | 1.278 | 0.119 |
| 13 | 0.65 | 1.260 | 0.104 |
| 14 | 0.70 | 1.299 | 0.047 |
| 15 | 0.75 | 1.268 | -0.060 |
| 16 | 0.80 | 1.278 | 0.064 |
| 17 | 0.85 | 1.287 | 0.869 |
| 18 | 0.90 | 1.276 | 2.568 |
| 19 | 0.95 | 1.271 | 7.901 |
| 20 | 1.00 | 2.875 | -0.228 |
| 21 | 1.05 | 1.247 | -8.191 |
| 22 | 1.10 | 1.233 | -2.489 |
| 23 | 1.15 | 1.224 | 0.233 |

# Unit 8- Energy
## Part 5
## Gravitational (Potential) Energy
## and
## Kinetic Energy

# Gravitational (Potential) Energy

Gravitational Energy is energy that is stored in a field. This is because $E_g$ is caused by the gravitational force which is a non-contact force. The gravitational Force ($\vec{F_g} = m\vec{g}$, where $\vec{g} = -10\,m/s^2$) is constant for a given mass near the surface of Earth.

While the gravitational force is approximately constant near the surface of Earth, the gravitational energy increases with distance from the surface of the Earth. This is similar to elastic potential energy – the further you stretch the spring the more energy it will have to snap back to its original length.

$$E_{el} = F_{el} \cdot \Delta x$$

Similarly...

$$E_g = F_g \cdot \Delta y$$

$$E_g = mg \cdot \Delta y$$

$$\boxed{E_g = mgh}$$

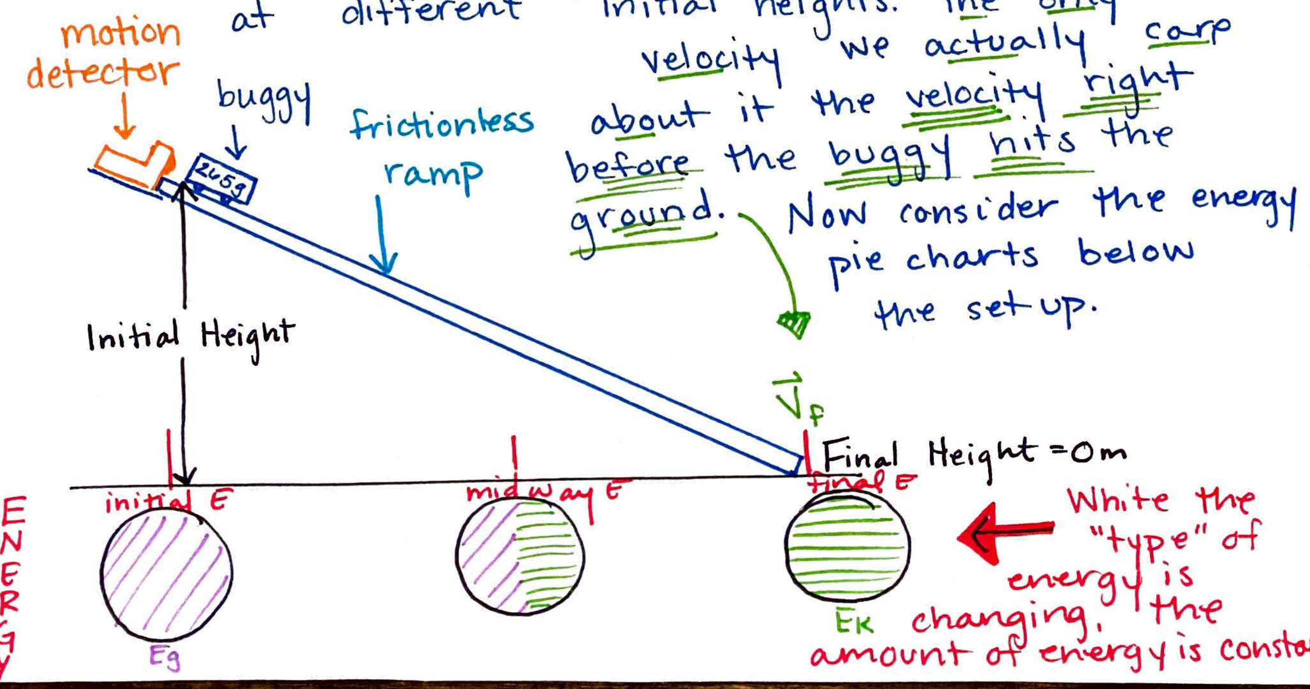
This is most often used, with $h$ = height = $\Delta y$

# Kinetic Energy

Finding an equation for kinetic energy is not going to be as simple. To do so we're going to do another lab.

We know $E_K$ is the energy of objects in motion (just like kinematics describes the motion of objects). So to find an equation for $E_K$, we will look at objects with different velocities. But here's the tricky part - how can we measure the energy associated to each velocity? The solution is... CLEVERNESS! And more specifically, the CONSERVATION OF ENERGY.

Here is our lab set up. We are measuring the velocity of a buggy on a frictionless ramp at different initial heights. The only velocity we actually carp about it the velocity right before the buggy hits the ground. Now consider the energy pie charts below the set up.



As the pie charts below the set up show, the energy before the car is released is ALL gravitational energy (and depends on height), and the energy right before the buggy hits the ground is ALL kinetic. Because energy is conserved and because this is a frictionless ramp ($\therefore$ no thermal energy transfer) we can say

$$E_{initial} = E_{final}$$

and ,importantly, $\boxed{E_{g,initial} = E_{K,final}.}$

We know how to calculate $E_g$ ($E_g = mgh$) and we can easily measure the initial height to make this calculation. So, once we know the initial $E_g$ WE ALSO KNOW THE $E_K$ at the bottom of the ramp. Using our data from the motion detector, we can find the velocity at the bottom of the ramp.

Now we have a way to relate Kinetic Energy and velocity! By collecting data at different heights of the ramp (and $\therefore$ different initial $E_g$ ... and $\therefore$ final $E_K$) we can come up with an equation!

I'll help you set up your lab.

