GreatHearts
Irving

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

April 13 - 17, 2020

Course: Math Fundamentals

Teacher(s): Ms. Schweizer rose.schweizer@greatheartsirving.org

Weekly Plan:

Monday, April 13
- ☐ Read pages 1-3
- ☐ Section 11.1, pg. 368, 9-30 mod 3, 40-42 all

Tuesday, April 14
- ☐ Read Pages 4-5
- ☐ Page 15, exercises 11-22 all

Wednesday, April 15
- ☐ Read Pages 6-8
- ☐ Page 16, exercises 1-25 odd

Thursday, April 16
- ☐ Read Pages 9-12
- ☐ Section 11.2, pg. 373, 3-27 mod 3
- ☐ pg. 374 Problems 1-3 all

Friday, April 17
- ☐ Read Pages 13-14
- ☐ Section 11.3, pag. 376, 7-29 odd

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____
Parent Signature

This is the first week we are really jumping into new material, but I'm sure you can all handle it. Last week we reviewed the properties of positive numbers so that this week we can talk about NEGATIVE NUMBERS! The types of numbers that we can work with are going to double!

Highlight or underline as you read my notes to help you understand as you go. Remember, you also have the section in the book as well. Email me if you have any questions.

Monday, April 13

What even are negative numbers? Find out by reading pages 1-3 in the packet and then completing the exercises from Section 11.1 in your book. Remember to complete the exercises on a separate piece of paper, copying down the original question and showing all your work. Then use the answer key to correct your work and make any corrections in pen.

Tuesday, April 14

We are not using the book today. Read pages 4-5 in the packet and then complete, on a separate piece of paper, the exercises found on page 15 of the packet. For numbers 11 and 12 it should say "a and b". Since your answers may be slightly different, I am not including an answer key for today.

Wednesday, April 15

Today's lesson is not in the book either. Read pages 6-8 in the packet about categorizing numbers and complete the exercises found on page 16 of the packet. I've included a limited answer key, but remember that your answers should include an explanation.

Thursday, April 16

Adding Integers! Now that we've discussed some of the properties of negative numbers, we are ready to work with them. Read pages 9-12 in the packet and then complete the exercises in Section 11.2. Remember to complete the exercises on a separate piece of paper, copying down the original question and showing all your work. Then use the answer key to correct your work and make any corrections in pen.

Friday, April 17

Subtracting Integers. You're a smart cookie, you know the drill. Read the pages in the packet and then complete the exercises in the book on a separate piece of paper. Correct them afterwards.

Answer Key:

Monday: Section 11.1

9. Number line

12. Number line

15. 3

18. 8

21. Number line

24. Number line

27. -3, -2, -1

30. -4,-3,-2,-1,0

40. Negative

41. Negative

42. Negative

Tuesday: Page 15

Answers may vary

Wednesday: Page 16

1. Whole, integer, rational, real

3. Irrational, real

5. Whole, integer, rational, real

7. Rational, real

9. Rational, real

11. Irrational, real

13. False

15. False

17. True

19. True

21. -- 26. May vary

Thursday: Section 11.2

3. -- 6. Number lines

9. -3

12. 9

15. 19

18. a.3 b. 3

21. 1

24. -10

27. True

Pg.374

1. -282+1200=918 ft

2. 12+(-7)+15=20

3. 25+3+(-8)+(-12)=8

Friday: Section 11.3

The answers are in the back of the textbook.

1 History of Negative Numbers

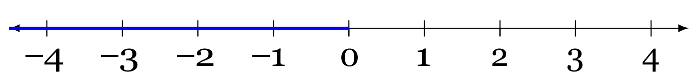
Negative numbers are commonly accepted today and most people work with them without giving a thought to the mathematical foundation of negative numbers, but this was not always the case. In ancient Greece, numbers were thought of geometrically, as distances, so they did not have the concept of a negative number. In fact, the first mention of negative numbers in Greek mathematics appears in the 3rd century, where they are dismissed as absurd. Since their idea of number was based on distance, they had no concept of what a negative number would mean.

