

Written Exercises

State whether each number is prime or composite.

- | | | | | | |
|--------|----------|--------|------------|----------|------------|
| 1. 39 | (2.) 41 | 3. 51 | (4.) 111 | 5. 124 | (6.) 321 |
| 7. 641 | (8.) 753 | 9. 894 | (10.) 1164 | 11. 2061 | (12.) 3001 |

Give the prime factorization of each whole number.

- | | | | | | |
|---------|----------|---------|----------|---------|----------|
| 13. 12 | (14.) 50 | 15. 24 | (16.) 28 | 17. 39 | (18.) 56 |
| 19. 66 | (20.) 51 | 21. 54 | (22.) 63 | 23. 84 | (24.) 90 |
| 25. 196 | 26. 360 | 27. 308 | 28. 693 | 29. 114 | 30. 1150 |

31. Explain why 2 is the only even prime number.
32. Write the prime factorizations of the square numbers 16, 36, 81, and 144 by using exponents. What do you think must be true of the exponents in the prime factorization of a square number?
33. Explain why the sum of two prime numbers greater than 2 can never be a prime number.
34. Explain how you know that each of the following numbers must be composite: 111; 111,111; 111,111,111; ...
35. List all the possible digits that can be the last digit of a prime number that is greater than 10.
36. Choose a six-digit number, such as 652,652, the last three digits of which are a repeat of the first three digits. Show that 7, 11, and 13 are all factors of the number you chose.
37. Since 7, 11, and 13 are factors of any number of the type defined in Exercise 36, what is the largest composite number that is always a factor of such a number? What is the other factor?
38. Give an example to show that the Fundamental Theorem of Arithmetic would be false if 1 were defined to be a prime number.

Review Exercises

Evaluate.

- | | | | | |
|-----------|-----------|--------------------|---------------------|---------------------|
| 1. 7^3 | 2. 3^5 | 3. 4^3 | 4. 2^6 | 5. 9^4 |
| 6. 10^3 | 7. 10^5 | 8. $3^4 \cdot 2^3$ | 9. $5^3 \cdot 10^2$ | 10. $6^3 \cdot 4^4$ |

Self-Test B

Find out whether each number is prime or composite. If it is composite, give its prime factorization.

1. 108 2. 79 3. 87 4. 109 [5-4]

Find the GCF of the numbers. If they are relatively prime, so state.

5. 30, 90 6. 98, 112 7. 45, 77 8. 75, 105 [5-5]

Find the LCM of each pair of numbers.

9. 28, 70 10. 21, 27 11. 39, 65 12. 45, 50 [5-6]

Self-Test answers and Extra Practice are at the back of the book.

COMPUTER INVESTIGATION: Factors

The computer can be used to find all the pairs of factors of a number. The program below will print all the pairs of factors of a given number. If the number is a perfect square, there will be a pair of equal factors.

```
10 PRINT "TO FACTOR A NUMBER N:"
20 PRINT "N = ";
30 INPUT N
40 PRINT "PAIRS OF FACTORS ARE:"
50 FOR F = 1 TO N
60 LET Q = N / F
70 IF Q < > INT (Q) THEN 90
80 PRINT F; TAB( 10);Q
90 NEXT F
100 END
```

RUN the program to print out the pairs of factors of the following.

1. 216 2. 256 3. 576 4. 672 5. 1680
6. 225 7. 625 8. 1450 9. 1840 10. 1059

You can see that we do not need to print all the pairs to find the factors of the number. We can stop as soon as the quotient becomes less than the factor being tested. Insert this line into the program.

```
65 IF Q < F THEN 100
```

11-20. Repeat Exercises 1-10 with the shorter print-out. If a number is a square number, state a square root of it.

Self-Test A

Add or subtract. Simplify.

1. $\frac{8}{17} + \frac{5}{17}$

2. $\frac{3}{8} + \frac{7}{32}$

3. $\frac{7}{9} - \frac{10}{27}$ [7-1]

4. $3\frac{5}{12} + 5\frac{1}{12}$

5. $6\frac{3}{4} - 4\frac{7}{16}$

6. $2\frac{5}{8} + 4\frac{7}{24}$ [7-2]

Multiply or divide. Simplify.