# Kinetic Energy versus Velocity Lab

Purpose: To find and understand the equation for Kinetic Energy and Kinetic Energy's relationship to motion (velocity).

Variables and Constants:

- Independent Variable: velocity
- Defendent Variable: Kinetic Energy
- Constants:
  - mass of the car (265 g)
  - length of ramp
  - position of motion detector

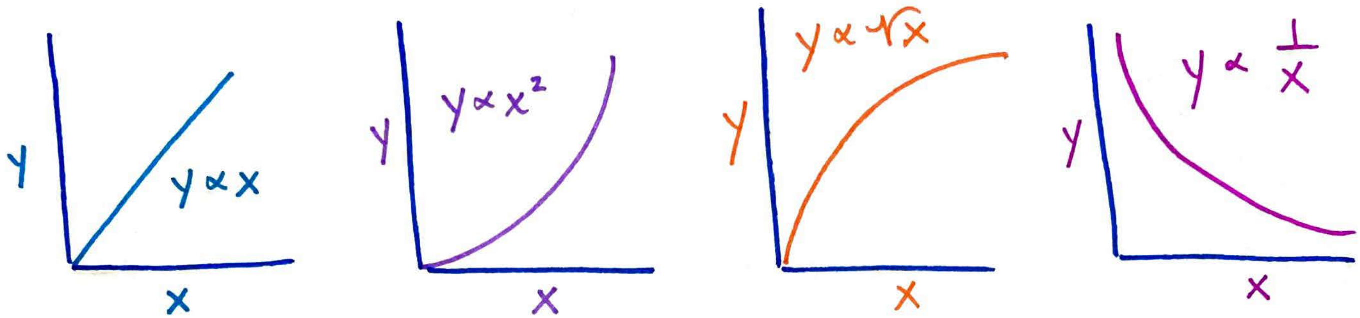
Technically in this lab we are controlling the Kinetic Energy NOT the velocity. And we're controlling $E_K$ by controlling $E_g$, which is determined by height.

In your lab you will be taking measurements of velocity (IV) and height (used to calculate energy - DV).

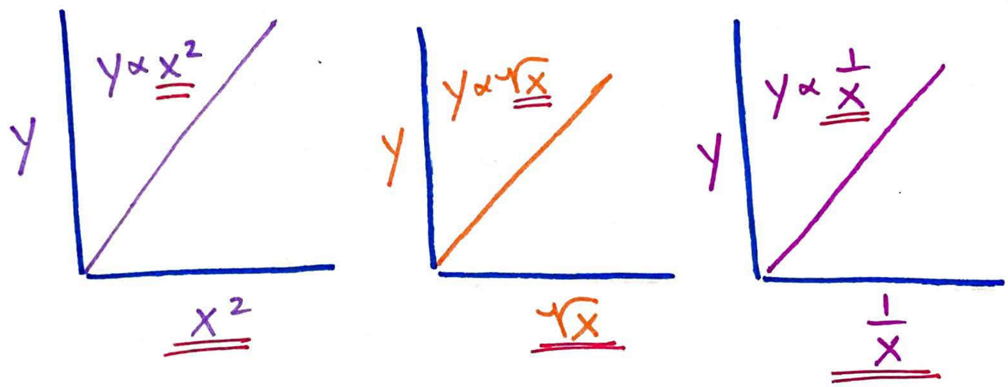
Your graph will have $E_K$ on the vertical axis and velocity on the horizontal axis.

Spoiler Alert! Your graph is not going to be linear! You will have to linearize your graph. Let me give you a reminder.

Common shapes of graphs include

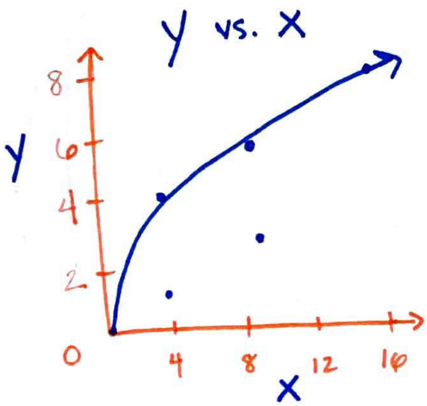



To Linearize, you recalculate the horizontal axis variables to match the shape of the graph.

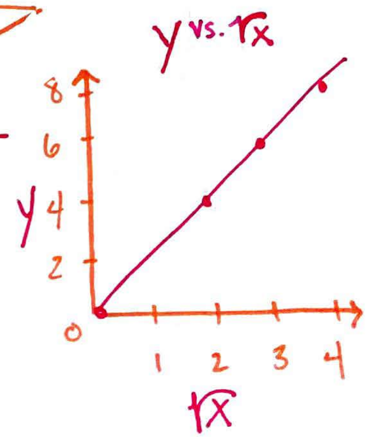



## FOR EXAMPLE

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 4 | 4 |
| 9 | 6 |
| 16 | 8 |




| $\sqrt{x}$ | $y$ |
|---|---|
| $\sqrt{0} = 0$ | 0 |
| $\sqrt{4} = 2$ | 4 |
| $\sqrt{9} = 3$ | 6 |
| $\sqrt{16} = 4$ | 8 |



GreatHearts Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: 11 Precalculus
**Teacher(s)**: Mr. Simmons

**Weekly Plan**:

Monday, May 4
☐ Story time!
☐ Problems 11-14 and 16 from “Relationship between Trig Functions”

Tuesday, May 5
☐ Read “Radian measure.”

Wednesday, May 6
☐ Problems 1-14

Thursday, May 7
☐ Read “Introduction to the Polar Plane

Friday, May 8
☐ Attend office hours
☐ Catch up or review the week’s work

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

_______________________________
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

_______________________________
Parent Signature

## Monday, May 4

1. Story time! If technologically feasible, email me with a story!

Also, please email me if you have questions about trigonometry. BUT. Please make your questions specific. At least give me a page number where you got confused. Better yet, tell me exactly what words you read that confused you.