The first widely accepted appearance of negative numbers was in the 7th century in India. The Indian mathematician Brahmagupta used negative numbers to represent debts, owing someone money. However, this interpretation was not sufficient for all mathematicians and many mathematicians still did not wholly accept negative number for hundreds of years.

All through the Renaissance mathematicians struggled with the concept of negative numbers and how we can work with them mathematically. They focused especially on how negative numbers effect which square roots can be found and how we find them. It was not until the 19th century that negative numbers were completely accepted by the mathematical community the way that they are today. So don't worry if you struggle with negative numbers, mathematicians struggled with them for over 1000 years!

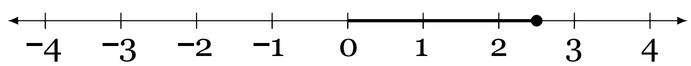
2 What are negative numbers?

As we know them today, **negative numbers** are numbers with a value less than zero, drawn to the *left* of zero on a number line.



Notice on the number line how -1 is the same distance from zero as 1 since they are both 1 away from zero. Likewise, -3 and 3 are both 3 away from zero. Since they are the same distance away from zero but in opposite directions, they are **opposites**.

On the number line below, we can count the distance from zero to find the **opposite** of the point.



Since the point is 2.5 units away from zero, it's opposite is 2.5 units away from zero in the opposite direction: -2.5.

Find the opposites of the following integers:

1. 7 **2.** -11

3 Absolute Value

This distance from zero to a number is called the **absolute value** of the number. Since a number and its opposite are the *same* distance from zero, they will have the *same* absolute value. Absolute value is written by two parallel lines around the number $|2| = 2$ (we read this as the absolute value of 2 equals 2).

Since the absolute value is a *distance*, it is always positive. (Remember the Greeks only had positive numbers because they were *distances*).

Ex.

$$|7| = 7$$

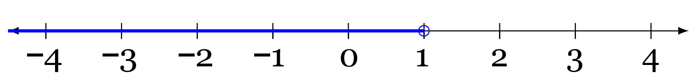
$$|-7| = 7$$

Find the absolute value of the numbers below.

1. $|31|$ **2.** $|0|$ **3.** $|-82|$

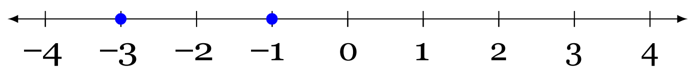
4 Comparing Negative Numbers

Consider the inequality $x < 1$. We know that x can be any number less than one. On a number line, we can graph the inequality as follows:



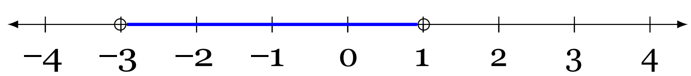
The blue line shows us all the different values for x. Notice how all the numbers *less than*, that is *smaller than*, one are to the *left* of one. This is true for any number. Anything to the **left** of that number is **less than** that number.

This includes the negative numbers. Look at the following number line:



From the number line we can conclude that $-3 < -1$ since -3 is to the left of -1.

We can also use a number line to solve inequalities like $-3 < x < 1$. Notice that x is in between -3 and 1. We can see all these values on a number line:



Thus, x can be any number between -3 and 1, or any number on the blue line.

Draw a number line to compare the two numbers. Place a < or > in each blank.

1. 0 5

2. -6 -8

5 Absolute Value Versus Opposites

Review yesterday's lesson to write down the definitions and give an example for the following vocabulary words:

opposites:

absolute value:

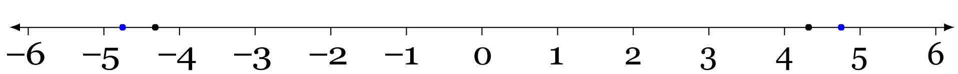
Now that you have those definitions in front of you, notice the differences between the **opposite** of a number and its **absolute value**. The opposite of a number may be negative, but the absolute value is *always* positive.

