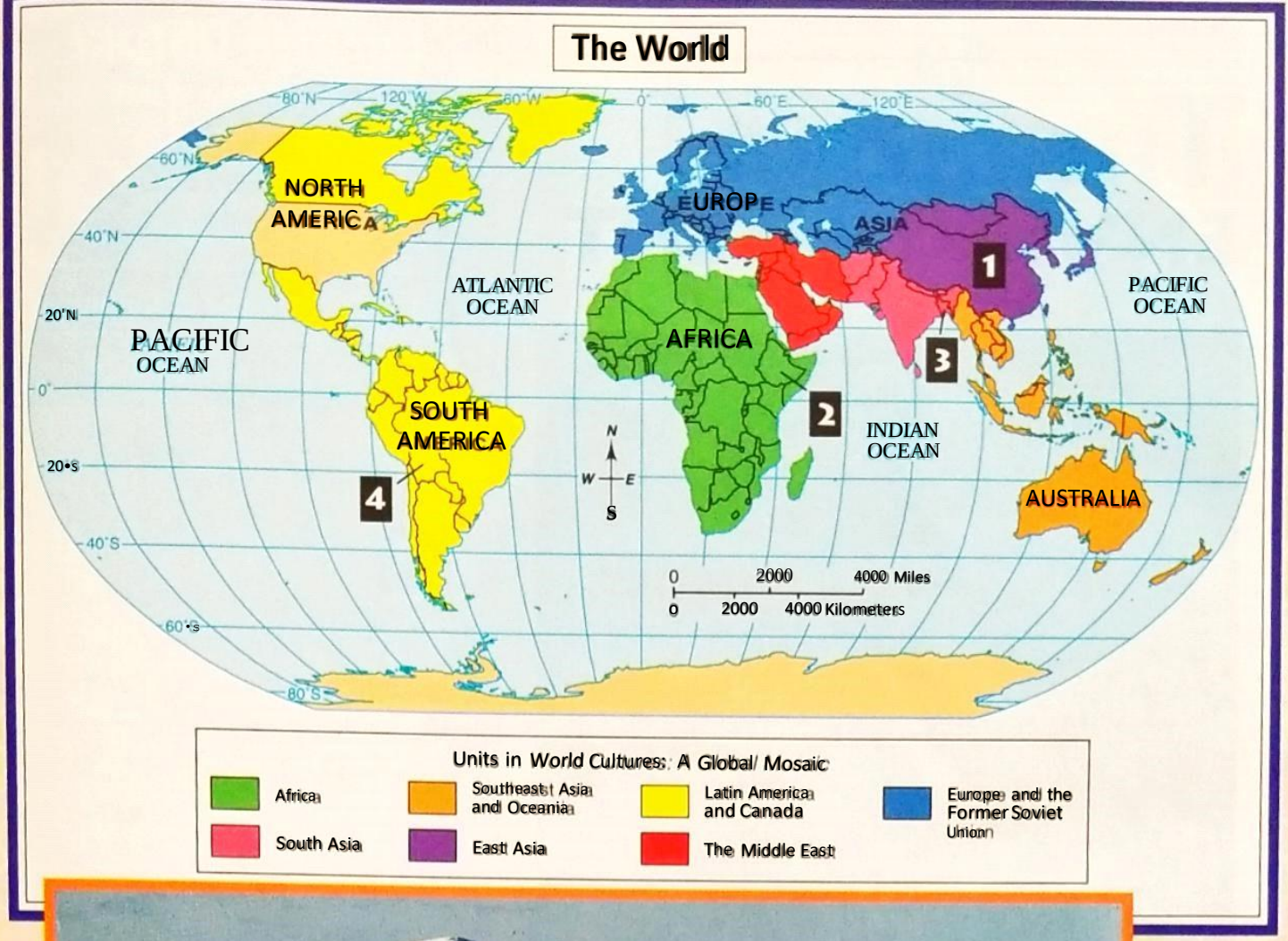
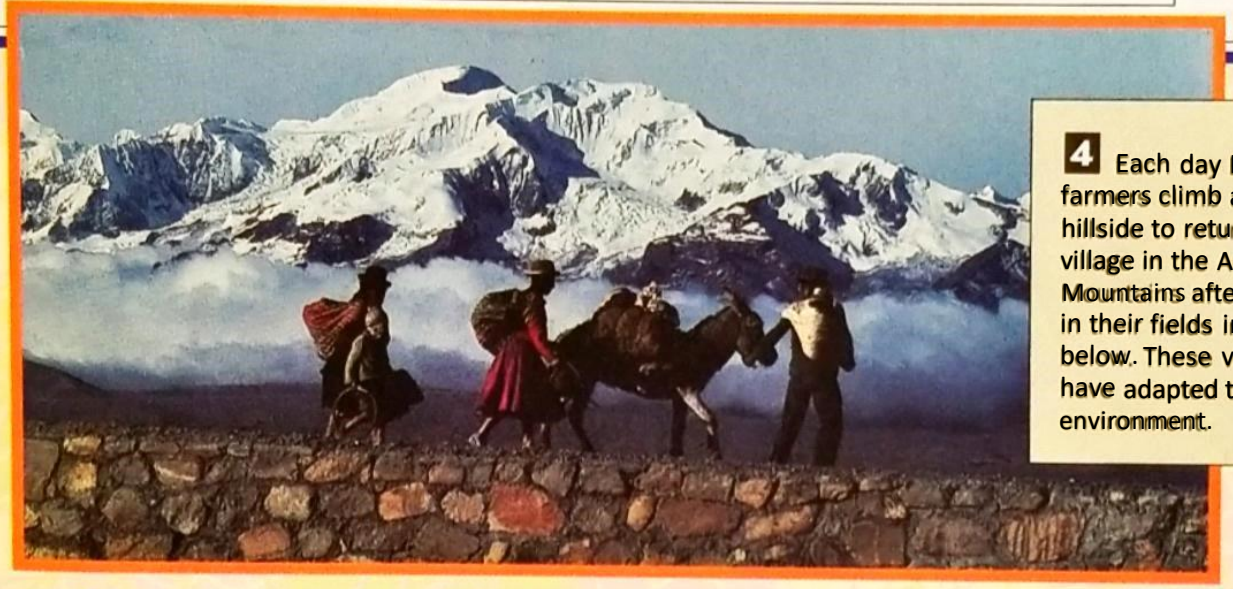
6th Grade Textbook Packet

4/6/2020-4/10/2020





4 Each day Bolivian farmers climb a steep hillside to return to their village in the Andes Mountains after working in their fields in the valley below. These villagers have adapted to a harsh environment.

1400 | 1600 | 1800 | 2000

▲ **Late 1400s**
Strong monarchs emerge in Europe and overseas expansion begins

▲ **1400s-1500s**
Timbuktu is center of Islamic learning

▲ **1569**
Mercator develops new map

▲ **1700s**
Industrial Revolution begins in Europe

▲ **1870s**
Age of Imperialism begins

▲ **1945**
United Nations formed

▲ **1975**
Helsinki Agreement signed

▲ **1957**
Soviet Union launches first artificial satellite

CAMBRIDGE
LATIN COURSE

Unit 1

Fifth Edition

THERMAE

Stage 9

About the language

1 Study the following examples:

Clēmēns **puellae** vīnum offerēbat.
*Clemens was offering wine **to the girl**.*

iuvenis **servō** pecūniam trādidit.
*The young man handed over money **to the slave**.*

dominus **mercātōrī** statuam ēmit.
*The master bought a statue **for the merchant**.*

Grumiō **ancillīs** cēnam parāvit.
*Grumio prepared a dinner **for the slave girls**.*

Quīntus **amīcīs** discum ostendit.
*Quintus showed the discus **to his friends**.*

servī **leōnibus** cibum dedērunt.
*The slaves gave food **to the lions**.*

The Latin words in **boldface** are nouns in the **dative case**.

2 You have now met three cases. Notice the different ways in which they are used:

nominative **servus** dormiēbat.
***The slave** was sleeping.*

dative dominus **servō** signum dedit.
*The master gave a sign **to the slave**.*

accusative dominus **servum** excitāvit
*The master woke **the slave**.*

About the language

3 Here is a full list of the noun endings that you have met.
The new dative cases are in **boldface**.

		first declension	*second declension*	*third declension*
SINGULAR	*nominative*	puella	servus	mercātor
	dative	**puellae**	**servō**	**mercātōrī**
	accusative	puellam	servum	mercātōrem
PLURAL	*nominative*	puellae	servī	mercātōrēs
	dative	**puellīs**	**servīs**	**mercātōribus**
	accusative	puellās	servōs	mercātōrēs

4 Further examples:

a ancilla dominō cibum ostendit.
b agricola uxōrī ānulum ēmit.
c servus Metellae togam trādidit.
d mercātor gladiātōribus pecūniam offerēbat.
e fēmina ancillīs tunicās quaerēbat.

5 Notice the different cases of the words for "I" and "you":

nominative	ego	tū
dative	mihi	tibi
accusative	mē	tē

ego senem salūtō.
senex **mihi** statuam ostendit.
senex **mē** salūtat.

__I__ greet the old man.
*The old man shows a statue **to me**.*
*The old man greets **me**.*

tū pictūram pingis.
āthlēta **tibi** pecūniam dat.
āthlēta **tē** laudat.

__You__ are painting a picture.
*The athlete gives money **to you**.*
*The athlete praises **you**.*

in tabernā

Metella et Melissa ē vīllā māne discessērunt. Metella fīliō togam quaerēbat. Metella et ancilla, postquam forum intrāvērunt, tabernam cōnspexērunt, ubi togae optimae erant. multae fēminae erant in tabernā. servī fēminīs stolās ostendēbant. duo gladiātōrēs quoque in tabernā erant. servī gladiātōribus tunicās ostendēbant.

mercātor in mediā tabernā stābat. mercātor erat Marcellus. Marcellus, postquam Metellam vīdit, rogāvit,

"quid quaeris, domina?"

"togam quaerō," inquit Metella. "ego fīliō dōnum quaerō, quod diem nātālem celebrat."

"ego multās togās habeō," respondit mercātor.

mercātor servīs signum dedit. servī mercātōrī togās celeriter trādidērunt. Marcellus fēminīs togās ostendit. Metella et ancilla togās īnspexērunt.

"hercle!" clāmāvit Melissa. "hae togae sunt sordidae."

Marcellus servōs vituperāvit.

"sunt intus togae splendidae," inquit Marcellus.

Marcellus fēminās intus dūxit. mercātor fēminīs aliās togās ostendit. Metella Quīntō mox togam splendidam ēlēgit.

"haec toga, quantī est?" rogāvit Metella.

"quīnquāgintā dēnāriōs cupiō," respondit Marcellus.

"quīnquāgintā dēnāriōs cupis! furcifer!" clāmāvit Melissa. "ego tibi decem dēnāriōs offerō."

"quadrāgintā dēnāriōs cupiō," respondit mercātor.

"tibi quīndecim dēnāriōs offerō," inquit ancilla.

"quid? haec est toga pulcherrima! quadrāgintā dēnāriōs cupiō," respondit Marcellus.

"tū nimium postulās," inquit Metella. "ego tibi trīgintā dēnāriōs dō."

"cōnsentiō," respondit Marcellus.

Melissa Marcellō pecūniam dedit. Marcellus Metellae togam trādidit.