For today, we're going to do a few more problems from "Relationships between Trig Functions" before moving on to radians:

2. Complete Problems 11-14 and 16 on pp. 122-123.

## Tuesday, May 5

1. Read "Radian Measure" on pp. 124-133.

## Wednesday, May 6

1. Complete Problems 1-14 on pp. 133-135.

## Thursday, May 7

1. Read "Introduction to the Polar Plane" on pp. 135-143.

2.) Let us complete Example 1c. If $\sin\alpha = a$, what are the six Trig ratios?
3.) Now let $\tan\alpha = d$.
   (A) What are the six Trig ratios given this?
   (B) Compare this answer to the previous.
4.) In Example 1c, we let the length of the hypotenuse of the triangle to equal 1. Is that OK? Why don't we let the hypotenuse equal $c$, to allow for any and all possibilities? Now suppose $\sin\alpha = \frac{x}{c}$, and the lengths of your triangle are $a, b$, and $c$, with the hypotenuse equaling $c$.
   (A) What are the six Trig ratios?
   (B) Compare this with the previous two results.
5.) Given the Trig functions and angle measure, write the equivalent cofunction.
   (A) $\sin 30°$
   (B) $\cos 10°$
   (C) $\cot 7°$
   (D) $\sec 64°$
   (E) $\cos 31°$
   (F) $\tan 14°$
   (G) $\csc 47°$
   (H) $\sin 25°$
   (I) $\tan(\beta + \gamma)$
   (J) $\sin\beta$
6.) Write out all of the cofunction identities, including the ones we discovered in the reading. Hint: There are six of them.
7.) Now write out all of the reciprocal identities. Hint: There are six of them.
8.) Evaluate the following.
   (A) $\sin^2 30°$
   (B) $\cos^2 30°$
   (C) $\tan^2 30°$
   (D) $\cos^2 45°$
   (E) $\tan^2 45°$
   (F) $\sin^2 60°$
   (G) $\cos^2 60°$
   (H) $\tan^2 60°$
   (I) $\sin(30°^2)$
   (J) $\cos^2\alpha$
9.) Write out a table of values for $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$, starting at $\alpha = 0$, going up by 5° each row, and ending at $\alpha = 90°$. You will need a calculator for this Exercise.
10.) Use your results from the previous Exercise to answer the following questions.
   (A) What is the maximum value of $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$?
   (B) What is the minimum value of $\sin^2\alpha$, $\cos^2\alpha$, and $\tan^2\alpha$?
   (C) Are there any similarities or differences between $\sin^2\alpha$ and $\sin\alpha$? Compare your results from the previous section.
11.) One of the most important relationships in Trigonometry is the Pythagorean Identity we discussed in the reading. Write this identity down now.
12.) Evaluate the following.
   (A) $\sin^2 60° + \cos^2 60°$
   (B) $\cos^2 30° + \sin^2 30°$
   (C) $\sin^2\left(\frac{\alpha}{2}\right) + \cos^2\left(\frac{\alpha}{2}\right)$
   (D) $\sin^2(3\alpha + \pi) + \cos^2(3\alpha + \pi)$
13.) It is often helpful to rewrite $\sin^2\alpha$ or $\cos^2\alpha$. Use the Pythagorean Identity to rewrite $\sin^2\alpha$ and $\cos^2\alpha$.
14.) Simplify the following.



   (A) $\tan\alpha \cdot \csc\alpha$
   (B) $(\sin\alpha + \cos\alpha)^2$
15.) Are there any other Pythagorean Identities? To find this out, use a calculator and try the following for different values of $\alpha$.
   (A) $\sec^2\alpha + \csc^2\alpha$
   (B) $\tan^2\alpha + \cot^2\alpha$
   (C) There are two other Pythagorean Identities. First, using the previous two, guess what they might be. Then, if you can't figure it out, look them up and write them down now. We'll discover how to arrive at these results when we have some better tools.
16.) Answer True or False.
   (A) $\sin\alpha = \cos(\alpha - 90°)$
   (B) $\sin^2\alpha + \cos^2\beta = 1$ iff $\alpha + \beta = 90°$
   (C) $\sin^2\alpha = \sin\alpha \cdot \alpha$
   (D) $\sin^2\alpha$ is sometimes negative.[vi]

[vi] Assume $\alpha \in \mathbb{R}$.

# Unit five

Radians and the Unit Circle

"Degrees are fine for everyday measurements. But Trigonometry marks a turning point in math, when the student lifts his gaze from the everyday towards larger, more distant ideas. You begin exploring basic relationships, deep symmetries, the kinds of patterns that make the universe tick. And to navigate that terrain, you need a notion of angles that's more natural, more fundamental, than slicing up the circle into an arbitrary number of pieces. The number π, strange though it may seem, lies at the heart of mathematics. The number 360 doesn't. Clinging to that Babylonian artifact will only distract you and obscure the elegant truths you're searching for."

Ben Orlin



Up to this point, we've measured all of our angles using degrees. In this unit, we'll endeavor to find a different and perhaps better method of measuring angles. Then we'll use that to graph points in a new type of plane. Finally, after this, we introduce perhaps the most important thing in Trigonometry: The Unit Circle.

## §1 Radian measure

Degrees were invented millennia ago, perhaps by the ancient peoples living in modern day Iraq. Knowing the origins of this unit could shed some light on its usefulness, and whether there isn't a more useful unit to use.

There are various theories as to why degrees were used and why they are the way that are. Almost certainly, however, it has to do with a circle. As with anything, it's often useful to consider portions or fractions of the whole.[i] The ancients chose to chop the circle up into 360 equal portions, calling the angle created by each portion a degree, as (partially) shown in Figure 47.

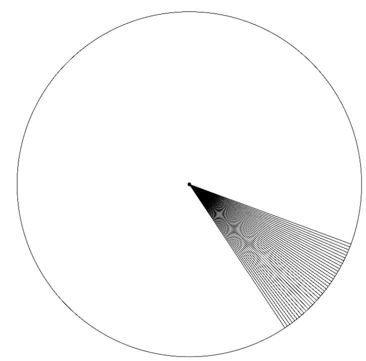


Figure 47

Each of the individual spokes above measures a single degree. If we were to continue creating these spokes, there would be 360 of them.