Find the opposite and the absolute value of the following numbers:
1. 88
2. -1243

6 Fractions

Yesterday we introduced the idea of negative numbers by considering the integers, the whole numbers. But what about fractions? Fortunately, fractions work just like whole numbers do: the opposite of $\frac{3}{4}$ is $-\frac{3}{4}$. With this in mind, list 4 values that are greater than -5 and less than -4:

One way to solve this problem is to consider all the numbers between positive 4 and positive 5. This includes numbers like 4.32, 4.5, $\frac{19}{4}$ and many more (infinitely many more). Now consider the opposite of these numbers. See the number line below:



Notice how since 4.32 (the black dot) is closer to zero than $\frac{19}{4}$ (the blue dot) in the positive numbers, the same is true of its opposite. -4.32 is closer to zero than $\frac{19}{4}$. Thus in the negative numbers, the less **absolute value** a number has, the closer a number is to zero and the *larger* it is. Zero is like a mirror reflecting the positive numbers, everything is flipped in the negative numbers.

List numbers in between each pair of numbers:

1. -1, 0

2. -2.5, -1.5

7 Fractions in General

What is the opposite and the absolute value of $\frac{a}{b}$?

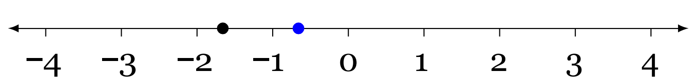
Just like with the integers, the opposite is the number that is the same distance away from zero. Thus, the *opposite* of $\frac{a}{b}$ is $-\frac{a}{b}$. The absolute value of the fraction is still its distance away from zero, so $|\frac{a}{b}| = \frac{a}{b}$.

Ex. Find the opposite and the absolute value of $-\frac{13}{2}$.

$$\textit{opposite} : \frac{13}{25}$$

$$-\frac{13}{25} = \frac{13}{25}$$

Ex. Compare $-\frac{2}{3}$, $-\frac{5}{3}$



Since $\frac{2}{3}$ is to the *right* of $\frac{5}{3}$ then we can conclude

$$-\frac{2}{3} > -\frac{5}{3}$$

Compare the following fractions by writing an inequality:

1. $-\frac{4}{5}$, $\frac{1}{4}$

2. $-3\frac{5}{7}$, $-3\frac{1}{2}$

8 Number Groups

Let's compare and contrast the following sets of numbers:

A: $\frac{2}{5}$, 9
Clearly, one of these numbers is a whole number and one of them is a fraction. However, recall that 9 can also be written as a fraction as $\frac{9}{1}$. So both numbers are fractions, but only 9 is a **whole number**.

B: -17, 13
In this case one is negative and less than zero. However, both numbers are considered **integers** because they can be written as a fraction over 1.

C: 0.333333333..., 0.39284717898082091000031 65...
Both decimals continue on forever and do not terminate, but notice that the first decimal repeats the same digit (3) over and over while the second decimal does not repeat. In fact, 0.33333333... is just another way to write $\frac{1}{3}$, so the first number can also be considered a fraction but the second cannot!

D: π, $\frac{1}{2}$
Here we have an **irrational** number like π which continues forever and does not repeat any digits. In comparison, if we write $\frac{1}{2}$ in decimal form (0.5) it terminates. However, both numbers have established values on the number line.

9 Categorizing

In the previous section we noted that we have different types of numbers with different properties. In order to know what types of properties a certain number has, mathematicians have established the following categories.

whole numbers: This includes the counting numbers and zero (0,1,2,3,4,...)

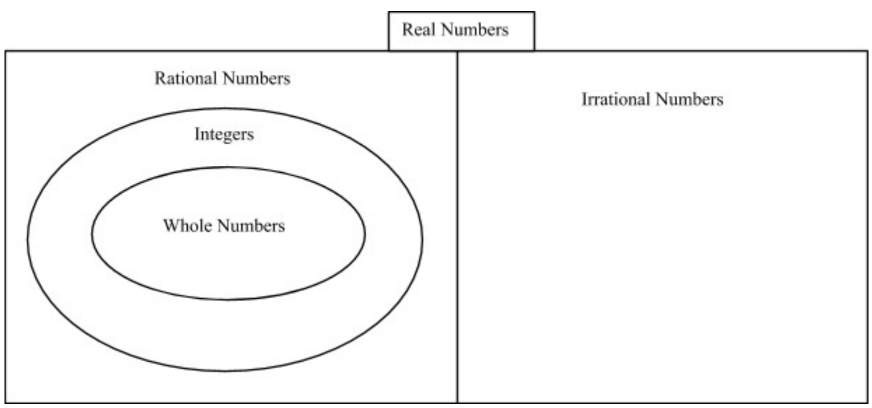
integers: whole numbers and their opposites, numbers who can be written as a fraction over 1 (..,-3,-2,-1,0,1,2,3,...)

