GreatHearts
Irving

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

April 6-10, 2020
Course: Math Fundamentals
Teacher: Ms. Schweizer rose.schweizer@greatheartsirving.org

Weekly Plan:

Monday, April 6
- ☐ Read Prime Factorization: Page 1
- ☐ Exercises: Section 5.4 pg. 162 13-33 odd

Tuesday, April 7
- ☐ Read GCF and LCM: Pages 2-3
- ☐ Exercises: Self-Test B pg. 169 1-12 all

Wednesday, April 8
- ☐ Read Fractions: Pages 4-5
- ☐ Exercises: Self-Test A pg. 225 1-15 all

Thursday, April 9
- ☐ Read Order of Operations: Pages 6-7
- ☐ Exercises: Section 1.5 pg. 17 27-37 odd
- ☐ Exercises: Section 8.5 pg. 262 19-31 odd

Friday, April 10
- ☐ No School!

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____
Parent Signature

Monday, April 6

This week we are reviewing various properties of numbers. Read Page 1 of the packet on Prime Factorization and complete the exercises in Section 5.4 on a separate sheet of paper, just as you normally do for your homework. You may also look over your old notes with the examples that we did together in class to help refresh your memory. Remember to show all of your work, and once you have completed the assignment, use either the back of the book or the attached answer key to correct your work and show any corrections in PEN.

Tuesday, April 7

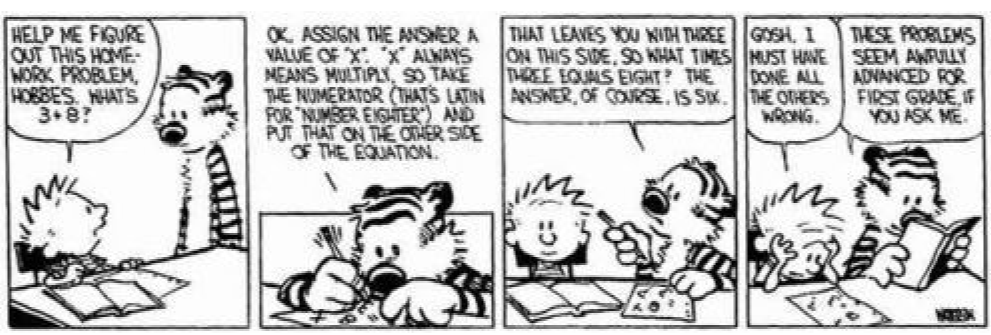
Continuing with the properties of numbers, read pages 2-3 in the packet to refresh your knowledge of the GCF and LCM, then complete the exercises in the book. Correct them with the attached answer key and show corrections in pen.

Wednesday, April 8

Use your knowledge of the GCF and LCM to make your work with fractions easier. Read pages 4-5 in the packet to review and then complete the exercises in the book, correcting them when you are done.

Thursday, April 9

Properties of numbers are related to properties of operations, so read pages 6-7 in the packet on the order of operations and inverse operations. Join the mathematical world and use this knowledge to complete the exercises. Note that the exercises come from two separate sections today. If you use the same piece of paper, make sure you label each section clearly. As always, show all your work and make corrections in pen.



Answer Key:

Section 5.4 pg. 162 13-33 odd

13. $2^2 \cdot 3$
15. $2^3 \cdot 3$
17. $3 \cdot 13$
19. $2 \cdot 3 \cdot 11$
21. $2 \cdot 3^3$
23. $2^2 \cdot 3 \cdot 7$
25. $2^2 \cdot 7^2$
27. $2^2 \cdot 7 \cdot 11$
29. $2 \cdot 3 \cdot 19$
31. All other even numbers have 2 as a factor.
33. odd+odd=even

Self-Test B pg. 169 1-12 all

1. Composite $2^2 \cdot 3^3$
2. Prime
3. Composite $3 \cdot 29$
4. Prime
5. 30
6. 14
7. 1; relatively prime
8. 15
9. 140
10. 189
11. 195
12. 450

Self-Test A pg. 225 1-15 all

1. 13/17
2. 19/32
3. 11/27
4. 8 ½
5. 2 5/16
6. 6 11/12
7. 25
8. 7/32
9. ⅜
10. 1/25
11. 9/20
12. 1 ⅕
13. 4 13/18
14. 4 ⅘
15. 11 3/7