Why 360? Perhaps because it is a nice number with many factors. So cutting a circle in half gives you a nice number of 180°, in thirds 120°, fourths 90°, and so on. This means

[i] This is why, for example, we have yards. Could you imagine measuring things if the smallest unit we could get was miles? And even that isn't enough, which is why continue to subdivide the units smaller and smaller.

that commonly used ratios are left with a whole number. This wouldn't be the case if the number, say, 10, was used. Then only a half-circle and a fifth-of-a-circle would have whole numbers. Another supposition is that there are approximately 360 days in a year. And since, each year, seasons repeat themselves, a circle makes a nice representation of a calendar.

Whatever the reason, however, we want to see if there is a better way of measuring angles. Of course, "better" is relative, and different situations might call for different units. So when we say "better," perhaps what we should say is more appropriate for our work in Trigonometry.

Consider the circle shown in Figure 48. What is the length of the **arc** from $A$ to $B$?

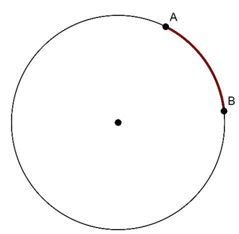



Figure 48

There are a few ways we could answer this question. One is to measure it the old-fashioned way. That, however, leaves room for error, and wouldn't help us to measure an arc from a different circle. Another way we could do it is to find the circumference of the circle, then multiply by the fraction of the outside of the circle represented by $\widehat{AB}$.[ii] This isn't the worst thing in the world, but then… How will we measure the angle which will allow us to find the fraction of the outside of the circle that $\widehat{AB}$ takes up? As you can see, we have a bit of an issue.

As we've done a few times in this course, we should go back to what we know for certain. We know that the circumference of a circle is

$$C = 2\pi r,$$

[ii] For example, if the arc were half of the outside of the circle, you would multiply the circumference of the circle by $\frac{1}{2}$, right?



where $r$ is the radius of a circle and $\pi$ is the mathematical constant approximately equal to 3.14. We also know that every radius in a circle is congruent. And that's about it. But this does show us that if we're trying to figure stuff out about a circle, it is usually a good idea to involve a radius. That's what we'll do in Figure 49.

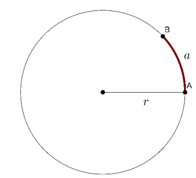



Figure 49

In keeping with our tradition, we've used the Latin letters $r$ and $a$ for the lengths of the radius and arc respectively.

Now, let's see what happens when we relate the radius to the arc length. Let's assume for a moment that $r = a$, i.e., that the radius is the same length as $\widehat{AB}$. This would allow us to create that angle seen in Figure 50, right?

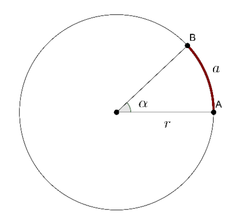



Figure 50

This angle, which we'll call $\alpha$, is unique. In other words, there is one and only one angle for which the radius is the same length as $\widehat{AB}$. Figure 51 shows this to be true.

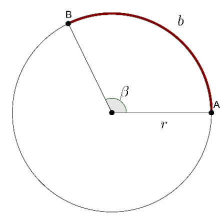



Figure 51

Here, $b = 1.5r$. As a consequence of this increased arc size, the angle is larger, and therefore $\alpha \neq \beta$.

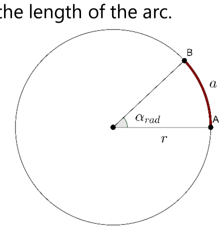
This is interesting for a couple of reasons. First of all, notice that there is one and only one angle that comes out as a consequence of the comparison to the radius and arc length. Secondly, and perhaps more importantly, the size of the circle (and, by extension), the lengths of the radius and arc, won't matter.[iii] Thus we make the following definition.

Radian measure

The angle formed by the ratio of the arc length $a$ to the radius $r$ in a circle. Symbolically,

$$\alpha_{rad} = \frac{a}{r},$$

where $r$ is the radius and $a$ is the length of the arc.




Example 1a

What does an angle measure of 1 radian look like?

This is an excellent question, and a great starting point to see the intuition behind radians. A radian is a ratio between the arc length and the radius, or $\frac{a}{r}$, where $r$ is the radius of a circle and $a$ is the length of the arc we're concerned with. Using our definition, then, we have

$$1 = \frac{a}{r}.$$

But this will only be true if $a = r$. Therefore, a measure of 1 radian will look like Figure 52, where the radius and $\widehat{AB}$ are equal.

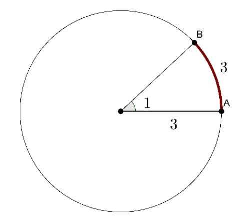



Figure 52

We've chosen a radius and arc length of 3, but we could have easily chosen any other length so long as $r = a$.

Notice that our angle measure is 1? You might be wondering what the units of this angle measure are, but there aren't any! You could say 1 radian, but if an angle measure is reported with no unit, it is assumed to be measured in radians.[iv]

Did you notice that our circle had a radius and arc length of 3? We wanted an angle of 1, and that is only true when $r = a$. So we could have also chosen $r = a = 5$, or $r = a = 100$, (and so on) if we wanted to. Do you see why?

Example 1b

What does an angle of 2 look like?

This is a similar question, so we again go back to the definition. The equation

$$2 = \frac{a}{r}$$

must be true. There is an infinite amount of possibilities for both $a$ and $r$, such as $a = 6, r = 3$ (which we show in Figure 53).

[iii] We'll show this explicitly in the forthcoming Examples.

[iv] This is another reason to prefer radians.

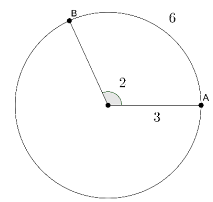



Figure 53

As you can see, the ratio is what's important. The fact that the arc length is twice the length of the radius is what tells us we have an angle measure of 2. Appreciate, also, how the angle is clearly different from the previous Example.

The previous two examples were there to help you get a grasp on radians, but we still haven't seen its best feature. We explore that now.

Example 2a

What is the length of the radius given Figure 54?