rational numbers: numbers that can be written as a ratio of two integers (i. e. fractions), including the integers. This includes terminating decimals like 0.5 and repeating decimals like 0.3333333..... since they also can be written as fractions.

irrational numbers: a number that is not rational (it doesn't fit into the other categories). Any decimal that does not terminate or repeat is considered an irrational number. There are many famous irrational numbers like the number π. Another famous irrational number is the 2, which was the first irrational number discovered by that famous Greek cult, the Pythagoreans.

real numbers: The real numbers is the largest category, it includes all the different types of numbers on the real number line, both rational and irrational.

An easy way to picture how all these categories are connected is by the following Venn Diagram:



When categorizing a number, make sure to put it in the *smallest* category to which it belongs. While it is true to say that 24 is a rational number, it is more precise to say that it is a whole number. To say it is a whole number gives us more information about the number's properties and how it behaves.

10 For Curious Minds

You will not be tested on this material, but it is interesting to know!

Each category of numbers is called a *ring* among mathematicians and each one has a specific letter.

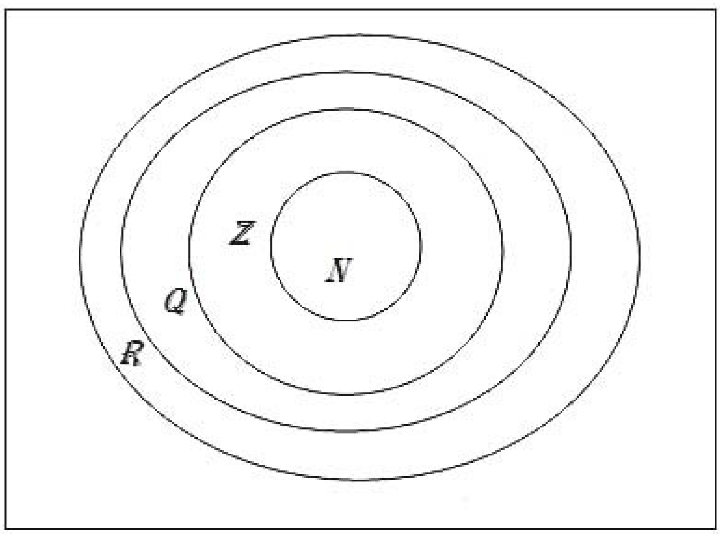
The whole numbers (without zero) are called the *natural* numbers and are denoted by .

The integers are written as , which comes from the German word Zahl, for number. This is a result of significant work from German-speaking mathematicians, like Leonhard Euler.

The rational numbers are written as , for quotient. Remember that the fraction bar is another way of writing the division symbol.

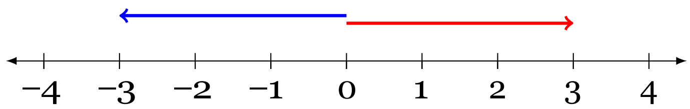
The real numbers are written as the letter , for real.

Another way to see this relationship is in the following diagram:



11 Arrow diagrams

Look at the following arrow diagram:



The red arrow starts at zero and moves to the right 3 units, which can be written as

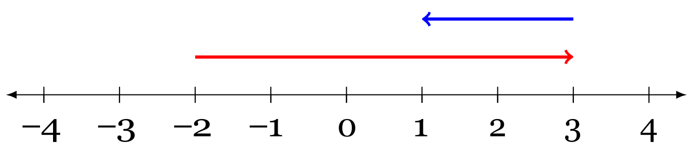
$$0 + 3 = 3$$

The blue arrow, on the other hand, starts at zero and moves to the *left* three units. Recall that addition and subtraction are **inverse** operations, so one way to interpret this arrow is with subtraction:

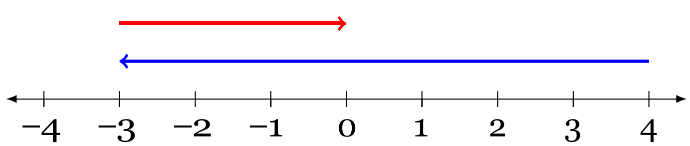
$$0 - 3 = -3$$

Write an equation for each number line using both arrows:

1.