"ego tibi grātiās maximās agō, domina," inquit Marcellus.

māne *in the morning*
togam *toga*

domina *my lady, ma'am*
dōnum *present, gift*
hae togae *these togas*
sordidae *dirty*
intus *inside*
aliās *other*
ēlēgit *chose*
haec *this*
quantī est? *how much is it?*
quīnquāgintā dēnāriōs *fifty denarii*
cupiō *I want*
decem *ten*
quadrāgintā *forty*
quīndecim *fifteen*
pulcherrima *very beautiful*
nimium *too much*
trīgintā *thirty*
cōnsentiō *I agree*
ego tibi grātiās maximās agō *I thank you very much*

A fabric shop.

Practicing the language

1 Complete each sentence with the verb that makes good sense.
Then translate the sentence, taking care with the different forms of the noun.

For example mercātōrēs fēminīs tunicās . . . . . (audīvērunt, ostendērunt, timuērunt)
mercātōrēs fēminīs tunicās **ostendērunt**.
The merchants showed the tunics to the women.

a ancilla dominō vīnum . . . . . (timuit, dedit, salūtāvit)
b iuvenis puellae stolam . . . . . (ēmit, vēnit, prōcessit)
c fēminae servīs tunicās . . . . . (intrāvērunt, quaesīvērunt, contendērunt)
d cīvēs āctōrī pecūniam . . . . . (laudāvērunt, vocāvērunt, trādidērunt)
e centuriō mercātōribus decem dēnāriōs . . . . . (trādidit, ēmit, vīdit)

2 Complete each sentence with the correct form of the verb. Then translate the sentence.

For example gladiātor amīcīs togam . . . . . (ostendit, ostendērunt)
gladiātor amīcīs togam **ostendit**.
The gladiator showed the toga to his friends.

a puella gladiātōribus tunicās . . . . . (dedit, dedērunt)
b cīvēs Milōnī statuam . . . . . (posuit, posuērunt)
c mercātor amīcō vīnum . . . . . (trādidit, trādidērunt)
d coquus ancillae ānulum . . . . . (ēmit, ēmērunt)
e Clēmēns et Grumiō Metellae cēnam optimam . . . . . (parāvit, parāvērunt)

3 This exercise is based on the story **in tabernā**, opposite. Read the story again.
Write out each sentence, completing it with the correct noun or phrase. Then translate the sentence.

a Metella . . . . . ad forum ambulāvit. (cum Quīntō, cum Grumiōne, cum Melissā)
b postquam forum intrāvērunt, ... cōnspexērunt. (portum, tabernam, vīllam)
c Metella gladiātōrēs et . . . . . in tabernā vīdit. (āctōrēs, fēminās, centuriōnēs)
d servī fēminīs . . . . . ostendēbant. (tunicās, stolās, togās)
e servī gladiātōribus . . . . . ostendēbant. (togās, stolās, tunicās)
f mercātor servīs . . . . . dedit. (signum, togam, gladium)
g servī mercātōrī . . . . . trādidērunt. (togam, togās, stolās)
h mercātor . . . . . vituperāvit, quod togae erant sordidae. (gladiātōrēs, fēminās, servōs)

The baths

About the middle of the afternoon, Caecilius would make his way, with a group of friends, to the public baths. The great majority of Pompeians did not have bathrooms in their houses, so they went regularly to the public baths to keep themselves clean. As at a leisure center, city pool, or health club today, they could also take exercise, meet friends, and have a snack. Let us imagine that Caecilius decides to visit the baths situated just to the north of the forum, and let us follow him through the various rooms and activities.

At one of the entrances, he pays a small admission fee to the doorkeeper and then goes to the **palaestra** (exercise area). This is an open space surrounded by a colonnade, rather like a large peristylium. Here he spends a little time greeting other friends and taking part in some of the popular exercises, which included throwing a large ball from one to another, wrestling, and fencing with wooden swords. These games were not taken too seriously but were a pleasant preparation for the bath which followed.

From the palaestra, Caecilius and his friends walk along a passage into a large hall known as the **apodytērium** (changing room). Here they undress and hand their clothes to one of the slave attendants who places them in recesses arranged in rows along the wall.

Leaving the apodyterium, they pass through an arched doorway into the **tepidārium** (warm room) and spend a little time sitting on benches round the wall in a warm, steamy atmosphere, perspiring gently and preparing for the higher temperatures in the next room.

This is the **caldārium** (hot room). At one end of the caldarium there was a large marble bath, rectangular in shape, and stretching across the full width of the room. This bath was filled with hot water in which the bathers sat or wallowed. The Romans did not have soap, but used olive oil instead. After soaking in the bath, Caecilius summons a slave to rub him down with the oil that he has brought with him in a little pot. For this rubbing down, Caecilius lies on a marble slab while the slave works the oil into his skin, and then gently removes it and the dirt with a blunt metal scraper known as a **strigil**. Next comes the masseur to massage skin and muscles. Refreshed by this treatment, Caecilius then goes to the large stone basin at the other end of the caldarium for a rinse down with cold water.

A visit to the baths

These pictures show us one route which a bather might take through the baths after he leaves the palaestra.

They are taken from several different sets of baths, as no one set has all its rooms well preserved today.

1 The entrance hall with the apodyterium beyond.
Stabian Baths, Pompeii.

2 The tepidarium. This sometimes had recesses for clothes like the apodyterium.
Forum Baths, Pompeii.

3 The hot tub in the caldarium.
Herculaneum.

Strigils and oil bottles.

4 The caldarium, showing a marble bench for sitting or massage.
Herculaneum.

5 The frigidarium: cold plunge bath.
Forum Baths, Pompeii.

in apodytēriō

duo servī in apodytēriō stant. servī sunt Sceledrus et Anthrāx.

Sceledrus: cūr nōn labōrās, Anthrāx? num dormīs?
Anthrāx: quid dīcis? dīligenter labōrō. ego cīvibus togās custōdiō.
Sceledrus: togās custōdīs? mendāx es!
Anthrāx: cūr mē vituperās? mendāx nōn sum. togās custōdiō.
Sceledrus: tē vituperō, quod fūr est in apodytēriō, sed tū nihil facis.
Anthrāx: ubi est fūr? fūrem nōn videō.
Sceledrus: ecce! homō ille est fūr. fūrem facile agnōscō. *(Sceledrus Anthrācī fūrem ostendit. fūr togam suam dēpōnit et togam splendidam induit. servī ad fūrem statim currunt.)*
Anthrāx: quid facis? furcifer! haec toga nōn est tua!
fūr: mendāx es! mea est toga! abī!
Sceledrus: tē agnōscō! pauper es, sed togam splendidam geris. *(mercātor intrat. togam frūstrā quaerit.)*
mercātor: ēheu! ubi est toga mea? toga ēvānuit! *(mercātor circumspectat.)* ecce! hic fūr togam meam gerit!
fūr: parce! parce! pauperrimus sum … uxor mea est aegra … decem līberōs habeō …

mercātor et servī fūrem nōn audiunt, sed eum ad iūdicem trahunt.

This mosaic of a squid is in an apodyterium in Herculaneum.

in apodytēriō *in the changing room*

num dormīs? *surely you are not asleep?*

suam *his*
induit *is putting on*

abī! *go away!*
pauper *poor*
geris *you are wearing*

parce! *have pity on me! spare me!*
pauperrimus *very poor*
aegra *sick, ill*
līberōs *children*
audiunt *listen to*

An apodyterium (changing room) in the women's section of the Stabian Baths at Pompeii.

The caldarium (hot room) in the Forum Baths, Pompeii. At the nearer end note the large rectangular marble bath, which was filled with hot water. At the far end there is a stone basin for cold water. Rooms in baths often had grooved, curved ceilings to channel condensation down the walls.

Before dressing again he might well visit the frigidārium (cold room) and there take a plunge in a deep circular pool of unheated water, followed by a brisk rub down with his towel.

Metella, too, would have visited public baths. Some baths had a separate suite of rooms for the use of female bathers; others may have given access to men and women at different times, or may have allowed mixed bathing. We do not know whether women were allowed to exercise in the palaestra. In the Forum and Stabian Baths, where separate facilities for men and women existed, those for the women were smaller, and had a pool of cold water in the apodyterium rather than a separate frigidarium. The smaller facilities may be an indication that fewer women attended the baths, or that women attended less regularly than men. Alternatively, it may indicate that women's needs were regarded as less important than those of men.

A visit to the baths was a leisurely social occasion. Men and women enjoyed a noisy, relaxed time in the company of friends. The Roman writer Seneca lived uncomfortably close to a set of baths in Rome and his description gives us a vivid impression of the atmosphere there:

> **I am surrounded by uproar. I live over a set of baths. Just imagine the babel of sounds that strikes my ears. When the athletic gentlemen below are exercising themselves, lifting lead weights, I can hear their grunts. I can hear the whistling of their breath as it escapes from their lungs. I can hear somebody enjoying a cheap rub down and the smack of the masseur's hands on his shoulders. If his hand comes down flat, it makes one sound; if it comes down hollowed, it makes another. Add to this the noise of a brawler or thief being arrested down below, the racket made by the man who likes to sing in his bath, or the sound of enthusiasts who hurl themselves into the water with a tremendous splash. Next I can hear the screech of the hair plucker, who advertises himself by shouting. He is never quiet except when he is plucking hair and making his victim shout instead. Finally, just imagine the cries of the cake seller, the sausage man, and the other food sellers as they advertise their goods round the bath, all adding to the din.**

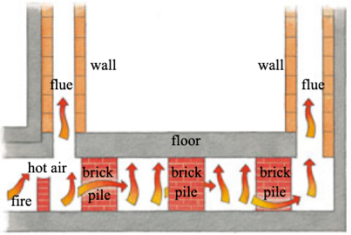
A bronze statue of a boxer from a set of baths in Rome. His training would no doubt have contributed to the din about which Seneca complains.

Heating the baths

The Romans were not the first people to build public baths. This was one of the many things they learned from the Greeks. But with their engineering skill the Romans greatly improved the methods of heating them. The previous method had been to heat the water in tanks over a furnace and to stand braziers (portable metal containers in which wood was burned) in the tepidarium and the caldarium to keep up the air temperature. The braziers were not very efficient and they failed to heat the floor.

In the first century BC, a Roman invented the first central heating system. The furnace was placed below the floor level; the floor was supported on small brick piles leaving space through which hot air from the furnace could circulate. In this way, the floor was warmed from below. The hot bath was placed near the furnace and a steady temperature was maintained by the hot air passing immediately below. Later, flues (channels) were built into the walls and warm air from beneath the floor was drawn up through them. This ingenious heating system was known as a **hypocaust**. It was used not only in baths but also in private houses, particularly in the colder parts of the Roman empire. Many examples have been found in Britain. Wood was the fuel most commonly burned in the furnaces.

Hypocaust in the Stabian Baths. Notice the floor suspended on brick piles, so that hot air can circulate beneath and warm both the room and the tank of water for bathing.

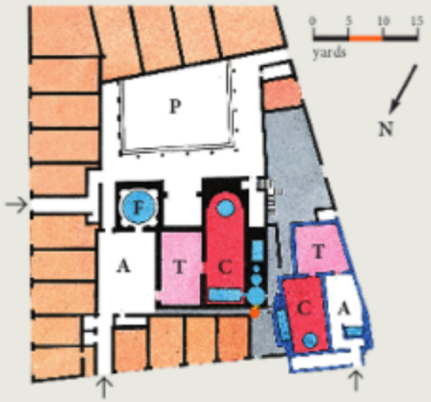


A hypocaust viewed from the side.

Plan of the Forum Baths, Pompeii

The men's section is outlined in black and the women's in blue. See how the hottest rooms (red) in both suites are arranged on either side of the one furnace (marked by an orange dot). The blue circles near this are boilers. After losing some heat to the hot rooms the hot air goes on to warm the warm rooms (pink).