7. $45 \times \frac{5}{9}$

8. $\frac{7}{12} \times \frac{3}{8}$

9. $\frac{15}{28} \times \frac{7}{10}$ [7-3]

10. $\frac{3}{5} \div 15$

11. $\frac{9}{25} \div \frac{4}{5}$

12. $\frac{18}{55} \div \frac{3}{11}$ [7-4]

13. $5\frac{2}{3} \times \frac{5}{6}$

14. $12\frac{3}{5} \div 2\frac{5}{8}$

15. $4\frac{6}{7} \times 2\frac{6}{17}$ [7-5]

Self-Test answers and Extra Practice are at the back of the book.

Class Exercises

State which operation is to be performed first.

- $12(18 + 36)$
- $(100 - 16) \div 7$
- $4[(6 + 17)2]$
- $[(54 - 12) + 26] \div 25$
- $15 + 12 \times 3$
- $87 - 16 \times 4$
- $31 + 88 \div 11$
- $96 \div 2 + 10$
- $96 \div (2 + 10)$
- $55 - 30 \div 5$
- $(55 - 30) \div 5$
- $84 \div (2 + 5)$

Written Exercises

Simplify.

- A**
- $7 + (9 \times 2)$
 - $(5 \times 7) + 4$
 - $(19 - 3) \div 4$
 - $(12 + 9)3$
 - $7 + (15 - 3)$
 - $18 + (42 \div 6)$
 - $48 - (5 \times 4)$
 - $12(11 - 5)$
 - $72 \div (8 - 2)$
 - $3 \times 9 + 18$
 - $64 \div 16 + 16$
 - $37 + 13 \times 2$
 - $20 + 6 \times 8$
 - $(37 + 13)4$
 - $102 \div 3 - 1$
 - $86 - 40 \div 2$
 - $(18 - 5)(23 - 9)$
 - $(32 + 24) \div (30 - 22)$
 - $(12 \times 9) - (8 \times 11)$
 - $(11 \times 7) + (90 \div 6)$
 - $(135 \div 3) - (17 \times 2)$
 - $(148 \div 4) + (16 \times 5)$
- B**
- $[48 - (4 \times 3)] \div 3$
 - $38 - [25 \div (43 - 18)]$
 - $[18 + (15 - 3)] + 14$
 - $[42 + (24 \div 6)] \div 2$
 - $53 + [64 - (10 \times 3)]$
 - $49 \div [81 - (37 \times 2)]$
 - $34 + 16 \times 2 - 29$
 - $126 - 9 \div 3 \times 28$
 - $181 - 24 \times 3 \div 8$
 - $8 + 6 \times 12 \div 4 + 6$
 - $19 + 84 \div 4 \times 8 - 11$
 - $97 - 75 \div 5 \times 3 + 68$

Evaluate if $t = 12$, $w = 10$, $x = 9$, $y = 4$, and $z = 3$.

- C**
- $tx - wz$
 - $x(y + w)$
 - $3z \div (y - z)$
 - $tw \div z + xy$
 - $ty + wx \div z$
 - $xy \div t + wz$

Review Exercises

Perform the indicated operation.

- $195 + 206 + 17$
- $5004 - 729$
- 607×74
- $1280 \div 16$
- $918 + 87 + 105$
- $9018 - 119$
- 727×102
- $6539 \div 13$

Class Exercises

State the two transformations you would use to find the solution of each equation. Be sure to specify which transformation you would use first.

1. $3n + 2 = 8$

2. $4n - 1 = 19$

3. $\frac{1}{2}n - 6 = 1$

4. $\frac{1}{3}n + 5 = 7$

5. $\frac{2}{3}n - 6 = 14$

6. $\frac{5}{2}n + 2 = 13$

7. $3n - 6 = 15$

8. $7n + 21 = 63$

9. $\frac{3}{4}n - 8 = 12$

10. $\frac{1}{2}n + 2 = 5$

11. $2\frac{1}{3}n - 2 = 8$

12. $1\frac{2}{3}n + 15 = 51$

Written Exercises

Solve each equation.