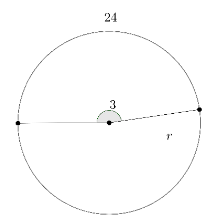



Figure 54

Now this is interesting. We are given an angle and an arc length, and are told to find the length of the radius. Using the definition of a radian, we can work backward and easily get the answer. Since, according to our definition, we have

$$3 = \frac{24}{r},$$

we simply solve the previous equation for $r$ and get

$$r = 8.$$

Easy! But to really appreciate radians, consider Figure 55, where we have used degrees instead.



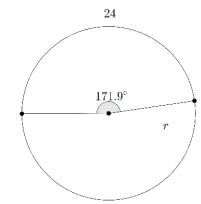



Figure 55

Could we find $r$? The answer is no – degrees are a measurement found completely independent of the size of a circle. Thus it offers us no help at all. Radians, therefore, give us free information!

Example 2b

What is the length of the arc subtended[v] by an angle of 6 and a radius of 10?

No picture is provided, and it would be helpful for you draw one, but it is not necessary. We simply use the definition:

$$6 = \frac{a}{10}.$$

Hence

$$a = 60.$$

There are a few more important questions which must be asked if we are to succeed with radians. For example, how many radians are in a full rotation? We know there are 360° in a full rotation, but what about radians?

Let us answer this question with a specific circle, and then generalize afterwards. Consider a circle with a radius of 1.[vi] Since the definition of a radian tells us that

$$\alpha_{rad} = \frac{a}{r},$$

And we have $r = 1$, we have

---

[v] This is a fancy, perhaps old-fashioned word which means formed or created by. So the arc is created by the angle.

[vi] We could have chosen any value for the radius, but we chose 1. Any thoughts on why we would choose this number and not, say, 23?

$$\alpha_{rad} = a.$$

If we are considering a full rotation, however, we are not looking at an arc, but the full circumference of the circle. Therefore, $a = 2\pi$ and hence

$$\alpha_{rad} = 2\pi.$$

This is an important fact, and we list it below for your convenience.

| |
|---|
| Radians in various rotations<br><br>Full rotation: $2\pi$<br>Half rotation: $\pi$<br>Quarter rotation: $\frac{\pi}{2}$ |

Thus, there are $2\pi$ radians in a full rotation.

This allows us to answer our next most important question: How do radians relate to degrees? In other words, how does one convert from one to the other?

To find this answer, let us find out how many degrees are in one radian. To do this, we just need to convert. We will use dimensional analysis to help us do this, as we show below.

$$\frac{360°}{1\ \cancel{\text{rotation}}} \left| \frac{1\ \cancel{\text{rotation}}}{2\pi \text{ radians}} \right| = \left| \frac{180°}{\pi \text{ radians}} \right| \approx \left| \frac{57.30°}{1 \text{ radian}} \right.$$

The rotations cancel, and leave us with our result of approximately 57.3° for every 1 radian.

This is a strange number, and we will rarely use it. Instead, the fraction $\frac{180}{\pi}$ is what you should memorize and become comfortable with. That said, it is helpful to know how much 1 radian is in degrees, since it will help you get a picture of what you're working with.

Example 3a

Convert 6 radians into degrees.

Since there are $\frac{180}{\pi}$ degrees for every one radian, we simply multiply this number by 6. Thus 6 radians is

$$343.8°.$$

Example 3b

Convert 75° into radians.

This is the opposite of the previous problem. Thus, we need a new conversion factor. We apply the same principle to obtain a conversion factor:



$$\frac{2\pi}{1\ \cancel{\text{rotation}}} \left| \frac{1\ \cancel{\text{rotation}}}{360°} \right| = \left| \frac{\pi}{180} \right.$$

This tells us that one degree is $\frac{\pi}{180}$ radians. And since we want to find out how many radians 75° is, we just multiply the previous by $\frac{\pi}{180}$. We get

$$75 \cdot \frac{\pi}{180} = \frac{75\pi}{180} = \frac{5\pi}{12}.$$

When working with radians, we never want an approximation.

Did you notice how similar the conversion factors were? You should have them memorized, as you'll need them in this section and beyond.

**§1 Exercises**

1.) Determine the measure of $\alpha$ (in radians) given the following radii and arc lengths.
   - (A) $r = 10, \alpha = 20$
   - (B) $r = 30, \alpha = 10$
   - (C) $r = 15, \alpha = 100$
   - (D) $r = \frac{3}{4}, \alpha = 16$

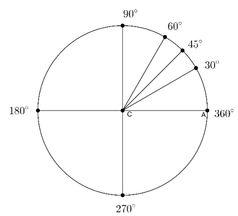
2.) Determine the length of $\widehat{AB}$ given the following radii and angles. Then sketch a circle with the given information and the length of the arc.
   - (A) $r = 5, \alpha = 2$
   - (B) $r = 3, \alpha = 3$
   - (C) $r = \frac{5}{3}, \alpha = \frac{3}{4}$
   - (D) $r = 10, \alpha = \frac{1}{2}$

3.) Determine the length of the radius given the following arc lengths and angles.
   - (A) $\widehat{AB} = 35, \alpha = 7$
   - (B) $\widehat{AB} = 5, \alpha = 5$
   - (C) $\widehat{AB} = 10, \alpha = 3$
   - (D) $\widehat{AB} = 6, \alpha = \frac{1}{2}$

4.) Sketch the following angles.
   - (A) 1
   - (B) 2
   - (C) 3
   - (D) 4
   - (E) 5
   - (F) 6
   - (G) 0.5
   - (H) 6.28

5.) Convert the following angle measure from angles to radians or vice versa.
   - (A) 200°
   - (B) 100°
   - (C) 50°
   - (D) 1.5
   - (E) $\frac{\pi}{12}$
   - (F) 720°
   - (G) $\frac{\pi}{5}$
   - (H) $\pi$

6.) The following Figure is a circle with various rotations on it. Assume that $C$ is located at the origin, and that there are four Quadrants, as normally defined on a coordinate plane. Each angle begins with $\overline{AC}$, then rotates counter-clockwise up to the next point.




(A) Write each of the angle measures in radians.[vii]
(B) Which is the larger angle, $\frac{\pi}{4}$ or $\frac{\pi}{2}$?
(C) Assume that we continue this pattern, so that in Quadrant II the next angle is 30° more than 90°, then 45° more than 90°, and so on (including Quadrants III and IV). Write each of these angles in degrees.
(D) Now write each of the angles you found in (C) in radians.