Key:
P: palaestra
A: apodytērium
T: tepidārium
C: caldārium
F: frigidārium
The small arrows mark public entrances.
The orange spaces are shops.



Vocabulary checklist 9

agnōscit: agnōvit	*recognizes*
celeriter	*quickly*
cupit: cupīvit	*wants*
dat: dedit	*gives*
diēs	*day*
ēmittit: ēmīsit	*throws, sends out*
fert: tulit	*brings, carries*
homō	*human being, man*
hospes	*guest*
ille	*that*
īnspicit: īnspexit	*looks at, examines*
iterum	*again*
manet: mānsit	*remains, stays*
medius	*middle*
mox	*soon*
offert: obtulit	*offers*
ostendit: ostendit	*shows*
post	*after*
prōcēdit: prōcessit	*proceeds, advances*
pulcher	*beautiful*
revenit: revēnit	*comes back, returns*
trādit: trādidit	*hands over*

The floors of baths often had marine themes. This mosaic of an octopus is in the women's baths at Herculaneum.

KENNETH GRAHAME

The Wind in the Willows

Introduction and Notes by
GILLIAN AVERY

PENGUIN BOOKS

porter twopence to keep a strict eye on him. They then left the horse at an inn stable, and gave what directions they could about the cart and its contents. Eventually, a slow train having landed them at a station not very far from Toad Hall, they escorted the spell-bound, sleep-walking Toad to his door, put him inside it, and instructed his housekeeper to feed him,[6] undress him, and put him to bed. Then they got out their boat from the boat-house, sculled down the river home, and at a very late hour sat down to supper in their own cosy riverside parlour, to the Rat's great joy and contentment.

The following evening the Mole, who had risen late and taken things very easy all day, was sitting on the bank fishing, when the Rat, who had been looking up his friends and gossiping, came strolling along to find him. "Heard the news?" he said. "There's nothing else being talked about, all along the river bank. Toad went up to Town by an early train this morning. And he has ordered a large and very expensive motor-car."

III
THE WILD WOOD

The Mole had long wanted to make the acquaintance of the Badger. He seemed, by all accounts, to be such an important personage and, though rarely visible, to make his unseen influence felt by everybody about the place. But whenever the Mole mentioned his wish to the Water Rat he always found himself put off. "It's all right," the Rat would say. "Badger'll turn up some day or other—he's always turning up—and then I'll introduce you. The best of fellows! But you must not only take him *as* you find him, but *when* you find him."

"Couldn't you ask him here—dinner or something?" said the Mole.

"He wouldn't come," replied the Rat simply. "Badger hates Society, and invitations, and dinner, and all that sort of thing."

"Well, then, supposing we go and call on *him?*" suggested the Mole.

"O, I'm sure he wouldn't like that at *all,*" said the Rat, quite alarmed. "He's so very shy, he'd be sure to be offended. I've never even ventured to call on him at his own home myself, though I know him so well. Besides, we can't. It's quite out of the question, because he lives in the very middle of the Wild Wood."

"Well, supposing he does," said the Mole. "You told me the Wild Wood was all right, you know."

"O, I know, I know, so it is," replied the Rat evasively. "But I think we won't go there just now. Not *just* yet. It's a long way, and he wouldn't be at home at this time of year anyhow, and he'll be coming along some day, if you'll wait quietly."

The Mole had to be content with this. But the Badger never

came along, and every day brought its amusements, and it was not till summer was long over, and cold and frost and miry ways kept them much indoors, and the swollen river raced past outside their windows with a speed that mocked at boating of any sort or kind, that he found his thoughts dwelling again with much persistence on the solitary grey Badger, who lived his own life by himself, in his hole in the middle of the Wild Wood.

In the winter time the Rat slept a great deal, retiring early and rising late. During his short day he sometimes scribbled poetry or did other small domestic jobs about the house; and, of course, there were always animals dropping in for a chat, and consequently there was a good deal of story-telling and comparing notes on the past summer and all its doings.

Such a rich chapter it had been, when one came to look back on it all! With illustrations so numerous and so very highly coloured! The pageant of the river bank[7] had marched steadily along, unfolding itself in scene-pictures that succeeded each other in stately procession. Purple loosestrife arrived early, shaking luxuriant tangled locks along the edge of the mirror whence its own face laughed back at it. Willow-herb, tender and wistful, like a pink sunset cloud, was not slow to follow. Comfrey, the purple hand-in-hand with the white, crept forth to take its place in the line; and at last one morning the diffident and delaying dog-rose stepped delicately on the stage, and one knew, as if string-music had announced it in stately chords that strayed into a gavotte, that June at last was here. One member of the company was still awaited; the shepherd-boy for the nymphs to woo, the knight for whom the ladies waited at the window, the prince that was to kiss the sleeping summer back to life and love. But when meadow-sweet, debonair and odorous in amber jerkin, moved graciously to his place in the group, then the play was ready to begin.

And what a play it had been! Drowsy animals, snug in their holes while wind and rain were battering at their doors, recalled still keen mornings, an hour before sunrise, when the white mist, as yet undispersed, clung closely along the surface of the water; then the shock of the early plunge, the scamper along the bank, and the radiant transformation of earth, air, and

water, when suddenly the sun was with them again, and grey was gold and colour was born and sprang out of the earth once more. They recalled the languorous siesta of hot midday, deep in green undergrowth, the sun striking through in tiny golden shafts and spots; the boating and bathing of the afternoon, the rambles along dusty lanes and through yellow cornfields; and the long, cool evening at last, when so many threads were gathered up, so many friendships rounded, and so many adventures planned for the morrow. There was plenty to talk about on those short winter days when the animals found themselves round the fire; still, the Mole had a good deal of spare time on his hands, and so one afternoon, when the Rat in his arm-chair before the blaze was alternately dozing and trying over rhymes that wouldn't fit, he formed the resolution to go out by himself and explore the Wild Wood, and perhaps strike up an acquaintance with Mr. Badger.

It was a cold still afternoon with a hard steely sky overhead, when he slipped out of the warm parlour into the open air. The country lay bare and entirely leafless around him, and he thought that he had never seen so far and so intimately into the insides of things as on that winter day when Nature was deep in her annual slumber and seemed to have kicked the clothes off. Copses, dells, quarries and all hidden places, which had been mysterious mines for exploration in leafy summer, now exposed themselves and their secrets pathetically, and seemed to ask him to overlook their shabby poverty for a while, till they could riot in rich masquerade as before, and trick and entice him with the old deceptions. It was pitiful in a way, and yet cheering—even exhilarating. He was glad that he liked the country undecorated, hard, and stripped of its finery. He had got down to the bare bones of it, and they were fine and strong and simple. He did not want the warm clover and the play of seeding grasses; the screens of quickset,[8] the billowy drapery of beech and elm seemed best away; and with great cheerfulness of spirit he pushed on towards the Wild Wood, which lay before him low and threatening, like a black reef in some still southern sea.

There was nothing to alarm him at first entry. Twigs crackled

under his feet, logs tripped him, funguses on stumps resembled caricatures, and startled him for the moment by their likeness to something familiar and far away; but that was all fun, and exciting. It led him on, and he penetrated to where the light was less, and trees crouched nearer and nearer, and holes made ugly mouths at him on either side.

Everything was very still now. The dusk advanced on him steadily, rapidly, gathering in behind and before; and the light seemed to be draining away like flood-water.

Then the faces began.

It was over his shoulder, and indistinctly, that he first thought he saw a face; a little evil wedge-shaped face, looking out at him from a hole. When he turned and confronted it, the thing had vanished.

He quickened his pace, telling himself cheerfully not to begin imagining things, or there would be simply no end to it. He passed another hole, and another, and another; and then—yes!—no!—yes! certainly a little narrow face, with hard eyes, had flashed up for an instant from a hole, and was gone. He hesitated—braced himself up for an effort and strode on. Then suddenly, and as if it had been so all the time, every hole, far and near, and there were hundreds of them, seemed to possess its face, coming and going rapidly, all fixing on him glances of malice and hatred: all hard-eyed and evil and sharp.

If he could only get away from the holes in the banks, he thought, there would be no more faces. He swung off the path and plunged into the untrodden places of the wood.

Then the whistling began.

Very faint and shrill it was; and far behind him, when first he heard it; but somehow it made him hurry forward. Then, still very faint and shrill, it sounded far ahead of him, and made him hesitate and want to go back. As he halted in indecision it broke out on either side, and seemed to be caught up and passed on throughout the whole length of the wood to its farthest limit. They were up and alert and ready, evidently, whoever they were! And he—he was alone, and unarmed, and far from any help; and the night was closing in.

Then the pattering began.

He thought it was only falling leaves at first, so slight and delicate was the sound of it. Then as it grew it took a regular rhythm, and he knew it for nothing else but the pat-pat-pat of little feet still a very long way off. Was it in front or behind? It seemed to be first one, and then the other, then both. It grew and it multiplied, till from every quarter as he listened anxiously, leaning this way and that, it seemed to be closing in on him. As he stood still to hearken, a rabbit came running hard towards him through the trees. He waited, expecting it to slacken pace, or to swerve from him into a different course. Instead, the animal almost brushed him as it dashed past, his face set and hard, his eyes staring. "Get out of this, you fool, get out!" the Mole heard him mutter as he swung round a stump and disappeared down a friendly burrow.

The pattering increased till it sounded like sudden hail on the dry leaf-carpet spread around him. The whole wood seemed running now, running hard, hunting, chasing, closing in round something or—somebody? In panic, he began to run too, aimlessly, he knew not whither. He ran up against things, he fell over things and into things, he darted under things and dodged round things. At last he took refuge in the deep dark hollow of an old beech tree, which offered shelter, concealment—perhaps even safety, but who could tell? Anyhow, he was too tired to run any further, and could only snuggle down into the dry leaves which had drifted into the hollow and hope he was safe for a time. And as he lay there panting and trembling, and listened to the whistlings and the patterings outside, he knew it at last, in all its fullness, that dread thing which other little dwellers in field and hedgerow had encountered here, and known as their darkest moment—that thing which the Rat had vainly tried to shield him from—the Terror of the Wild Wood!

Meantime the Rat, warm and comfortable, dozed by his fireside. His paper of half-finished verses slipped from his knee, his head fell back, his mouth opened, and he wandered by the verdant banks of dream-rivers. Then a coal slipped, the fire crackled and sent up a spurt of flame, and he woke with a start. Remembering what he had been engaged upon, he reached down to the floor for his verses, pored over them for a minute,

and then looked round for the Mole to ask him if he knew a good rhyme for something or other.

But the Mole was not there.

He listened for a time. The house seemed very quiet.

Then he called "Moly!" several times, and, receiving no answer, got up and went out into the hall.

The Mole's cap was missing from its accustomed peg. His goloshes, which always lay by the umbrella-stand, were also gone.

The Rat left the house, and carefully examined the muddy surface of the ground outside, hoping to find the Mole's tracks. There they were, sure enough. The goloshes were new, just bought for the winter, and the pimples on their soles were fresh and sharp. He could see the imprints of them in the mud, running along straight and purposeful, leading direct to the Wild Wood.

The Rat looked very grave, and stood in deep thought for a minute or two. Then he re-entered the house, strapped a belt round his waist, shoved a brace of pistols into it, took up a stout cudgel that stood in a corner of the hall, and set off for the Wild Wood at a smart pace.