1. $2n - 5 = 17$

2. $3n + 8 = 23$

3. $5n + 6 = 41$

4. $4n - 15 = 9$

5. $6n + 11 = 77$

6. $8n - 13 = 51$

7. $50 - 3n = 20$

8. $42 - 5n = 7$

9. $29 - 6n = 11$

10. $79 - 8n = 15$

11. $\frac{1}{4}n + 5 = 25$

12. $\frac{1}{8}n - 11 = 21$

13. $\frac{1}{2}n + 3 = 18$

14. $\frac{1}{3}n - 7 = 11$

15. $\frac{1}{5}n - 2 = 9$

16. $\frac{1}{4}n + 3 = 8$

17. $\frac{2}{3}n + 12 = 28$

18. $\frac{3}{5}n - 11 = 7$

19. $6n - 7 = 19$

20. $10n + 4 = 39$

21. $\frac{6}{5}n - 7 = 20$

22. $\frac{15}{4}n + 7 = 32$

23. $2\frac{2}{5}n + 5 = 23$

24. $1\frac{1}{7}n - 9 = 27$

25. $\frac{3}{5}n + \frac{2}{3} = \frac{8}{3}$

26. $\frac{2}{3}n - \frac{5}{8} = \frac{7}{8}$

27. $\frac{3}{4}n - \frac{11}{15} = \frac{3}{5}$

28. $\frac{5}{6}n + \frac{1}{10} = \frac{29}{30}$

29. $\frac{7}{8}n - \frac{5}{6} = \frac{3}{4}$

30. $\frac{1}{3}n - \frac{11}{25} = \frac{7}{10}$

31. $\frac{2}{5}n + \frac{3}{7} = \frac{11}{5}$

32. $\frac{1}{6}n + \frac{1}{5} = \frac{7}{11}$

33. $1\frac{5}{8}n + \frac{13}{12} = \frac{51}{4}$

34. $2\frac{2}{3}n - \frac{4}{7} = \frac{8}{9}$

35. $\frac{11}{3}n - \frac{5}{9} = \frac{5}{6}$

36. $\frac{7}{2}n - \frac{11}{12} = \frac{5}{9}$

37. $1\frac{3}{8}n + \frac{1}{4} = \frac{7}{8}$

38. $\frac{3}{4}n - \frac{1}{12} = \frac{7}{3}$

39. $\frac{3}{7}n + \frac{4}{5} = \frac{6}{7}$

5-4 Prime Numbers and Composite Numbers

Consider the list of counting numbers and their factors given at the right. Notice that each of the numbers 2, 3, 5, 7, and 11 has *exactly* two factors: 1 and the number itself. A number with this property is called a **prime number**. A counting number that has more than two factors is called a **composite number**. In the list 4, 6, 8, 9, 10, and 12 are all composite numbers. Since 1 has exactly one factor, it is neither prime nor composite.

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12

About 230 B.C. Eratosthenes, a Greek mathematician, suggested a way to find prime numbers in a list of all the counting numbers up to a certain number. Eratosthenes first crossed out all multiples of 2, except 2 itself. Next he crossed out all multiples of the next remaining number, 3, except 3 itself. He continued crossing out multiples of each successive remaining number except the number itself. The numbers remaining at the end of this process are the primes.

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	...	

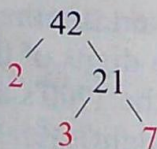
The method just described is called the **Sieve of Eratosthenes**, because it picks out the prime numbers as a strainer, or sieve, picks out solid particles from a liquid.

Every counting number greater than 1 has at least one prime factor, which may be the number itself. You can factor a number into prime factors by using either of the following methods.

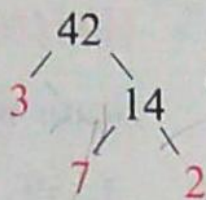
Inverted short division

$$\begin{array}{r} 2 \overline{)42} \\ 3 \overline{)21} \\ \quad 7 \end{array}$$

Factor tree



Another factor tree for the number 42 is shown at the right. Notice that the prime factors of 42 are the same in either factor tree except for their order. Every whole number is similar to 42 in this respect. This fact is expressed in the following theorem.



Fundamental Theorem of Arithmetic

Every composite number greater than 1 can be written as a product of prime factors in exactly one way, except for the order of the factors.

When we write 42 as $2 \cdot 3 \cdot 7$, this product of prime factors is called the **prime factorization** of 42.

EXAMPLE Give the prime factorization of 60.