Section 1.5 pg. 17 27-37 odd

27. 87
29. 37
31. 172
33. 176
35. 78
37. 9

Section 8.5 pg. 262 19-31 odd

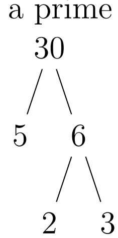
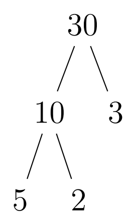
19. 4 ⅓
21. 22 ½
23. 7 ½
25. 3 ⅓
27. 1 7/9
29. 1 17/21
31. 4 3/7

1 Prime Factorization

Today is a review of how to find the prime factorization of a number. Recall that the prime factorization of a number is the product of prime factors expressing a number. In essence, it is all the prime numbers we multiply together to get the composite number.

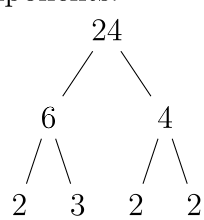
Take for example the prime factorization of $30 = 2 \times 3 \times 5$. Note that each factor is *prime*, it has no other factors.

The prime factorization can be found using a *factor tree*, where the bottom of each strand is a prime number.

For example:  or 

Notice how the final primes are the same in both factor trees.

Sometimes a prime factor may appear more than once, which can be written using exponents.

Ex: 

The prime factorization of $24 = 2 \times 2 \times 2 \times 3$ or $24 = 2^3 \times 3$.

The **Fundamental Theorem of Arithmetic** tells us that any whole number can be broken down into prime factors.

Theorem 1 (Fundamental Theorem of Arithmetic) *Every composite number greater than 1 can be written as a product of prime factors in exactly one way, except for the order of the factors.*

2 Greatest Common Factor

The greatest common factor is the factor that two or more numbers have in common that is the greatest. Somewhat self-explanatory there. One way, of course, to find the GCF is to list all the factors of both numbers.

Ex. Find the GCF of 18 and 32
18: 1, 2, 3, 6, 9, 18
32: 1, 2, 4, 8, 16, 32
Thus, the GCF is 2.

However, this approach is less useful the more factors a number has. The number 1050 seems nice, but has 24 factors! It's also hard to tell if you have actually found all the factors or are still missing some. Fortunately, we can use the *prime factorization* of both numbers to solve this problem.

Ex. Find the GCF of 540 and 495.
1. Find the prime factorization of both numbers.
$540 = 2^2 \times 3^3 \times 5$
$495 = 3^2 \times 5 \times 11$
2. Next see which prime factors the numbers have in common. In this case they have $3^2 \times 5$ in common, so their GCF is 45.

Look at each prime factorization and use the *lowest* power of each prime factor that is in *both* numbers.

A final note:

Definition 1 *Two numbers are* ***relatively prime*** *if their GCF is one.*

3 Least Common Multiple

The least common multiple is the multiple that two or more numbers have in common that is the least (except zero). Helpfully-named. In order to find the LCM, you can certainly find the LCM by listing of multiples of each number until you reach the least common one.

Ex. Find the LCM of 8 and 6.

1. Multiples of each number:
8: 0, 8, 16, 24, 32, 40 ...
6: 0, 6, 12, 18, 24 ...
2. LCM (8,6)=24

Like the GCF, this method becomes more difficult for larger numbers, so again we turn to the prime factorization to help us out. With the GCF, we were looking for a *factor*, so something smaller than the number, hence the reason we used the lowest powers. With the LCM we are looking for a *multiple*, so something larger than the numbers so we will use the largest powers.

Ex. LCM(250, 56)
1. Find the prime factorization of each number.
$250 = 2 \times 5^3$
$56 = 2^3 \times 7$
2. Now take the largest power of each prime factor that appears. In this case, LCM(250, 56)= $2^3 \times 5^3 \times 7 = 7000$.

Look at each prime factorization and take the *greatest* power of all the prime factors that appear in *either* prime factorization. If a number appears in either prime factorization, it should appear in the LCM as well.