It was already getting towards dusk when he reached the first fringe of trees and plunged without hesitation into the wood, looking anxiously on either side for any sign of his friend. Here and there wicked little faces popped out of holes, but vanished immediately at sight of the valorous animal, his pistols, and the great ugly cudgel in his grasp; and the whistling and pattering, which he had heard quite plainly on his first entry, died away and ceased, and all was very still. He made his way manfully through the length of the wood, to its furthest edge; then, forsaking all paths, he set himself to traverse it, laboriously working over the whole ground, and all the time calling out cheerfully, "Moly, Moly, Moly! Where are you? It's me—it's old Rat!"

He had patiently hunted through the wood for an hour or more, when at last to his joy he heard a little answering cry. Guiding himself by the sound, he made his way through the gathering darkness to the foot of an old beech tree, with a hole

in it, and from out of the hole came a feeble voice, saying "Ratty! Is that really you?"

The Rat crept into the hollow, and there he found the Mole, exhausted and still trembling. "O Rat!" he cried, "I've been so frightened, you can't think!"

"O, I quite understand," said the Rat soothingly. "You shouldn't really have gone and done it, Mole. I did my best to keep you from it. We river-bankers, we hardly ever come here by ourselves. If we have to come, we come in couples, at least; then we're generally all right. Besides, there are a hundred things one has to know, which we understand all about and you don't, as yet. I mean passwords, and signs, and sayings which have power and effect, and plants you carry in your pocket, and verses you repeat, and dodges and tricks you practise; all simple enough when you know them, but they've got to be known if you're small, or you'll find yourself in trouble. Of course if you were Badger or Otter, it would be quite another matter."

"Surely the brave Mr. Toad wouldn't mind coming here by himself, would he?" inquired the Mole.

"Old Toad?" said the Rat, laughing heartily. "He wouldn't show his face here alone, not for a whole hatful of golden guineas, Toad wouldn't."

The Mole was greatly cheered by the sound of the Rat's careless laughter, as well as by the sight of his stick and his gleaming pistols, and he stopped shivering and began to feel bolder and more himself again.

"Now then," said the Rat presently, "we really must pull ourselves together and make a start for home while there's still a little light left. It will never do to spend the night here, you understand. Too cold, for one thing."

"Dear Ratty," said the poor Mole, "I'm dreadfully sorry, but I'm simply dead beat and that's a solid fact. You *must* let me rest here a while longer, and get my strength back, if I'm to get home at all."

"O, all right," said the good-natured Rat, "rest away. It's pretty nearly pitch dark now, anyhow; and there ought to be a bit of a moon later."

So the Mole got well into the dry leaves and stretched himself

out, and presently dropped off into sleep, though of a broken and troubled sort; while the Rat covered himself up, too, as best he might, for warmth, and lay patiently waiting, with a pistol in his paw.

When at last the Mole woke up, much refreshed and in his usual spirits, the Rat said, "Now then! I'll just take a look outside and see if everything's quiet, and then we really must be off."

He went to the entrance of their retreat and put his head out. Then the Mole heard him saying quietly to himself, "Hullo! hullo! here—*is*—a—go!"

"What's up, Ratty?" asked the Mole.

"*Snow* is up," replied the Rat briefly; "or rather, *down*. It's snowing hard."

The Mole came and crouched beside him, and, looking out, saw the wood that had been so dreadful to him in quite a changed aspect. Holes, hollows, pools, pitfalls, and other black menaces to the wayfarer were vanishing fast, and a gleaming carpet of faery was springing up everywhere, that looked too delicate to be trodden upon by rough feet. A fine powder filled the air and caressed the cheek with a tingle in its touch, and the black boles of the trees showed up in a light that seemed to come from below.

"Well, well, it can't be helped," said the Rat, after pondering. "We must make a start, and take our chance, I suppose. The worst of it is, I don't exactly know where we are. And now this snow makes everything look so very different."

It did indeed. The Mole would not have known that it was the same wood. However, they set out bravely, and took the line that seemed most promising, holding on to each other and pretending with invincible cheerfulness that they recognized an old friend in every fresh tree that grimly and silently greeted them, or saw openings, gaps, or paths with a familiar turn in them, in the monotony of white space and black tree-trunks that refused to vary.

An hour or two later—they had lost all count of time—they pulled up, dispirited, weary, and hopelessly at sea, and sat down on a fallen tree-trunk to recover their breath and consider what

was to be done. They were aching with fatigue and bruised with tumbles; they had fallen into several holes and got wet through; the snow was getting so deep that they could hardly drag their little legs through it, and the trees were thicker and more like each other than ever. There seemed to be no end to this wood, and no beginning, and no difference in it, and, worst of all, no way out.

"We can't sit here very long," said the Rat. "We shall have to make another push for it, and do something or other. The cold is too awful for anything, and the snow will soon be too deep for us to wade through." He peered about him and considered. "Look here," he went on, "this is what occurs to me. There's a sort of dell down here in front of us, where the ground seems all hilly and humpy and hummocky. We'll make our way down into that, and try and find some sort of shelter, a cave or hole with a dry floor to it, out of the snow and the wind, and there we'll have a good rest before we try again, for we're both of us pretty dead beat. Besides, the snow may leave off, or something may turn up."

So once more they got on their feet, and struggled down into the dell, where they hunted about for a cave or some corner that was dry and a protection from the keen wind and the whirling snow. They were investigating one of the hummocky bits the Rat had spoken of, when suddenly the Mole tripped up and fell forward on his face with a squeal.

"O my leg!" he cried. "O my poor shin!" and he sat up on the snow and nursed his leg in both his front paws.

"Poor old Mole!" said the Rat kindly. "You don't seem to be having much luck to-day, do you? Let's have a look at the leg. Yes," he went on, going down on his knees to look, "you've cut your shin, sure enough. Wait till I get at my handkerchief, and I'll tie it up for you."

"I must have tripped over a hidden branch or a stump," said the Mole miserably. "O, my! O, my!"

"It's a very clean cut," said the Rat, examining it again attentively. "That was never done by a branch or a stump. Looks as if it was made by a sharp edge of something in metal. Funny!" He pondered awhile, and examined the humps and slopes that surrounded them.

"Well, never mind what done it," said the Mole, forgetting his grammar in his pain. "It hurts just the same, whatever done it."

But the Rat, after carefully tying up the leg with his handkerchief, had left him and was busy scraping in the snow. He scratched and shovelled and explored, all four legs working busily, while the Mole waited impatiently, remarking at intervals, "O, *come* on, Rat!"

Suddenly the Rat cried "Hooray!" and then "Hooray-oo-ray-oo-ray-oo-ray!" and fell to executing a feeble jig in the snow.

"What *have* you found, Ratty?" asked the Mole, still nursing his leg.

"Come and see!" said the delighted Rat, as he jigged on.

The Mole hobbled up to the spot and had a good look.

"Well," he said at last, slowly, "I *see* it right enough. Seen the same sort of thing before, lots of times. Familiar object, I call it. A door-scraper! Well, what of it? Why dance jigs around a door-scraper?"

"But don't you see what it *means,* you—you dull-witted animal?" cried the Rat impatiently.

"Of course I see what it means," replied the Mole. "It simply means that some *very* careless and forgetful person has left his door-scraper lying about in the middle of the Wild Wood, *just* where it's *sure* to trip *everybody* up. Very thoughtless of him, I call it. When I get home I shall go and complain about it to—to somebody or other, see if I don't!"

"O, dear! O, dear!" cried the Rat, in despair at his obtuseness. "Here, stop arguing and come and scrape!" And he set to work again and made the snow fly in all directions around him.

After some further toil his efforts were rewarded, and a very shabby door-mat lay exposed to view.

"There, what did I tell you?" exclaimed the Rat in great triumph.

"Absolutely nothing, whatever," replied the Mole, with perfect truthfulness. "Well now," he went on, "you seem to have found another piece of domestic litter, done for and thrown away, and I suppose you're perfectly happy. Better go ahead and dance your jig round that if you've got to, and get it over, and then perhaps we can go on and not waste any more time over

rubbish-heaps. Can we *eat* a door-mat? Or sleep under a door-mat? Or sit on a door-mat and sledge home over the snow on it, you exasperating rodent?"

"Do—you—mean—to—say," cried the excited Rat, "that this door-mat doesn't *tell* you anything?"

"Really, Rat," said the Mole, quite pettishly, "I think we'd had enough of this folly. Who ever heard of a door-mat *telling* anyone anything? They simply don't do it. They are not that sort at all. Door-mats know their place."

"Now look here, you—you thick-headed beast," replied the Rat, really angry, "this must stop. Not another word, but scrape—scrape and scratch and dig and hunt round, especially on the sides of the hummocks, if you want to sleep dry and warm to-night, for it's our last chance!"

The Rat attacked a snow-bank beside them with ardour, probing with his cudgel everywhere and then digging with fury; and the Mole scraped busily too, more to oblige the Rat than for any other reason, for his opinion was that his friend was getting light-headed.

Some ten minutes' hard work, and the point of the Rat's cudgel struck something that sounded hollow. He worked till he could get a paw through and feel; then called the Mole to come and help him. Hard at it went the two animals, till at last the result of their labours stood full in view of the astonished and hitherto incredulous Mole.

In the side of what had seemed to be a snow-bank stood a solid-looking little door, painted a dark green. An iron bell-pull hung by the side, and below it, on a small brass plate, neatly engraved in square capital letters, they could read by the aid of moonlight

MR. BADGER.

The Mole fell backwards on the snow from sheer surprise and delight. "Rat!" he cried in penitence, "you're a wonder! A real wonder, that's what you are. I see it all now! You argued it out, step by step, in that wise head of yours, from the very moment that I fell and cut my shin, and you looked at the cut, and

at once your majestic mind said to itself, 'Door-scraper!' And then you turned to and found the very door-scraper that done it! Did you stop there? No. Some people would have been quite satisfied; but not you. Your intellect went on working. 'Let me only just find a door-mat,' says you to yourself, 'and my theory is proved!' And of course you found your door-mat. You're so clever, I believe you could find anything you liked. 'Now,' says you, 'that door exists, as plain as if I saw it. There's nothing else remains to be done but to find it!' Well, I've read about that sort of thing in books, but I've never come across it before in real life. You ought to go where you'll be properly appreciated. You're simply wasted here, among us fellows. If I only had your head, Ratty——"

"But as you haven't," interrupted the Rat, rather unkindly, "I suppose you're going to sit on the snow all night and *talk?* Get up at once and hang on to that bell-pull you see there, and ring hard, as hard as you can, while I hammer!"

While the Rat attacked the door with his stick, the Mole sprang up at the bell-pull, clutched it and swung there, both feet well off the ground, and from quite a long way off they could faintly hear a deep-toned bell respond.

IV
MR. BADGER

They waited patiently for what seemed a very long time, stamping in the snow to keep their feet warm. At last they heard the sound of slow shuffling footsteps approaching the door from the inside. It seemed, as the Mole remarked to the Rat, like some one walking in carpet slippers that were too large for him and down at heel; which was intelligent of Mole, because that was exactly what it was.

There was the noise of a bolt shot back, and the door opened a few inches, enough to show a long snout and a pair of sleepy blinking eyes.

"Now, the *very* next time this happens," said a gruff and suspicious voice, "I shall be exceedingly angry. Who is it *this* time, disturbing people on such a night? Speak up!"

"Oh, Badger," cried the Rat, "let us in, please. It's me, Rat, and my friend Mole, and we've lost our way in the snow."

"What, Ratty, my dear little man!" exclaimed the Badger, in quite a different voice. "Come along in, both of you, at once. Why, you must be perished. Well I never! Lost in the snow! And in the Wild Wood, too, and at this time of night! But come in with you."

The two animals tumbled over each other in their eagerness to get inside, and heard the door shut behind them with great joy and relief.