Solution

Method 1

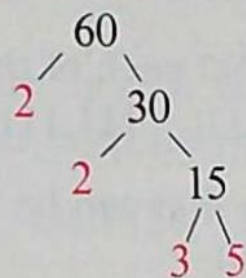
$$2 \overline{)60}$$

$$2 \overline{)30}$$

$$3 \overline{)15}$$

5

Method 2



Using either method, we find that the prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

5-5 Greatest Common Factor

If the factors of the numbers 30 and 42 are listed, the numbers 1, 2, 3, and 6 appear in both lists.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

These numbers are called **common factors** of 30 and 42. The number 6 is the greatest of these and is therefore called the **greatest common factor** of the two numbers. We write

$$\text{GCF}(30, 42) = 6$$

to denote the greatest common factor of 30 and 42.

EXAMPLE 1 Find $\text{GCF}(54, 72)$.

Solution

List the factors of each number.

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

The common factors are 1, 2, 3, 6, 9, and 18.

The greatest number in both lists is 18. Therefore,

$$\text{GCF}(54, 72) = 18.$$

Another way to find the GCF of two numbers is to use their prime factorizations. To find the GCF, multiply together the greatest power of each prime factor that occurs in *both* prime factorizations.

EXAMPLE 2 Find $\text{GCF}(54, 72)$ using the prime factorization method.

Solution

First find the prime factorizations of 54 and 72.

$$54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$$

$$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$$

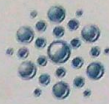
Find the greatest power of 2 that occurs in both prime factorizations.

The greatest power of 2 that occurs in both prime factorizations is 2.

Find the greatest power of 3 that occurs in both prime factorizations.

The greatest power of 3 that occurs in both prime factorizations is 3^2 .

Therefore, $\text{GCF}(54, 72) = 2 \cdot 3^2 = 18$.



EXAMPLE 3 Find GCF(45, 60) using the prime factorization method.

Solution

$$45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

Since 2 is not a factor of 45, there is no greatest power of 2 that occurs in both prime factorizations.

The greatest power of 3 that occurs in both prime factorizations is 3.

The greatest power of 5 that occurs in both prime factorizations is 5.

Therefore, $\text{GCF}(45, 60) = 3 \cdot 5 = 15$.

The number 1 is a common factor of any two whole numbers. If 1 is the GCF, then the two numbers are said to be **relatively prime**. As the next example shows, two numbers can be relatively prime even if one or both are composite.

EXAMPLE 4 Show that 15 and 16 are relatively prime.

Solution

List the factors of each number.

Factors of 15: 1, 3, 5, 15

Factors of 16: 1, 2, 4, 8, 16

Since $\text{GCF}(15, 16) = 1$, the two numbers are relatively prime.

Class Exercises

Give the GCF of each pair of numbers.

1. 4, 10

2. 15, 35

3. 6, 12

4. 9, 14

5. 18, 27

State whether the numbers in each pair are relatively prime.

6. 16, 20

7. 5, 15

8. 8, 9

9. 6, 35

10. 22, 26

Written Exercises

Find the GCF of each pair of numbers.

1. 16, 24

2. 18, 45

3. 24, 36

4. 26, 39

5. 15, 28

6. 44, 55

7. 28, 42

8. 75, 175

9. 60, 105

10. 54, 81

11. 56, 84

12. 63, 100

5-6 Least Common Multiple

If the first few multiples of 8 and 12 are listed in order, certain numbers appear in both lists:

Multiples of 8: 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, . . .

Multiples of 12: 0, 12, 24, 36, 48, 60, 72, 84, . . .

The numbers 0, 24, 48, 72, . . . are called **common multiples** of 8 and 12. Excluding 0, the least of these multiples is 24 and is therefore called the **least common multiple**. We write the least common multiple of 8 and 12 as

$$\text{LCM}(8, 12) = 24.$$

To find the LCM of two whole numbers, we can write out lists of multiples of the two numbers as above. The LCM will be the first multiple, excluding 0, that occurs in both lists.

A second method is to write out the first few multiples of the larger of the two numbers and then test each multiple for divisibility by the smaller number. The first multiple of the larger number that is divisible by the smaller number is the LCM.

EXAMPLE 1 Find $\text{LCM}(12, 15)$.

Solution Write out the first few multiples of 15 (excluding 0).