2.



12 Inverses

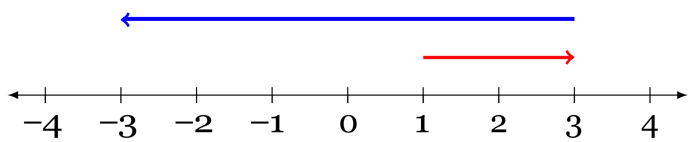
However, we also know that 3 and -3 are **opposites**, so their arrows are opposite directions as well. All negative numbers are to the *left* of zero, so this arrow also represents:

$$0 + (-3) = -3$$

Think back to division of fractions. In order to divide, we use the reciprocal (flip the number) and use the inverse operation, multiplication. In the case of negative numbers, instead of subtracting a positive number, we use its opposite and use the inverse operation. Thus, **adding a negative number is the same as subtracting a positive number**. The operations are flipped.

Now we know that any left facing arrow can either represent subtraction *or* addition of a negative number. Look at the following example:

Ex.



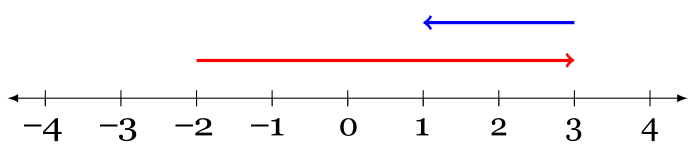
The first arrow starts at 1 and goes to the *right* two units: 1 + 2. The next arrow goes to the *left* six units. So we are *subtracting* positive six or *adding* negative six. the final equation would look like this:

$$1 + 2 + (-6) = -3$$

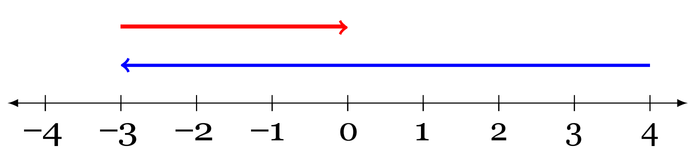
.

Now write equations for these number lines using negative numbers:

1.

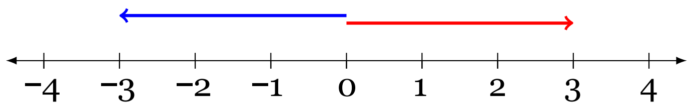


2.

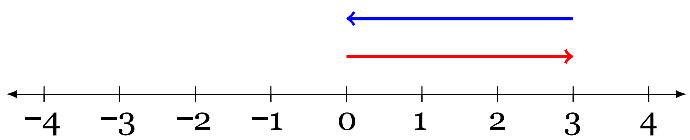


13 Absolute Value as Referee

Let's return to the first number line.



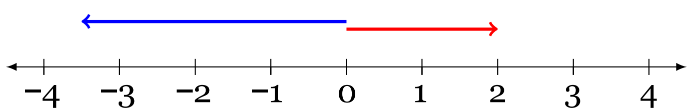
As we talked about, the arrow pointing to the *right* represents positive 3 and the arrow pointing to the *left* represents negative 3. Now, if we change the number line slightly, what equation does it represent now?



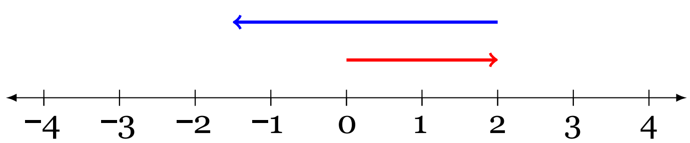
$$3 + (-3) = 0$$

This is because 3 and -3 are opposites: they are the same distance from zero and have the same **absolute value**. (Opposites are also called *additive inverses*, adding 3 and -3 "undoes" each other).