The Badger, who wore a long dressing-gown, and whose slippers were indeed very down at heel, carried a flat candlestick in his paw and had probably been on his way to bed when their summons sounded. He looked kindly down on them and patted both their heads. "This is not the sort of night for small animals

to be out," he said paternally. "I'm afraid you've been up to some of your pranks again, Ratty. But come along; come into the kitchen. There's a first-rate fire there, and supper and everything."

He shuffled on in front of them, carrying the light, and they followed him, nudging each other in an anticipating sort of way, down a long, gloomy, and, to tell the truth, decidedly shabby passage, into a sort of a central hall, out of which they could dimly see other long tunnel-like passages branching, passages mysterious and without apparent end. But there were doors in the hall as well—stout oaken comfortable-looking doors. One of these the Badger flung open, and at once they found themselves in all the glow and warmth of a large fire-lit kitchen.

The floor was well-worn red brick, and on the wide hearth burnt a fire of logs, between two attractive chimney-corners tucked away in the wall, well out of any suspicion of draught. A couple of high-backed settles, facing each other on either side of the fire, gave further sitting accommodation for the sociably disposed. In the middle of the room stood a long table of plain boards placed on trestles, with benches down each side. At one end of it, where an arm-chair stood pushed back, were spread the remains of the Badger's plain but ample supper. Rows of spotless plates winked from the shelves of the dresser at the far end of the room, and from the rafters overhead hung hams, bundles of dried herbs, nets of onions, and baskets of eggs. It seemed a place where heroes could fitly feast after victory, where weary harvesters could line up in scores along the table and keep their Harvest Home[9] with mirth and song, or where two or three friends of simple tastes could sit about as they pleased and eat and smoke and talk in comfort and contentment. The ruddy brick floor smiled up at the smoky ceiling; the oaken settles, shiny with long wear, exchanged cheerful glances with each other; plates on the dresser grinned at pots on the shelf, and the merry firelight flickered and played over everything without distinction.

The kindly Badger thrust them down on a settle to toast themselves at the fire, and bade them remove their wet coats

and boots. Then he fetched them dressing-gowns and slippers, and himself bathed the Mole's shin with warm water and mended the cut with sticking-plaster till the whole thing was just as good as new, if not better. In the embracing light and warmth, warm and dry at last, with weary legs propped up in front of them, and a suggestive clink of plates being arranged on the table behind, it seemed to the storm-driven animals, now in safe anchorage, that the cold and trackless Wild Wood just left outside was miles and miles away, and all that they had suffered in it a half-forgotten dream.

When at last they were thoroughly toasted, the Badger summoned them to the table, where he had been busy laying a repast. They had felt pretty hungry before, but when they actually saw at last the supper that was spread for them, really it seemed only a question of what they should attack first where all was so attractive, and whether the other things would obligingly wait for them till they had time to give them attention. Conversation was impossible for a long time; and when it was slowly resumed, it was that regrettable sort of conversation that results from talking with your mouth full. The Badger did not mind that sort of thing at all, nor did he take any notice of elbows on the table, or everybody speaking at once. As he did not go into Society himself, he had got an idea that these things belonged to the things that didn't really matter. (We know of course that he was wrong, and took too narrow a view; because they do matter very much, though it would take too long to explain why.) He sat in his arm-chair at the head of the table, and nodded gravely at intervals as the animals told their story; and he did not seem surprised or shocked at anything, and he never said, "I told you so," or, "Just what I always said," or remarked that they ought to have done so-and-so, or ought not to have done something else. The Mole began to feel very friendly towards him.

When supper was really finished at last, and each animal felt that his skin was now as tight as was decently safe, and that by this time he didn't care a hang for anybody or anything, they gathered round the glowing embers of the great wood fire, and thought how jolly it was to be sitting up *so* late, and *so* independent, and *so* full; and after they had chatted for a time about

things in general, the Badger said heartily, "Now then! tell us the news from your part of the world. How's old Toad going on?"

"Oh, from bad to worse," said the Rat gravely, while the Mole, cocked up on a settle and basking in the firelight, his heels higher than his head, tried to look properly mournful. "Another smash-up only last week, and a bad one. You see, he will insist on driving himself, and he's hopelessly incapable. If he'd only employ a decent, steady, well-trained animal, pay him good wages, and leave everything to him, he'd get on all right. But no; he's convinced he's a heaven-born driver, and nobody can teach him anything; and all the rest follows."

"How many has he had?" inquired the Badger gloomily.

"Smashes, or machines?" asked the Rat. "Oh, well, after all, it's the same thing—with Toad. This is the seventh. As for the others—you know that coach-house of his? Well, it's piled up—literally piled up to the roof—with fragments of motor-cars, none of them bigger than your hat! That accounts for the other six—so far as they can be accounted for."

"He's been in hospital three times," put in the Mole; "and as for the fines he's had to pay, it's simply awful to think of."

"Yes, and that's part of the trouble," continued the Rat. "Toad's rich, we all know; but he's not a millionaire. And he's a hopelessly bad driver, and quite regardless of law and order. Killed or ruined—it's got to be one of the two things, sooner or later. Badger! we're his friends—oughtn't we to do something?"

The Badger went through a bit of hard thinking. "Now look here!" he said at last, rather severely; "of course you know I can't do anything *now?*"

His two friends assented, quite understanding his point. No animal, according to the rules of animal-etiquette, is ever expected to do anything strenuous, or heroic, or even moderately active during the off-season of winter. All are sleepy—some actually asleep. All are weather-bound, more or less; and all are resting from arduous days and nights, during which every muscle in them has been severely tested, and every energy kept at full stretch.

"Very well then!" continued the Badger. "*But,* when once the year has really turned, and the nights are shorter, and halfway

through them one rouses and feels fidgety and wanting to be up and doing by sunrise, if not before—*you* know!——"

Both animals nodded gravely. *They* knew!

"Well, *then,*" went on the Badger, "we—that is, you and me and our friend the Mole here—we'll take Toad seriously in hand. We'll stand no nonsense whatever. We'll bring him back to reason, by force if need be. We'll *make* him be a sensible Toad. We'll—you're asleep, Rat!"

"Not me!" said the Rat, waking up with a jerk.

"He's been asleep two or three times since supper," said the Mole, laughing. He himself was feeling quite wakeful and even lively, though he didn't know why. The reason was, of course, that he being naturally an underground animal by birth and breeding, the situation of Badger's house exactly suited him and made him feel at home; while the Rat, who slept every night in a bedroom the windows of which opened on a breezy river, naturally felt the atmosphere still and oppressive.

"Well, it's time we were all in bed," said the Badger, getting up and fetching flat candlesticks. "Come along, you two, and I'll show you your quarters. And take your time to-morrow morning—breakfast at any hour you please!"

He conducted the two animals to a long room that seemed half bedchamber and half loft. The Badger's winter stores, which indeed were visible everywhere, took up half the room—piles of apples, turnips, and potatoes, baskets full of nuts, and jars of honey; but the two little white beds on the remainder of the floor looked soft and inviting, and the linen on them, though coarse, was clean and smelt beautifully of lavender; and the Mole and the Water Rat, shaking off their garments in some thirty seconds, tumbled in between the sheets in great joy and contentment.

In accordance with the kindly Badger's injunctions, the two tired animals came down to breakfast very late next morning, and found a bright fire burning in the kitchen, and two young hedgehogs sitting on a bench at the table, eating oatmeal porridge out of wooden bowls. The hedgehogs dropped their spoons, rose to their feet, and ducked their heads respectfully as the two entered.

"There, sit down, sit down," said the Rat pleasantly, "and go on with your porridge. Where have you youngsters come from? Lost your way in the snow, I suppose?"

"Yes, please, sir," said the elder of the two hedgehogs respectfully. "Me and little Billy here, we was trying to find our way to school—mother *would* have us go, was the weather ever so—and of course we lost ourselves, sir, and Billy he got frightened and took and cried, being young and faint-hearted. And at last we happened up against Mr. Badger's back door, and made so bold as to knock, sir, for Mr. Badger he's a kind-hearted gentleman, as everyone knows——"

"I understand," said the Rat, cutting himself some rashers from a side of bacon, while the Mole dropped some eggs into a saucepan. "And what's the weather like outside? You needn't 'sir' me quite so much," he added.

"O, terrible bad, sir, terrible deep the snow is," said the hedgehog. "No getting out for the likes of you gentlemen to-day."

"Where's Mr. Badger?" inquired the Mole, as he warmed the coffee-pot before the fire.

"The master's gone into his study, sir," replied the hedgehog, "and he said as how he was going to be particular busy this morning, and on no account was he to be disturbed."

This explanation, of course, was thoroughly understood by every one present. The fact is, as already set forth, when you live a life of intense activity for six months in the year, and of comparative or actual somnolence for the other six, during the latter period you cannot be continually pleading sleepiness when there are people about or things to be done. The excuse gets monotonous. The animals well knew that Badger, having eaten a hearty breakfast, had retired to his study and settled himself in an arm-chair with his legs up on another and a red cotton handkerchief over his face, and was being "busy" in the usual way at this time of the year.

The front-door bell clanged loudly, and the Rat, who was very greasy with buttered toast, sent Billy, the smaller hedgehog, to see who it might be. There was a sound of much stamping in the hall, and presently Billy returned in front of the

Otter, who threw himself on the Rat with an embrace and a shout of affectionate greeting.

"Get off!" spluttered the Rat, with his mouth full.

"Thought I should find you here all right," said the Otter cheerfully. "They were all in a great state of alarm along River Bank when I arrived this morning. Rat never been home all night—nor Mole either—something dreadful must have happened, they said; and the snow had covered up all your tracks, of course. But I knew that when people were in any fix they mostly went to Badger, or else Badger got to know of it somehow, so I came straight off here, through the Wild Wood and the snow! My! it was fine, coming through the snow as the red sun was rising and showing against the black tree-trunks! As you went along in the stillness, every now and then masses of snow slid off the branches suddenly with a flop! making you jump and run for cover. Snow-castles and snow-caverns had sprung up out of nowhere in the night—and snow-bridges, terraces, ramparts—I could have stayed and played with them for hours. Here and there great branches had been torn away by the sheer weight of the snow, and robins perched and hopped on them in their perky conceited way, just as if they had done it themselves. A ragged string of wild geese passed overhead, high on the grey sky, and a few rooks whirled over the trees, inspected, and flapped off homewards with a disgusted expression; but I met no sensible being to ask the news of. About halfway across I came on a rabbit sitting on a stump, cleaning his silly face with his paws. He was a pretty scared animal when I crept up behind him and placed a heavy fore-paw on his shoulder. I had to cuff his head once or twice to get any sense out of it at all. At last I managed to extract from him that Mole had been seen in the Wild Wood last night by one of them. It was the talk of the burrows, he said, how Mole, Mr. Rat's particular friend, was in a bad fix; how he had lost his way, and 'They' were up and out hunting, and were chivvying him round and round. 'Then why didn't any of you *do* something?' I asked. 'You mayn't be blest with brains, but there are hundreds and hundreds of you, big, stout fellows, as fat as butter, and your burrows running in all directions, and you could have

taken him in and made him safe and comfortable, or tried to, at all events.' 'What, *us?*' he merely said: '*do* something? us rabbits?' So I cuffed him again and left him. There was nothing else to be done. At any rate, I had learnt something; and if I had had the luck to meet any of 'Them' I'd have learnt something more—or *they* would."

"Weren't you at all—er—nervous?" asked the Mole, some of yesterday's terror coming back to him at the mention of the Wild Wood.

"Nervous?" The Otter showed a gleaming set of strong white teeth as he laughed. "I'd give 'em nerves if any of them tried anything on with me. Here, Mole, fry me some slices of ham, like the good little chap you are. I'm frightfully hungry, and I've got any amount to say to Ratty here. Haven't seen him for an age."