15, 30, 45, 60, 75, 90

Test each for divisibility by 12.

$$\begin{array}{r} 1 \text{ R } 3 \\ 12 \overline{)15} \end{array}$$

$$\begin{array}{r} 2 \text{ R } 6 \\ 12 \overline{)30} \end{array}$$

$$\begin{array}{r} 3 \text{ R } 9 \\ 12 \overline{)45} \end{array}$$

$$\begin{array}{r} 5 \\ 12 \overline{)60} \end{array}$$

Therefore, $\text{LCM}(12, 15) = 60$.

The LCM of two whole numbers can be found using their prime factorizations. For each prime factor of the two numbers, multiply together the greatest power of that factor that occurs in *either* prime factorization. The product will be the LCM.

EXAMPLE 2 Find $\text{LCM}(54, 60)$.

Solution

$$54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$$

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

The greatest power of 2 that occurs in either prime factorization is 2^2 .

7-4 Division of Fractions

Certain numbers, when multiplied together, have the product 1. For example,

$$5 \times \frac{1}{5} = 1 \quad \text{and} \quad \frac{3}{4} \times \frac{4}{3} = 1.$$

Two numbers whose product is 1 are called **reciprocals** of each other. Thus $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$, and $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$. Since the product of 0 and any number is always 0, the number 0 has no reciprocal. This suggests the following property.

Property

Every nonzero number has a unique (exactly one) reciprocal.

COMMUNICATION IN MATHEMATICS: Study Skills

It is important to know the meanings of all the words in a mathematical definition or property. If you are not sure about the mathematical meaning, check the glossary at the back of the book, or look up the meaning in a mathematics dictionary or reference book.

Recall that multiplication and division are inverse operations. Now study the following examples.

$$18 = 3 \times 6, \quad \text{so} \quad 18 \div 6 = 3. \quad \text{Also, } 18 \times \frac{1}{6} = 3.$$

$$7 = 28 \times \frac{1}{4}, \quad \text{so} \quad 7 \div \frac{1}{4} = 28. \quad \text{Also, } 7 \times 4 = 28.$$

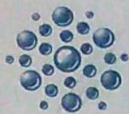
$$20 = 24 \times \frac{5}{6}, \quad \text{so} \quad 20 \div \frac{5}{6} = 24. \quad \text{Also, } 20 \times \frac{6}{5} = 24.$$

These examples suggest that dividing a number by a fraction is the same as multiplying the number by the reciprocal of the fraction.

Rule

If a , b , c , and d are whole numbers with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$



7-5 Multiplication and Division of Mixed Numbers

One method of finding the product of two mixed numbers is to first change the mixed numbers to improper fractions and then multiply. A second method makes use of the distributive property. This method is convenient for doing simple products mentally. Both methods are illustrated in the following example.

EXAMPLE 1 Multiply $6 \times 3\frac{1}{12}$. Simplify.

Solution

Method 1

$$6 \times 3\frac{1}{12} = \frac{6}{1} \times \frac{37}{12} = \frac{\overset{1}{6}}{1} \times \frac{37}{\underset{2}{12}} = \frac{37}{2} = 18\frac{1}{2}$$

Method 2

$$\begin{aligned} 6 \times 3\frac{1}{12} &= 6 \times \left(3 + \frac{1}{12}\right) = (6 \times 3) + \left(6 \times \frac{1}{12}\right) \\ &= 18 + \frac{6}{12} = 18\frac{1}{2} \end{aligned}$$

Estimates may be used to check our computation. We can estimate a product or a quotient by rounding each number to the nearest whole number.

EXAMPLE 2 Multiply $5\frac{3}{4} \times 4\frac{2}{3}$. Simplify.

Solution

Estimate: $6 \times 5 = 30$

$$5\frac{3}{4} \times 4\frac{2}{3} = \frac{23}{4} \times \frac{14}{3} = \frac{23}{\underset{2}{4}} \times \frac{\overset{7}{14}}{3} = \frac{161}{6} = 26\frac{5}{6}$$

To divide one mixed number by another, we change the mixed numbers to improper fractions and use the method of the previous lesson.

EXAMPLE 3 Divide $2\frac{2}{3} \div 10\frac{2}{3}$. Simplify.