Now think about the the following two numbers:



In this case, it is clear that the blue arrow is much longer and therefore the **absolute value** of the negative number is greater, $|-3.5| > |2|$. Let's see what happens when we add the two numbers together:

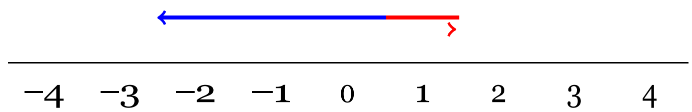


$$2 + (-3.5) = -1.5$$

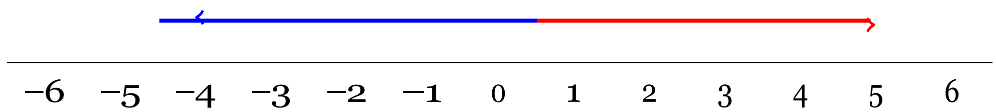
Since the absolute value of the negative number is greater than the absolute value of the positive number, the result is negative.

Look at the following number lines and state whether the result would be positive, negative, or zero.

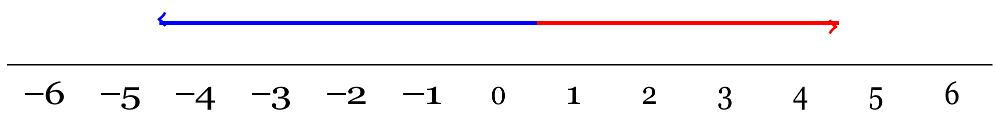
1.



2.



3.



14 Final Notes

Remember that addition is associative and commutative. If you've forgotten those properties, go look them up! They will help simplify solving addition problems with negative numbers.

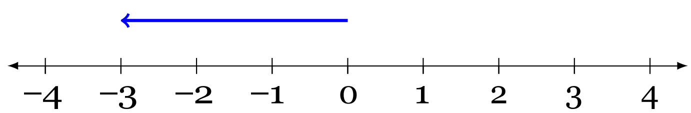
Fill in the blank:

1. The sum of two positive integers is a integer.
2. The sum of two negative integers is a integer.

15 Review

Yesterday we talked about adding negative numbers. Fill in the blank: **Adding a negative number is the same as________________a positive number**.

Recall that this is true because addition and subtraction are **inverse operations** and positive and negative numbers are opposites.



In the number line above, the blue arrow can either be seen as subtraction of a *positive* number **or** addition of a *negative* number.

$$0 - 3 = -3$$

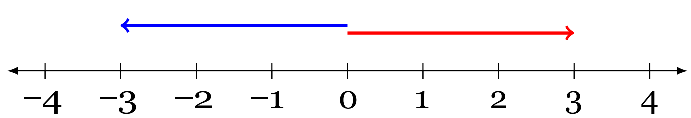
$$0 + (-3) = -3$$

Rewrite the following equations changing the subtraction of a positive number into the addition of a negative number.

1. $8 - 5 = 3$
2. $13 - 16 = -3$

16 Number Lines

Let's look at the number line from yesterday:



We already said the left facing blue arrow can represent two things:

1. The subtraction of a positive number.
2. The addition of a negative number.

Since a positive number is to the *right* of zero, adding a positive number is shown by a *right* facing arrow. Since negative numbers are to the *left* of zero, adding a negative number is shown by a *left* facing arrow.

Now, if *subtracting* a positive number goes the opposite way, to the *left*, then what do you think happens when we subtract a *negative* number? The arrow also goes the opposite direction! Since *adding* a negative number goes to the left, *subtracting* a negative number is shown by a right facing arrow.

That tells us we can also interpret the red arrow two ways as well!

1. The addition of a positive number.
2. The subtraction of a negative number.

17 Subtracting Integers

Today we are focusing on the inverse of addition: **subtraction**. Now, we still have the same two basic facts from yesterday:

1. Addition and subtraction are inverse operations.
2. Positive and negative numbers are opposites.

From these facts we were able to transform the addition of a negative number into subtraction of a positive number.