7.) One of the problems that students have with radians is that they're terrible with fractions. Accordingly, let us practice our fraction intuition.
(A) Which number is larger, $\frac{1}{4}$ or $\frac{1}{2}$? How can you tell without having to divide the numerator and denominator?
(B) Make an argument for why $\frac{1}{4}$ is less that $\frac{1}{2}$. (Hint: Try using money!)
(C) Likewise, which is larger: $\frac{\pi}{3}$ or $\frac{\pi}{6}$?
(D) When comparing a whole number to a fraction, it's often useful to convert the whole number into a fraction. For example, which is larger, 2 or $\frac{5}{3}$? To see, let's convert 2 into a fraction that has the same denominator as $\frac{5}{3}$. Now answer the question: Which is larger, 2 or $\frac{5}{3}$?
(E) Which is larger: $\pi$ or $\frac{5\pi}{6}$?
(F) Which is larger: $\frac{15\pi}{4}$ or $2\pi$?

8.) How can you tell that $\frac{5\pi}{6}$ is less than a half-rotation? (Hint: Try using common denominators in your fractions.)

9.) Which Quadrant is $\frac{11\pi}{6}$ in? How can you quickly tell? (Hint: Use the Figure from 2.) to help you visualize.)

10.) Of course, you can always convert the radians into degrees to check which one is larger. Convert $\frac{7\pi}{4}$ and $\frac{5\pi}{3}$ into degrees and then determine which one is larger.

11.) Now take $\frac{7\pi}{4}$ and $\frac{5\pi}{3}$ and get common denominators. Which one is larger?

[vii] And just to reiterate: Your answers must be in exact form.



12.) A wheel has a radius of 12 inches.[viii]
(A) If the wheel makes a full rotation, how far has the wheel traveled from its starting point?
(B) If the wheel makes a half rotation, how far has the wheel traveled from its starting point?
(C) Suppose the wheel has rotated $10\pi$. How far has it traveled?
(D) If the wheel has traveled 36 inches, how much has it rotated?

13.) Suppose a wheel is 24 inches around.
(A) If the wheel has rotated $\frac{5\pi}{6}$, how far has it traveled?
(B) If the wheel has traveled 10 feet, how much has it rotated?

14.) Suppose a wheel has traveled 10 feet.
(A) If it has rotated $\frac{13\pi}{6}$, what is its radius?
(B) If it has rotated $\frac{3\pi}{4}$, what is its circumference?

## §2 Introduction to the Polar Plane

After a brief hiatus, we now return to our Trig functions. We will use what you learned in Unit four extensively in this section; you will need to be able to calculate Trig ratios very quickly.[i] We will also begin to work with Trig functions where our angles are in radians.

To help us practice and memorize these Trig functions, we will now introduce a new way to graph.

Before we do this, a few words on what makes the coordinate plane so effective. There are numerous ways one could set up a graphing system, but, ideally, we would like it be simple to use and effective. The coordinate plane is great because it requires just two components (an $x$- and $y$-value) to plot any point. So it's effective and easy to use.

So if we're going to come up with a new way to plot points, it should be just as simple and effective. In other words, we should come up with a system that only requires two components to plot any point. Anything more than that will render our system far less effective.

So let us consider a system that uses circles instead of rectangular gridlines, as shown in Figure 56.

[viii] Assume that the wheel is a perfect circle.
[i] Or, as we suggested, you should simply memorize them.

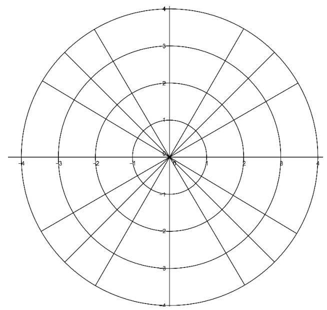


Figure 56

Let us emphasize that we have circles here, not rectangular gridlines. So using an $x$- and $y$-value will not suffice. We need two different components entirely. How about we use an angle measure and a radius? That should allow us to plot any point on this plane using only two components.

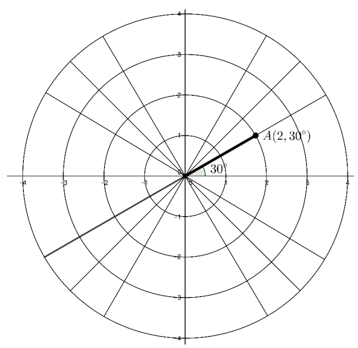



Figure 57

Consider Figure 57. We have two components, 2 and 30°. Let's start with the 2. Formally, it's a radius – that is, it is the distance from the center to the point. Another way of looking



at it is that our point must be on the second circle.[ii] The second component, 30°, tells us where upon that second circle we must put our point. So we place our point 2 away from the origin,[iii] then rotate the point 30° up. We show this process in Figures 58 a and b.

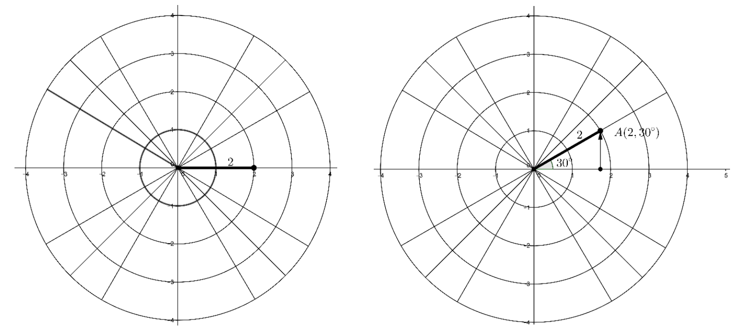



Figure 58a and b

We started by making a segment of length 2, then rotating that segment up 30°. Note that the segment only helps to place the point; what we really care about is the point.

This process works very well, so let us now define our new way of graphing

| Graphing on the Polar Plane |
|---|
| A point $A(r, \alpha)$ is plotted by creating a line segment of length $r$, and then rotating that segment $\alpha$. |

Just like with the coordinate plane, we should label our circles. You may have noticed lines extruding from the **pole**, or origin of the Polar Plane. These lines form angles with positive $x$-axis, and look awfully similar to a problem from the previous section. We'll label them below in Figure 59.