So the good-natured Mole, having cut some slices of ham, set the hedgehogs to fry it, and returned to his own breakfast, while the Otter and the Rat, their heads together, eagerly talked river-shop, which is long shop and talk that is endless, running on like the babbling river itself.

A plate of fried ham had just been cleared and sent back for more, when the Badger entered, yawning and rubbing his eyes, and greeted them all in his quiet, simple way, with kind enquiries for every one. "It must be getting on for luncheon time," he remarked to the Otter. "Better stop and have it with us. You must be hungry, this cold morning."

"Rather!" replied the Otter, winking at the Mole. "The sight of these greedy young hedgehogs stuffing themselves with fried ham makes me feel positively famished."

The hedgehogs, who were just beginning to feel hungry again after their porridge, and after working so hard at their frying, looked timidly up at Mr. Badger, but were too shy to say anything.

"Here, you two youngsters be off home to your mother," said the Badger kindly. "I'll send some one with you to show you the way. You won't want any dinner to-day, I'll be bound."

He gave them sixpence apiece and a pat on the head, and they went off with much respectful swinging of caps and touching of forelocks.

Presently they all sat down to luncheon together. The Mole found himself placed next to Mr. Badger, and, as the other two were still deep in river-gossip from which nothing could divert them, he took the opportunity to tell Badger how comfortable and home-like it all felt to him. "Once well underground," he said, "you know exactly where you are. Nothing can happen to you, and nothing can get at you. You're entirely your own master, and you don't have to consult anybody or mind what they say. Things go on all the same overhead, and you let 'em, and don't bother about 'em. When you want to, up you go, and there the things are, waiting for you."

The Badger simply beamed on him. "That's exactly what I say," he replied. "There's no security, or peace and tranquillity, except underground. And then, if your ideas get larger and you want to expand—why, a dig and a scrape, and there you are! If you feel your house is a bit too big, you stop up a hole or two, and there you are again! No builders, no tradesmen, no remarks passed on you by fellows looking over your wall, and, above all, no *weather*. Look at Rat, now. A couple of feet of flood water, and he's got to move into hired lodgings; uncomfortable, inconveniently situated, and horribly expensive. Take Toad. I say nothing against Toad Hall; quite the best house in these parts, *as* a house. But supposing a fire breaks out—where's Toad? Supposing tiles are blown off, or walls sink or crack, or windows get broken—where's Toad? Supposing the rooms are draughty—I *hate* a draught myself—where's Toad? No, up and out of doors is good enough to roam about and get one's living in; but underground to come back to at last—that's my idea of *home!*"

The Mole assented heartily; and the Badger in consequence got very friendly with him. "When lunch is over," he said, "I'll take you all round this little place of mine. I can see you'll appreciate it. You understand what domestic architecture ought to be, you do."

After luncheon, accordingly, when the other two had settled themselves into the chimney-corner and had started a heated argument on the subject of *eels*, the Badger lighted a lantern and bade the Mole follow him. Crossing the hall, they passed down one of the principal tunnels, and the wavering light of the

lantern gave glimpses on either side of rooms both large and small, some mere cupboards, others nearly as broad and imposing as Toad's dining-hall. A narrow passage at right angles led them into another corridor, and here the same thing was repeated. The Mole was staggered at the size, the extent, the ramifications of it all; at the length of the dim passages, the solid vaultings of the crammed store-chambers, the masonry everywhere, the pillars, the arches, the pavements. "How on earth, Badger," he said at last, "did you ever find time and strength to do all this? It's astonishing!"

"It *would* be astonishing indeed," said the Badger simply, "if I *had* done it. But as a matter of fact I did none of it—only cleaned out the passages and chambers, as far as I had need of them. There's lots more of it, all round about. I see you don't understand, and I must explain it to you. Well, very long ago, on the spot where the Wild Wood waves now, before ever it had planted itself and grown up to what it now is, there was a city—a city of people,[10] you know. Here, where we are standing, they lived, and walked, and talked, and slept, and carried on their business. Here they stabled their horses and feasted, from here they rode out to fight or drove out to trade. They were a powerful people, and rich, and great builders. They built to last, for they thought their city would last for ever."

"But what has become of them all?" asked the Mole.

"Who can tell?" said the Badger. "People come—they stay for a while, they flourish, they build—and they go. It is their way. But we remain. There were badgers here, I've been told, long before that same city ever came to be. And now there are badgers here again. We are an enduring lot, and we may move out for a time, but we wait, and are patient, and back we come. And so it will ever be."

"Well, and when they went at last, those people?" said the Mole.

"When they went," continued the Badger, "the strong winds and persistent rains took the matter in hand, patiently, ceaselessly, year after year. Perhaps we badgers too, in our small way, helped a little—who knows? It was all down, down, down, gradually—ruin and levelling and disappearance. Then it was

all up, up, up, gradually, as seeds grew to saplings, and saplings to forest trees, and bramble and fern came creeping in to help. Leaf-mould rose and obliterated, streams in their winter freshets brought sand and soil to clog and to cover, and in course of time our home was ready for us again, and we moved in. Up above us, on the surface, the same thing happened. Animals arrived, liked the look of the place, took up their quarters, settled down, spread, and flourished. They didn't bother themselves about the past—they never do; they're too busy. The place was a bit humpy and hillocky, naturally, and full of holes; but that was rather an advantage. And they don't bother about the future, either—the future when perhaps the people will move in again—for a time—as may very well be. The Wild Wood is pretty well populated by now; with all the usual lot, good, bad, and indifferent—I name no names. It takes all sorts to make a world. But I fancy you know something about them yourself by this time."

"I do indeed," said the Mole, with a slight shiver.

"Well, well," said the Badger, patting him on the shoulder, "it was your first experience of them, you see. They're not so bad really; and we must all live and let live. But I'll pass the word around to-morrow, and I think you'll have no further trouble. Any friend of *mine* walks where he likes in this country, or I'll know the reason why!"

When they got back to the kitchen again, they found the Rat walking up and down, very restless. The underground atmosphere was oppressing him and getting on his nerves, and he seemed really to be afraid that the river would run away if he wasn't there to look after it. So he had his overcoat on, and his pistols thrust into his belt again. "Come along, Mole," he said anxiously, as soon as he caught sight of them. "We must get off while it's daylight. Don't want to spend another night in the Wild Wood again."

"It'll be all right, my fine fellow," said the Otter. "I'm coming along with you, and I know every path blindfold; and if there's a head that needs to be punched, you can confidently rely upon me to punch it."

"You really needn't fret, Ratty," added the Badger placidly.

"My passages run further than you think, and I've bolt-holes to the edge of the wood in several directions, though I don't care for everybody to know about them. When you really have to go, you shall leave by one of my short cuts. Meantime, make yourself easy, and sit down again."

The Rat was nevertheless still anxious to be off and attend to his river, so the Badger, taking up his lantern again, led the way along a damp and airless tunnel that wound and dipped, part vaulted, part hewn through solid rock, for a weary distance that seemed to be miles. At last daylight began to show itself confusedly through tangled growth overhanging the mouth of the passage; and the Badger, bidding them a hasty good-bye, pushed them hurriedly through the opening, made everything look as natural as possible again, with creepers, brushwood, and dead leaves, and retreated.

They found themselves standing on the very edge of the Wild Wood. Rocks and brambles and tree-roots behind them, confusedly heaped and tangled; in front, a great space of quiet fields, hemmed by lines of hedges black on the snow, and, far ahead, a glint of the familiar old river, while the wintry sun hung red and low on the horizon. The Otter, as knowing all the paths, took charge of the party, and they trailed out on a bee-line for a distant stile. Pausing there a moment and looking back, they saw the whole mass of the Wild Wood, dense, menacing, compact, grimly set in vast white surroundings; simultaneously they turned and made swiftly for home, for firelight and the familiar things it played on, for the voice, sounding cheerily outside their window, of the river that they knew and trusted in all its moods, that never made them afraid with any amazement.

As he hurried along, eagerly anticipating the moment when he would be at home again among the things he knew and liked, the Mole saw clearly that he was an animal of tilled field and hedgerow, linked to the ploughed furrow, the frequented pasture, the lane of evening lingerings, the cultivated garden-plot. For others the asperities, the stubborn endurance, or the clash of actual conflict, that went with Nature in the rough; he must be wise, must keep to the pleasant places in which his lines were laid and which held adventure enough, in their way, to last for a lifetime.

V

DULCE DOMUM[11]

The sheep ran huddling together against the hurdles, blowing out thin nostrils and stamping with delicate fore-feet, their heads thrown back and a light steam rising from the crowded sheep-pen into the frosty air, as the two animals hastened by in high spirits, with much chatter and laughter. They were returning across country after a long day's outing with Otter, hunting and exploring on the wide uplands where certain streams tributary to their own River had their first small beginnings; and the shades of the short winter day were closing in on them, and they had still some distance to go. Plodding at random across the plough, they had heard the sheep and had made for them; and now, leading from the sheep-pen, they found a beaten track that made walking a lighter business, and responded, moreover, to that small inquiring something which all animals carry inside them, saying unmistakably, "Yes, quite right; *this* leads home!"

"It looks as if we were coming to a village," said the Mole somewhat dubiously, slackening his pace, as the track, that had in time become a path and then had developed into a lane, now handed them over to the charge of a well-metalled road. The animals did not hold with villages, and their own highways, thickly frequented as they were, took an independent course, regardless of church, post office, or public-house.

"Oh, never mind!" said the Rat. "At this season of the year they're all safe indoors by this time, sitting round the fire; men, women, and children, dogs and cats and all. We shall slip through all right, without any bother or unpleasantness, and we can have a look at them through their windows if you like, and see what they're doing."

Written Exercises

State whether each number is prime or composite.

1. 39	**2.** 41	**3.** 51	**4.** 111	**5.** 124
6. 321	**7.** 641	**8.** 753	**9.** 894	**10.** 1164
11. 2061	**12.** 3001			

Give the prime factorization of each whole number.

13. 12	**14.** 50	**15.** 24	**16.** 28	**17.** 39
18. 56	**19.** 66	**20.** 51	**21.** 54	**22.** 63
23. 84	**24.** 90	**25.** 196	**26.** 360	**27.** 308
28. 693	**29.** 114	**30.** 1150		

31. Explain why 2 is the only even prime number.

32. Write the prime factorizations of the square numbers 16, 36, 81, and 144 by using exponents. What do you think must be true of the exponents in the prime factorization of a square number?

33. Explain why the sum of two prime numbers greater than 2 can never be a prime number.

34. Explain how you know that each of the following numbers must be composite: 111; 111,111; 111,111,111; . . .

35. List all the possible digits that can be the last digit of a prime number that is greater than 10.

36. Choose a six-digit number, such as 652,652, the last three digits of which are a repeat of the first three digits. Show that 7, 11, and 13 are all factors of the number you chose.

37. Since 7, 11, and 13 are factors of any number of the type defined in Exercise 36, what is the largest composite number that is always a factor of such a number? What is the other factor?

38. Give an example to show that the Fundamental Theorem of Arithmetic would be false if 1 were defined to be a prime number.

Review Exercises

Evaluate.

1. 7^3	**2.** 3^5	**3.** 4^3	**4.** 2^6	**5.** 9^4
6. 10^3	**7.** 10^5	**8.** $3^4 \cdot 2^3$	**9.** $5^3 \cdot 10^2$	**10.** $6^3 \cdot 4^4$

Self-Test B

Find out whether each number is prime or composite. If it is composite, give its prime factorization.