What if we are *subtracting* a negative number? Just like with addition, we can use the inverse operation: change the subtraction into addition. Since we are using the inverse *operation* we also must use the opposite *number*. Thus, **subtracting a negative number becomes adding a positive number**!

Ex.

$$8 - (-4)$$

$$8 + 4 = 12$$

Notice how we use the inverse operation (addition) and the opposite number (-4). Practice using the inverse operation and the opposite number on the following exercises:

1. $7 - (-9)$
2. $-3 - (-5)$
3. $-2 - 8$

Notice in the last problem how changing the subtraction to addition $(-2) + (-8)$ makes the problem easier. 2+8=10, so $(-2) + (-8) = (-10)$.

8. The Great Barrier Reef is the world's largest reef system and is located off the coast of Australia. Tt reaches from the surface of the ocean to a depth of 150 meters. What does -150 represent in this situation? (Why is it negative?) What does l-150l represent in this situation?

9. Give an example of a and b where a is negative, b is positive, and lal < lbl.

10. Give an example of a and b where a is negative, b is positive, and lal > lbl.

11. Give an example of a and are both negative, and lal < lbl.

12. Give an example of a and are both positive, and lal < lbl.

13. Give an example of a and b where $a < b$, but lal > lbl.

14. Give an example of a and b where a is a greater distance from zero, but $a < b$. (What other problem is this identical to?

15. Explain why ll$x$ll is the same as lxl.

16. Simplify the expression l-l-$x$ll.

17. Tf Sue owe John \$50, interpret the meaning of -\$50 and l-\$50l.

18. Give an example to show that l$a + b$l is not always the same as lal + lbl.

19. Give an example to show that la - bl is not always the same as lal - lbl.

20. Explain why $|a \times b| = |a| \times |b|$.

21. Tf a is 5 units from 2, what are all the possible values of a?

22. Tf b is 7 units from -1, what are all the possible values of b?

23. Tf lxl represents the distance from x to 0, what do you think lx -yl represents?

24. Tf Object A is at an elevation of 500 feet and Object B is at an elevation of -700 feet, which object is farther from the surface of the earth? (What does an elevation of -700 feet mean?

25. CHALLENGE: What is $\sqrt{4^2}$? What is $\sqrt{(-4)^2}$? Simplify the expression $\sqrt{n^2}$.

15

9. Give a real number that is not irrational.
10. Give an integer that is not a whole number.
11. Give a whole number that is not rational.
12. Give a number that is both rational and irrational.
13. Give a number that is both rational and an integer.
14. Give a number that is both a whole number and an integer.

Exercises

Classify each of the following numbers in as many ways as possible.

1. 6
2. -8
3. $\sqrt{5}$
4. $-\frac{6}{7}$
5. 0
6. π
7. 56.789
8. 36.789898989...
9. $6\frac{1}{2}$
10. $-\frac{18}{2}$
11. 0.34334333433334.
12. $\frac{125}{10}$

Label each of the following statements as True or False. If the statement is false, explain why it is false.

13. All real numbers are irrational numbers.
14. Some rational numbers are irrational numbers.
15. All rational numbers are whole numbers.
16. All whole number are integers.
17. Some integers are negative numbers.
18. Some positive numbers are irrational numbers.
19. All integers are rational numbers.
20. 0 is a rational number.

For each of the following, given an example or explain that it is not possible.

21. Give an irrational number that is not a real number.
22. Give a whole number that is not an irrational number.
23. Give a rational number that is not a whole number.
24. Give a number that is both an integer and irrational.
25. Give a number that is both rational and a whole number.
26. Give a number that is both a negative and an irrational number.

Challenge Questions

Label each of the following statements as True or False. If the statement is false, explain why it is false.

27. A whole number plus a whole number is always a whole number.
28. An integer times an integer is always an integer.
29. An irrational number plus an irrational number is always an irrational number.
30. A rational number plus a rational number is always a rational number.
31. A whole number minus a whole number is always a whole number.