Solution

$$2\frac{2}{3} \div 10\frac{2}{3} = \frac{8}{3} \div \frac{32}{3} = \frac{8}{3} \times \frac{3}{32} = \frac{\overset{1}{8}}{\underset{1}{3}} \times \frac{\underset{4}{3}}{\underset{32}{32}} = \frac{1}{1} \times \frac{1}{4} = \frac{1}{4}$$

1-5 Order of Operations

Symbols, such as parentheses and brackets, [], that show which operations are to be performed first are called **grouping symbols**. When one pair of grouping symbols is enclosed in another, we always perform the operation enclosed in the inner pair of symbols first.

EXAMPLE 1 Simplify.

a. $(14 + 77) \div 7$

b. $[3 + (4 \times 5)] \times 10$

Solution

$$\begin{array}{r} (14 + 77) \div 7 \\ \hline 91 \div 7 \\ \hline 13 \end{array}$$

$$\begin{array}{r} [3 + (4 \times 5)] \times 10 \\ \hline [3 + 20] \times 10 \\ \hline 23 \times 10 \\ \hline 230 \end{array}$$

Some expressions, such as

$$8 + 3 - 9 \times 2 \div 3,$$

are written without grouping symbols. In order to simplify these expressions we use the following rule.

Rule

When there are no grouping symbols:

1. Do all multiplications and divisions in order from left to right.
2. Then do all additions and subtractions in order from left to right.

EXAMPLE 2 Simplify.

a. $72 - 24 \div 3$

b. $8 + 3 - 9 \times 2 \div 3$

Solution

$$\begin{array}{r} 72 - 24 \div 3 \\ \hline 72 - 8 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 8 + 3 - 9 \times 2 \div 3 \\ \hline 8 + 3 - 18 \div 3 \\ \hline 8 + 3 - 6 \\ \hline 11 - 6 \\ \hline 5 \end{array}$$

8-5 Combined Operations

In order to solve an equation of the form

$$ax + b = c \quad \text{or} \quad ax - b = c \quad \text{or} \quad b - ax = c,$$

where a , b , and c are given numbers and x is a variable, we must use more than one transformation.

EXAMPLE 1 Solve the equation $3n - 5 = 10 + 6$.

Solution

Simplify the numerical expression.

$$3n - 5 = 10 + 6$$

$$3n - 5 = 16$$

Add 5 to both sides.

$$3n - 5 + 5 = 16 + 5$$

$$3n = 21$$

Divide both sides by 3.

$$\frac{3n}{3} = \frac{21}{3}$$

$$n = 7$$

The solution is 7.

Example 1 suggests the following general procedure for solving equations.

1. Simplify each side of the equation.
2. If there are still indicated additions or subtractions, use the inverse operations to undo them.
3. If there are indicated multiplications or divisions involving the variable, use the inverse operations to undo them.

It is important to remember that in using the procedure outlined above you must *always perform the same operation on both sides of the equation*. Also, you must use the steps in the procedure in the order indicated. That is, you first simplify each side of the equation, then undo additions and subtractions, and then undo multiplications and divisions.

EXAMPLE 2 Solve the equation $\frac{3}{2}n + 7 = 22$.

Solution

Subtract 7 from both sides.

$$\frac{3}{2}n + 7 = 22$$

$$\frac{3}{2}n + 7 - 7 = 22 - 7$$

$$\frac{3}{2}n = 15$$

Multiply both sides by $\frac{2}{3}$, the reciprocal of $\frac{3}{2}$.

$$\frac{2}{3} \times \frac{3}{2}n = \frac{2}{3} \times 15$$

$$n = \frac{2}{\cancel{3}^1} \times \overset{5}{15}$$

$$n = 10$$

The solution is 10.

EXAMPLE 3 Solve the equation $40 - \frac{5}{3}n = 15$.

Solution

Add $\frac{5}{3}n$ to both sides.

$$40 - \frac{5}{3}n = 15$$

$$40 - \frac{5}{3}n + \frac{5}{3}n = 15 + \frac{5}{3}n$$

$$40 = 15 + \frac{5}{3}n$$

Subtract 15 from both sides.

$$40 - 15 = 15 + \frac{5}{3}n - 15$$

$$25 = \frac{5}{3}n$$

Multiply both sides by $\frac{3}{5}$.

$$\frac{3}{5} \times 25 = \frac{3}{5} \times \frac{5}{3}n$$

$$\frac{3}{\cancel{5}^1} \times 25 = n$$

$$15 = n$$

The solution is 15.