1. 108 **2.** 79 **3.** 87 **4.** 109 [5-4]

Find the GCF of the numbers. If they are relatively prime, so state.

5. 30, 90 **6.** 98, 112 **7.** 45, 77 **8.** 75, 105 [5-5]

Find the LCM of each pair of numbers.

9. 28, 70 **10.** 21, 27 **11.** 39, 65 **12.** 45, 50 [5-6]

Self-Test answers and Extra Practice are at the back of the book.

COMPUTER INVESTIGATION: *Factors*

The computer can be used to find all the pairs of factors of a number. The program below will print all the pairs of factors of a given number. If the number is a perfect square, there will be a pair of equal factors.

```
10  PRINT "TO FACTOR A NUMBER N:"
20  PRINT "N = ";
30  INPUT N
40  PRINT "PAIRS OF FACTORS ARE:"
50  FOR F = 1 TO N
60  LET Q = N / F
70  IF Q <  > INT (Q) THEN 90
80  PRINT F; TAB( 10);Q
90  NEXT F
100  END
```

RUN the program to print out the pairs of factors of the following.

1. 216 **2.** 256 **3.** 576 **4.** 672 **5.** 1680

6. 225 **7.** 625 **8.** 1450 **9.** 1840 **10.** 1059

You can see that we do not need to print all the pairs to find the factors of the number. We can stop as soon as the quotient becomes less than the factor being tested. Insert this line into the program.

65 IF Q < F THEN 100

11-20. Repeat Exercises 1–10 with the shorter print-out. If a number is a square number, state a square root of it.

Self-Test A

Add or subtract. Simplify.

1. $\frac{8}{17} + \frac{5}{17}$
2. $\frac{3}{8} + \frac{7}{32}$
3. $\frac{7}{9} - \frac{10}{27}$ [7-1]
4. $3\frac{5}{12} + 5\frac{1}{12}$
5. $6\frac{3}{4} - 4\frac{7}{16}$
6. $2\frac{5}{8} + 4\frac{7}{24}$ [7-2]

Multiply or divide. Simplify.

7. $45 \times \frac{5}{9}$
8. $\frac{7}{12} \times \frac{3}{8}$
9. $\frac{15}{28} \times \frac{7}{10}$ [7-3]
10. $\frac{3}{5} \div 15$
11. $\frac{9}{25} \div \frac{4}{5}$
12. $\frac{18}{55} \div \frac{3}{11}$ [7-4]
13. $5\frac{2}{3} \times \frac{5}{6}$
14. $12\frac{3}{5} \div 2\frac{5}{8}$
15. $4\frac{6}{7} \times 2\frac{6}{17}$ [7-5]

Self-Test answers and Extra Practice are at the back of the book.

Class Exercises

State which operation is to be performed first.

1. $12(18 + 36)$
2. $(100 - 16) \div 7$
3. $4[(6 + 17)2]$
4. $[(54 - 12) + 26] \div 25$
5. $15 + 12 \times 3$
6. $87 - 16 \times 4$
7. $31 + 88 \div 11$
8. $96 \div 2 + 10$
9. $96 \div (2 + 10)$
10. $55 - 30 \div 5$
11. $(55 - 30) \div 5$
12. $84 \div (2 + 5)$

Written Exercises

Simplify.

A

1. $7 + (9 \times 2)$
2. $(5 \times 7) + 4$
3. $(19 - 3) \div 4$
4. $(12 + 9)3$
5. $7 + (15 - 3)$
6. $18 + (42 \div 6)$
7. $48 - (5 \times 4)$
8. $12(11 - 5)$
9. $72 \div (8 - 2)$
10. $3 \times 9 + 18$
11. $64 \div 16 + 16$
12. $37 + 13 \times 2$
13. $20 + 6 \times 8$
14. $(37 + 13)4$
15. $102 \div 3 - 1$
16. $86 - 40 \div 2$
17. $(18 - 5)(23 - 9)$
18. $(32 + 24) \div (30 - 22)$
19. $(12 \times 9) - (8 \times 11)$
20. $(11 \times 7) + (90 \div 6)$
21. $(135 \div 3) - (17 \times 2)$
22. $(148 \div 4) + (16 \times 5)$

B

23. $[48 - (4 \times 3)] \div 3$
24. $38 - [25 \div (43 - 18)]$
25. $[18 + (15 - 3)] + 14$
26. $[42 + (24 \div 6)] \div 2$
27. $53 + [64 - (10 \times 3)]$
28. $49 \div [81 - (37 \times 2)]$
29. $34 + 16 \times 2 - 29$
30. $126 - 9 \div 3 \times 28$
31. $181 - 24 \times 3 \div 8$
32. $8 + 6 \times 12 \div 4 + 6$
33. $19 + 84 \div 4 \times 8 - 11$
34. $97 - 75 \div 5 \times 3 + 68$

Evaluate if $t = 12$, $w = 10$, $x = 9$, $y = 4$, and $z = 3$.

C

35. $tx - wz$
36. $x(y + w)$
37. $3z \div (y - z)$
38. $tw \div z + xy$
39. $ty + wx \div z$
40. $xy \div t + wz$

Review Exercises

Perform the indicated operation.

1. $195 + 206 + 17$
2. $5004 - 729$
3. 607×74
4. $1280 \div 16$
5. $918 + 87 + 105$
6. $9018 - 119$
7. 727×102
8. $6539 \div 13$

Class Exercises

State the two transformations you would use to find the solution of each equation. Be sure to specify which transformation you would use first.

1. $3n + 2 = 8$
2. $4n - 1 = 19$
3. $\frac{1}{2}n - 6 = 1$
4. $\frac{1}{3}n + 5 = 7$
5. $\frac{2}{3}n - 6 = 14$
6. $\frac{5}{2}n + 2 = 13$
7. $3n - 6 = 15$
8. $7n + 21 = 63$
9. $\frac{3}{4}n - 8 = 12$
10. $\frac{1}{2}n + 2 = 5$
11. $2\frac{1}{3}n - 2 = 8$
12. $1\frac{2}{3}n + 15 = 51$

Written Exercises

Solve each equation.

A
1. $2n - 5 = 17$
2. $3n + 8 = 23$
3. $5n + 6 = 41$
4. $4n - 15 = 9$
5. $6n + 11 = 77$
6. $8n - 13 = 51$
7. $50 - 3n = 20$
8. $42 - 5n = 7$
9. $29 - 6n = 11$
10. $79 - 8n = 15$
11. $\frac{1}{4}n + 5 = 25$
12. $\frac{1}{8}n - 11 = 21$
13. $\frac{1}{2}n + 3 = 18$
14. $\frac{1}{3}n - 7 = 11$
15. $\frac{1}{5}n - 2 = 9$
16. $\frac{1}{4}n + 3 = 8$
17. $\frac{2}{3}n + 12 = 28$
18. $\frac{3}{5}n - 11 = 7$
19. $6n - 7 = 19$
20. $10n + 4 = 39$
21. $\frac{6}{5}n - 7 = 20$
22. $\frac{15}{4}n + 7 = 32$
23. $2\frac{2}{5}n + 5 = 23$
24. $1\frac{1}{7}n - 9 = 27$

B
25. $\frac{3}{5}n + \frac{2}{3} = \frac{8}{3}$
26. $\frac{2}{3}n - \frac{5}{8} = \frac{7}{8}$
27. $\frac{3}{4}n - \frac{11}{15} = \frac{3}{5}$
28. $\frac{5}{6}n + \frac{1}{10} = \frac{29}{30}$
29. $\frac{7}{8}n - \frac{5}{6} = \frac{3}{4}$
30. $\frac{1}{3}n - \frac{11}{25} = \frac{7}{10}$
31. $\frac{2}{5}n + \frac{3}{7} = \frac{11}{5}$
32. $\frac{1}{6}n + \frac{1}{5} = \frac{7}{11}$
33. $1\frac{5}{8}n + \frac{13}{12} = \frac{51}{4}$
34. $2\frac{2}{3}n - \frac{4}{7} = \frac{8}{9}$
35. $\frac{11}{3}n - \frac{5}{9} = \frac{5}{6}$
36. $\frac{7}{2}n - \frac{11}{12} = \frac{5}{9}$
37. $1\frac{3}{8}n + \frac{1}{4} = \frac{7}{8}$
38. $\frac{3}{4}n - \frac{1}{12} = \frac{7}{3}$
39. $\frac{3}{7}n + \frac{4}{5} = \frac{6}{7}$

5-4 Prime Numbers and Composite Numbers

Number	Factors
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1, 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12

Consider the list of counting numbers and their factors given at the right. Notice that each of the numbers 2, 3, 5, 7, and 11 has *exactly* two factors: 1 and the number itself. A number with this property is called a **prime number.** A counting number that has more than two factors is called a **composite number.** In the list 4, 6, 8, 9, 10, and 12 are all composite numbers. Since 1 has exactly one factor, it is neither prime nor composite.

About 230 B.C. Eratosthenes, a Greek mathematician, suggested a way to find prime numbers in a list of all the counting numbers up to a certain number. Eratosthenes first crossed out all multiples of 2, except 2 itself. Next he crossed out all multiples of the next remaining number, 3, except 3 itself. He continued crossing out multiples of each successive remaining number except the number itself. The numbers remaining at the end of this process are the primes.

	~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~
~~10~~	11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19
~~20~~	~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29
~~30~~	31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	. . .	

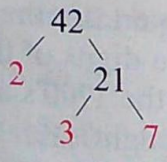
The method just described is called the **Sieve of Eratosthenes,** because it picks out the prime numbers as a strainer, or sieve, picks out solid particles from a liquid.

Every counting number greater than 1 has at least one prime factor, which may be the number itself. You can factor a number into prime factors by using either of the following methods.

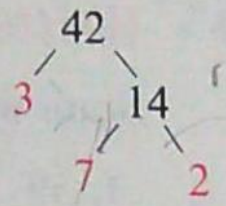
Inverted short division

$$2\overline{)42}$$
$$3\overline{)21}$$
$$7$$

Factor tree



Another factor tree for the number 42 is shown at the right. Notice that the prime factors of 42 are the same in either factor tree except for their order. Every whole number is similar to 42 in this respect. This fact is expressed in the following theorem.



Fundamental Theorem of Arithmetic

Every composite number greater than 1 can be written as a product of prime factors in exactly one way, except for the order of the factors.

When we write 42 as $2 \cdot 3 \cdot 7$, this product of prime factors is called the **prime factorization** of 42.

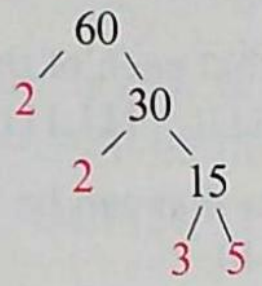
EXAMPLE Give the prime factorization of 60.

Solution

Method 1

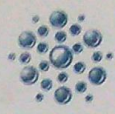
$$\begin{array}{r|l} 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

Method 2



Using either method, we find that the prime factorization of 60 is $2 \cdot 2 \cdot 3 \cdot 5$ or $2^2 \cdot 3 \cdot 5$.

5-5 Greatest Common Factor



▲ See Pages E7–E8

If the factors of the numbers 30 and 42 are listed, the numbers 1, 2, 3, and 6 appear in both lists.

Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30

Factors of 42: 1, 2, 3, 6, 7, 14, 21, 42

These numbers are called **common factors** of 30 and 42. The number 6 is the greatest of these and is therefore called the **greatest common factor** of the two numbers. We write

$$\text{GCF}(30, 42) = 6$$

to denote the greatest common factor of 30 and 42.

EXAMPLE 1 Find GCF(54, 72).