[ii] There's a disadvantage to this perspective, since we could also have 1.5 as the radius. In this case, we need to draw a circle halfway between the first and second circle. So it's not too much of a stretch to us this logic.
[iii] Which we'll give a different name shortly.

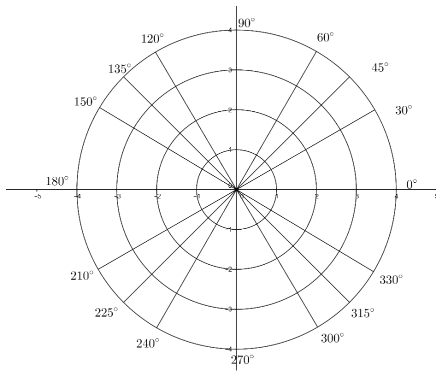



Figure 59

As with the coordinate plane, we don't *have* to choose these angles. However, there is good reason to choose these particular angles, and we'll reveal that answer shortly. [iv] Also of important note: The angle measures always start on what we normally call the positive $x$-axis and rotate up from there.[v]

Example 1a

Plot the point $A(2,45°)$.

To do this, we simply go to our second circle, then rotate up 45°. We show the plotted point in Figure 60.

[iv] If you've not already figured it out yourself.

[v] This is by convention. Someone somewhere decided that they were going to do it that way, and we've all followed suit.



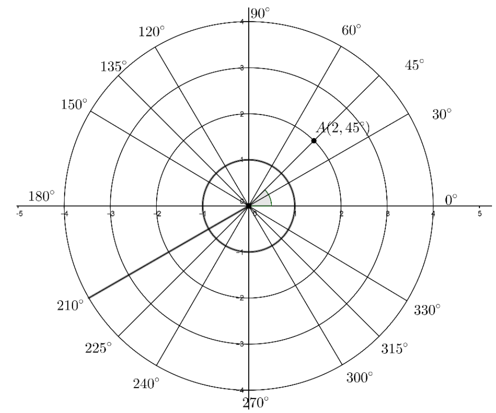



Figure 60

Example 1b

Graph the point $B(3.5,10°)$.

Neither of the two components in $B$ is on a line, but, like the coordinate plane, we can easily approximate their locations. We show this in Figure 61.

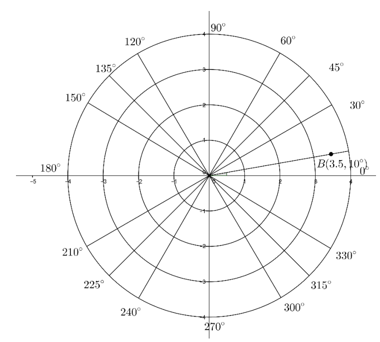



Figure 61

It might be helpful to draw a line for a 10° angle. It might also be helpful to draw a circle halfway between the third and fourth circle.

Example 1c

Graph the point $C\left(2.2, \frac{\pi}{2}\right)$.

In this example, we are using radians to measure our angles and not degrees. Recalling that 90° and $\frac{\pi}{2}$ are equivalent, we can easily graph Figure 62.

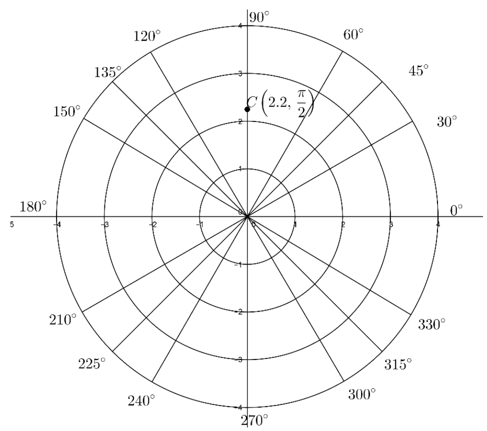



Figure 62

Many of your Exercises will use radians, so the above graph we've provided may not be the most helpful. You will want to create a Polar Plane with the radians listed and not degrees. But don't worry, this will be one of your Exercises.

We have not yet discussed negative angles. Up to this point, you might think that angles, like lengths, can only be positive. But angles (unlike lengths) have a direction. So far we've always rotated "up", which has amounted to a counter-clockwise rotation. Nothing is stopping us from rotating in the opposite direction, i.e., clockwise, but we haven't had a good way to label this other than spelling it out entirely. Let us therefore agree that a negative angle measure tells us to rotate clockwise, while a positive angle measure tells us to rotate counter-clockwise.

Example 2

Graph the point $D(3, -60°)$.



The negative angle tells us to rotate 60° clockwise. Since we know our point must be on the third circle (due to the radius being 3), we just need to determine how to rotate clockwise. Using the same Polar Plane as before, but counting in the opposite direction (and applying appropriate labels), we come up with Figure 63.

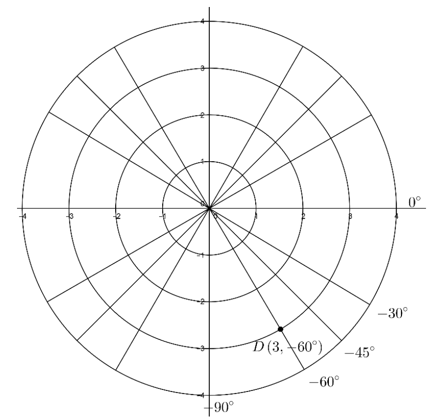



Figure 63

Perhaps you noticed that we did not complete labeling this Polar Plane. As you might have guessed, yes, this will be one of your Exercises.

But there's something quite curious as to the above: This is a point we could have made using our Polar Plane from before. If we take the same point but switch the labels back, we get Figure 64.