4 Adding and Subtracting Fractions

When adding and subtracting fractions, remember that you must first always have a **common denominator**. A good choice for a common denominator is the LCM of the denominators. This keeps the numbers as small as possible and easiest to work with.

Ex.

$$\frac{4}{9} + \frac{5}{12}$$

$$LCM(9, 12) = 36$$

$$\frac{8}{36} + \frac{15}{36} = \frac{23}{36}$$

Using the LCM also puts the fraction closer to lowest terms than if you used a larger multiple, so it makes simplifying the fraction easier.

5 Multiplying Fractions

When adding and subtracting, the LCM was helpful, and for multiplying or dividing fractions the GCF will be helpful. Multiplying fractions is simple: multiply across, multiplying the numerators together and multiplying the denominators together.

Ex.

$$\frac{4}{15} \times \frac{3}{10}$$

$$1.\ Multiply\ across : \frac{4 \times 3}{15 \times 10} = \frac{12}{150}$$

$$2.\ Simplify : \frac{12}{150} = \frac{2}{25}$$

However, it is often easier to simplify *before* multiplying across. Note that

as long as one number in the *numerator* and one number in the *denominator* share a common factor, you can **cross-cancel**.

Ex.

$$\frac{4}{15} \times \frac{3}{10}$$

$$1.\ Simplify: \frac{4}{\boxed{15}} \times \frac{\boxed{3}}{10} = \frac{4}{\boxed{5}} \times \frac{\boxed{1}}{10}$$

since GCF(3,15)=3

$$\frac{\boxed{4}}{5} \times \frac{1}{\boxed{10}} = \frac{\boxed{2}}{5} \times \frac{1}{\boxed{5}}$$

since GCF(4,10)=2

$$2.\ Multiply\ across: \frac{2}{5} \times \frac{1}{5} = \frac{2}{25}$$

Again, this becomes more useful as the numbers grow larger and harder to work with.

6 Dividing Fractions

Dividing fractions is related to multiplying fractions. Recall that multiplication and division are **inverse** operations. In order to divide fractions, we can use this fact to turn it into a multiplication problem, which we already know how to solve! In order to do this we use the **reciprocal** of the divisor (the second number). The reciprocal functions as the inverse number, so we can use it with our inverse operation.

Ex.

$$\frac{21}{32} \div \frac{7}{24}$$

1. Change the division to multiplication and use the reciprocal of the divisor. (Use the inverse operation and the inverse number.)

$$\frac{21}{32} \times \frac{\boxed{24}}{\boxed{7}}$$

2. Cross-cancel

$$\frac{21}{32} \times \frac{24}{7} = \frac{3}{4} \times \frac{3}{1}$$

3. Multiply

$$\frac{3}{4} \times \frac{3}{1} = \frac{9}{4} = 2\frac{1}{4}$$

7 Order of Operations

When solving an equation, it is important to follow the **order of operations**. Without an order of operations, the same equation might give multiple answers and mathematics would be frustrating and inconsistent. In order to avoid this terrible fate, remember to use the order of operations! Mathematicians have universally agreed to the following order in which to perform operations:

1. Grouping Symbols [parentheses (starting with the inside ones)]
2. Exponents
3. Multiplication and Division from left to right
4. Addition and Subtraction from left to right

An easy way to remember the order is with the acronym **PEMDAS**
Parentheses
Exponents
Multiplication
Division
Addition
Subtraction

The following example illustrates the importance of following the order of operations:
Ex.

$$10 \div 2 \div 2$$

$$5 \div 2$$

$$2.5$$

Ex.

$$10 \div (2 \div 2)$$

$$10 \div (1)$$

$$10$$

No matter whether the equation has fractions, decimals, or variables, you must always follow the order of operations.

8 Inverse Operations

Recall that we often use **inverse operations** to solve for a variable. Since we are using inverse operations, we must also use the *inverse* of the order of operations. Since we are "undoing" the equation to find the value of the variable, we must work in reverse order, that is, undo addition and subtraction, then multiplication and division and so on. See the following example.

Ex.

$$13a - 3 = 49$$

$$13a - 3 + 3 = 49 + 3$$

$$13a = 52$$

$$13a \div 13 = 52 \div 13$$

$$a = 4$$