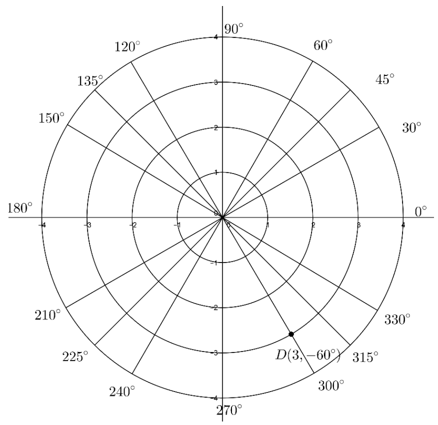



Figure 64

We've left the label from the previous Figure for you to help you compare.

Notice that our point is now located at $(3{,}300°)$. So it appears as though 300° is equivalent to $-60°$. This is interesting!

**Coterminal** angles

Two different angles that end up in the same spot are said to be coterminal.

So $-60°$ and $300°$ are coterminal, since they end up in the exact same spot. Another way of looking at this is that the point created by $(r, -60°)$ and $(r, 300°)$ will be the same (where $r \in \mathbb{R} > 0$).

Example 3

Plot the point $E(2, 840°)$.

This problem contains another strange angle. After all, there are only $360°$ in a rotation. But who's to say that we can only do one rotation? To account for multiple rotations, we can have angles that exceed $360°$.[vi] How many rotations is $840°$? Well, if $360°$ is one rotation, then two rotations would be

[vi] Likewise, we can also have angles that are less than $-360°$.



$$360° + 360° = 720°,$$

right? Likewise, we can see that three rotations would be 1,080°. So it seems as though we have two full rotations and then some left over. To account for this, we'll just subtract two full rotations from what we have, 840°:

$$840° - 720°$$

$$120°.$$

So we have two full rotations and then $120°$ more. This helps tremendously when we graph $840°$, since now all we have to do is identify the $120°$ angle on our Polar Plane. We show this in Figure 65.

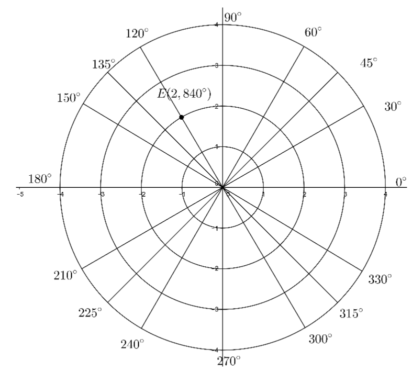



Figure 65

This also tells us that $840°$ and $120°$ are coterminal.

And this, in turn, tells us how many coterminal angles each angle has. Can you figure it out?

We'll further explore these ideas in the Exercises, as well as prepare for our more formal introduction to the Polar Plane in Unit seven. The brevity of this section will give you time to practice the basics and, perhaps more importantly, pave the way for success in the last section of this Unit, which is perhaps the most important.

GreatHearts
Irving

# Remote Learning Packet

*Please submit scans of written work in Google Classroom at the end of the week.*

**May 4-8, 2020**
**Course**: Spanish III
**Teacher(s)**: Mrs. Barrera anna.barrera@greatheartsirving.org
Supplemental links: www.spanishdict.com

**Weekly Plan**:

Monday, May 4
- [ ] Capítulo 5 - Story time. Listen to a story via video narrated in Spanish.
- [ ] Capítulo 5 - Writing assignment of the story that you listed in the video.

Tuesday, May 5
- [ ] Capítulo 5 - Write about skills and qualities needed in professions. .
- [ ] Capítulo 5 - Write about jobs and preferences

Wednesday, May 6
- [ ] Capítulo 5 - Reading job announcements.
- [ ] Capítulo 5 - Writing questions to interview a candidate

Thursday, May 7
- [ ] Capítulo 5 - Writing assignment using connected sentences with details and elaboration
- [ ] Capítulo 5 - Questions about everyday life in spoken conversation.

Friday, May 8
- [ ] attend office hours
- [ ] catch-up or review the week's work

## Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

## Monday, May 4

Capítulo 5 - Story time. Listen to a story via video narrated in Spanish. Writing assignment of the story.

I.**Listen Activity** link is in google classroom: Title of the book - ***Vida o muerte en el Cusco***. You will listen as I read Chapter 1 of a girls' adventure. Elena Garcia is vacationing in Cusco, Peru with her mother. Everything that Elena encounters bugs her - the food, the people, the clothing, the altitude. A casual hike to explore the ruins of Ollantaytambo turns into a life or death situation. With the help of a new friend, the experiences that follow have a transformative effect on Elena.
II.**Writing Assignment**: After you listen to chapter 1, you will write a paragraph that consists of 8 to 10 sentences explaining to me what happened in chapter 1. Use the mix of preterite, imperfect and present perfect tense.

## Tuesday, May 5

Capítulo 5 - Write about skills and qualities needed in professions. Write about jobs and preferences.

I.**Textbook p. 219** - **Activity 9** - *En qué te gustaría trabajar?* This activity will entail a writing assignment not a speaking assignment. Follow the videomodelo as to what kind of sentences you should write. There are 4 pictures in which there are two professions for each picture. You will write three questions and responses for each picture just like the model. For example; 1. Student A; asks a question and preferences. 1. Student B; Responds to A and then asks Student A, a question. 1. Student A responds. Follow the same for #2, 3, and 4.

## Wednesday, May 6

Capítulo 5 - Reading job announcements. Writing questions to interview a candidate.

I.**Textbook p. 220 - Activity 11** - *El mejor trabajo para ti*. **Number 1**: Make a list of the abilities and qualities necessary for each job mentioned in the classified. You can find these job descriptions below #3. There are four job announcements.
I.**Textbook p. 220 - Activity 11** - *El mejor trabajo para ti*. **Number 2:** Choose a job announcement and write a minimum of five questions for what you would ask a candidate relating to the job you chose from the four listed. You will also need to respond to how a candidate would respond to your question. For example; A. Your question B. Candidate's response. A. Your question. B. Candidate's response.
A.______B______. A____ B._______ A._______ B._________.

## Thursday, May 7

Capítulo 5 - Writing assignment using connected sentences with details and elaboration. Respond to questions about everyday life in spoken conversation.
**I.Textbook p.221** - **Activity 13** - *Y tu, que dices?* Answer in a complete sentence the five questions posed to you in this exercise. The last question refers to a job you have held, and if you haven't then refer to chores that you are responsible for at your house.