Solution List the factors of each number.

Factors of 54: 1, 2, 3, 6, 9, 18, 27, 54

Factors of 72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

The common factors are 1, 2, 3, 6, 9, and 18.
The greatest number in both lists is 18. Therefore,

$$\text{GCF}(54, 72) = 18.$$

Another way to find the GCF of two numbers is to use their prime factorizations. To find the GCF, multiply together the greatest power of each prime factor that occurs in *both* prime factorizations.

EXAMPLE 2 Find GCF(54, 72) using the prime factorization method.

Solution First find the prime factorizations of 54 and 72.

$$54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$$
$$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 2^3 \cdot 3^2$$

Find the greatest power of 2 that occurs in both prime factorizations.
The greatest power of 2 that occurs in both prime factorizations is 2.

Find the greatest power of 3 that occurs in both prime factorizations.
The greatest power of 3 that occurs in both prime factorizations is 3^2.

Therefore, $\text{GCF}(54, 72) = 2 \cdot 3^2 = 18$.

EXAMPLE 3 Find GCF(45, 60) using the prime factorization method.

Solution

$45 = 3 \cdot 3 \cdot 5 = 3^2 \cdot 5$
$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$

Since 2 is not a factor of 45, there is no greatest power of 2 that occurs in both prime factorizations.
The greatest power of 3 that occurs in both prime factorizations is 3.
The greatest power of 5 that occurs in both prime factorizations is 5.
Therefore, GCF(45, 60) $= 3 \cdot 5 = 15$.

The number 1 is a common factor of any two whole numbers. If 1 is the GCF, then the two numbers are said to be **relatively prime.** As the next example shows, two numbers can be relatively prime even if one or both are composite.

EXAMPLE 4 Show that 15 and 16 are relatively prime.

Solution

List the factors of each number.

Factors of 15: 1, 3, 5, 15

Factors of 16: 1, 2, 4, 8, 16

Since GCF(15, 16) = 1, the two numbers are relatively prime.

Class Exercises

Give the GCF of each pair of numbers.

1. 4, 10 **2.** 15, 35 **3.** 6, 12 **4.** 9, 14 **5.** 18, 27

State whether the numbers in each pair are relatively prime.

6. 16, 20 **7.** 5, 15 **8.** 8, 9 **9.** 6, 35 **10.** 22, 26

Written Exercises

Find the GCF of each pair of numbers.

1. 16, 24 **2.** 18, 45 **3.** 24, 36 **4.** 26, 39

5. 15, 28 **6.** 44, 55 **7.** 28, 42 **8.** 75, 175

9. 60, 105 **10.** 54, 81 **11.** 56, 84 **12.** 63, 100

5-6 Least Common Multiple

If the first few multiples of 8 and 12 are listed in order, certain numbers appear in both lists:

Multiples of 8: 0, 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, . . .

Multiples of 12: 0, 12, 24, 36, 48, 60, 72, 84, . . .

The numbers 0, 24, 48, 72, . . . are called **common multiples** of 8 and 12. Excluding 0, the least of these multiples is 24 and is therefore called the **least common multiple.** We write the least common multiple of 8 and 12 as

$$\text{LCM}(8, 12) = 24.$$

To find the LCM of two whole numbers, we can write out lists of multiples of the two numbers as above. The LCM will be the first multiple, excluding 0, that occurs in both lists.

A second method is to write out the first few multiples of the larger of the two numbers and then test each multiple for divisibility by the smaller number. The first multiple of the larger number that is divisible by the smaller number is the LCM.

EXAMPLE 1 Find LCM(12, 15).

Solution Write out the first few multiples of 15 (excluding 0).

15, 30, 45, 60, 75, 90

Test each for divisibility by 12.

$$12\overline{)15}\ 1\text{ R }3 \qquad 12\overline{)30}\ 2\text{ R }6 \qquad 12\overline{)45}\ 3\text{ R }9 \qquad 12\overline{)60}\ 5$$

Therefore, LCM(12, 15) = 60.

The LCM of two whole numbers can be found using their prime factorizations. For each prime factor of the two numbers, multiply together the greatest power of that factor that occurs in *either* prime factorization. The product will be the LCM.

EXAMPLE 2 Find LCM(54, 60).

Solution

$$54 = 2 \cdot 3 \cdot 3 \cdot 3 = 2 \cdot 3^3$$
$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

The greatest power of 2 that occurs in either prime factorization is 2^2.

7-4 Division of Fractions

◀ See Pages EI3–EI4

Certain numbers, when multiplied together, have the product 1. For example,

$$5 \times \frac{1}{5} = 1 \quad \text{and} \quad \frac{3}{4} \times \frac{4}{3} = 1.$$

Two numbers whose product is 1 are called **reciprocals** of each other. Thus $\frac{3}{4}$ is the reciprocal of $\frac{4}{3}$, and $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$. Since the product of 0 and any number is always 0, the number 0 has no reciprocal. This suggests the following property.

Property

Every nonzero number has a unique (exactly one) reciprocal.

COMMUNICATION IN MATHEMATICS: *Study Skills*

It is important to know the meanings of all the words in a mathematical definition or property. If you are not sure about the mathematical meaning, check the glossary at the back of the book, or look up the meaning in a mathematics dictionary or reference book.

Recall that multiplication and division are inverse operations. Now study the following examples.

$18 = 3 \times 6$, so $18 \div 6 = 3$. Also, $18 \times \frac{1}{6} = 3$.

$7 = 28 \times \frac{1}{4}$, so $7 \div \frac{1}{4} = 28$. Also, $7 \times 4 = 28$.

$20 = 24 \times \frac{5}{6}$, so $20 \div \frac{5}{6} = 24$. Also, $20 \times \frac{6}{5} = 24$.

These examples suggest that dividing a number by a fraction is the same as multiplying the number by the reciprocal of the fraction.

Rule

If a, b, c, and d are whole numbers with $b \neq 0$, $c \neq 0$, and $d \neq 0$, then

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}.$$

7-5 Multiplication and Division of Mixed Numbers

One method of finding the product of two mixed numbers is to first change the mixed numbers to improper fractions and then multiply. A second method makes use of the distributive property. This method is convenient for doing simple products mentally. Both methods are illustrated in the following example.

EXAMPLE 1 Multiply $6 \times 3\frac{1}{12}$. Simplify.

Solution *Method 1*

$$6 \times 3\frac{1}{12} = \frac{6}{1} \times \frac{37}{12} = \frac{\overset{1}{\cancel{6}}}{1} \times \frac{37}{\underset{2}{\cancel{12}}} = \frac{37}{2} = 18\frac{1}{2}$$

Method 2

$$6 \times 3\frac{1}{12} = 6 \times \left(3 + \frac{1}{12}\right) = (6 \times 3) + \left(6 \times \frac{1}{12}\right)$$

$$= 18 + \frac{6}{12} = 18\frac{1}{2}$$

Estimates may be used to check our computation. We can estimate a product or a quotient by rounding each number to the nearest whole number.

EXAMPLE 2 Multiply $5\frac{3}{4} \times 4\frac{2}{3}$. Simplify.

Solution Estimate: $6 \times 5 = 30$

$$5\frac{3}{4} \times 4\frac{2}{3} = \frac{23}{4} \times \frac{14}{3} = \frac{23}{\underset{2}{\cancel{4}}} \times \frac{\overset{7}{\cancel{14}}}{3} = \frac{161}{6} = 26\frac{5}{6}$$

To divide one mixed number by another, we change the mixed numbers to improper fractions and use the method of the previous lesson.

EXAMPLE 3 Divide $2\frac{2}{3} \div 10\frac{2}{3}$. Simplify.

Solution

$$2\frac{2}{3} \div 10\frac{2}{3} = \frac{8}{3} \div \frac{32}{3} = \frac{8}{3} \times \frac{3}{32} = \frac{\overset{1}{\cancel{8}}}{\underset{1}{\cancel{3}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{32}}} = \frac{1}{1} \times \frac{1}{4} = \frac{1}{4}$$

1-5 Order of Operations

Symbols, such as parentheses and brackets, [ ], that show which operations are to be performed first are called **grouping symbols.** When one pair of grouping symbols is enclosed in another, we always perform the operation enclosed in the inner pair of symbols first.

EXAMPLE 1 Simplify.

a. $(14 + 77) \div 7$ **b.** $[3 + (4 \times 5)] \times 10$

Solution

a. $\underbrace{(14 + 77)}_{91} \div 7$

$\underbrace{91 \div 7}_{13}$

b. $[3 + \underbrace{(4 \times 5)}_{20}] \times 10$

$\underbrace{[3 + 20]}_{23} \times 10$

$\underbrace{23 \times 10}_{230}$

Some expressions, such as

$$8 + 3 - 9 \times 2 \div 3,$$

are written without grouping symbols. In order to simplify these expressions we use the following rule.

Rule

When there are no grouping symbols:

1. Do all multiplications and divisions in order from left to right.
2. Then do all additions and subtractions in order from left to right.

EXAMPLE 2 Simplify.

a. $72 - 24 \div 3$ **b.** $8 + 3 - 9 \times 2 \div 3$

Solution

a. $72 - \underbrace{24 \div 3}_{8}$

$\underbrace{72 - 8}_{64}$

b. $8 + 3 - \underbrace{9 \times 2}_{18} \div 3$

$8 + 3 - \underbrace{18 \div 3}_{6}$

$\underbrace{8 + 3}_{11} - 6$

$\underbrace{11 - 6}_{5}$

8-5 Combined Operations

In order to solve an equation of the form

$$ax + b = c \quad \text{or} \quad ax - b = c \quad \text{or} \quad b - ax = c,$$

where a, b, and c are given numbers and x is a variable, we must use more than one transformation.

EXAMPLE 1 Solve the equation $3n - 5 = 10 + 6$.

Solution Simplify the numerical expression.

$$3n - 5 = 10 + 6$$
$$3n - 5 = 16$$

Add 5 to both sides.

$$3n - 5 + 5 = 16 + 5$$
$$3n = 21$$

Divide both sides by 3.

$$\frac{3n}{3} = \frac{21}{3}$$
$$n = 7$$

The solution is 7.

Example 1 suggests the following general procedure for solving equations.

1. Simplify each side of the equation.
2. If there are still indicated additions or subtractions, use the inverse operations to undo them.
3. If there are indicated multiplications or divisions involving the variable, use the inverse operations to undo them.

It is important to remember that in using the procedure outlined above you must *always perform the same operation on both sides of the equation.* Also, you must use the steps in the procedure in the order indicated. That is, you first simplify each side of the equation, then undo additions and subtractions, and then undo multiplications and divisions.

EXAMPLE 2 Solve the equation $\frac{3}{2}n + 7 = 22$.

Solution Subtract 7 from both sides.

$$\frac{3}{2}n + 7 = 22$$

$$\frac{3}{2}n + 7 - 7 = 22 - 7$$

$$\frac{3}{2}n = 15$$

Multiply both sides by $\frac{2}{3}$, the reciprocal of $\frac{3}{2}$.

$$\frac{2}{3} \times \frac{3}{2}n = \frac{2}{3} \times 15$$

$$n = \frac{2}{\cancel{3}_{1}} \times \cancel{15}^{5}$$

$$n = 10$$

The solution is 10.

EXAMPLE 3 Solve the equation $40 - \frac{5}{3}n = 15$.

Solution Add $\frac{5}{3}n$ to both sides.

$$40 - \frac{5}{3}n = 15$$

$$40 - \frac{5}{3}n + \frac{5}{3}n = 15 + \frac{5}{3}n$$

$$40 = 15 + \frac{5}{3}n$$

Subtract 15 from both sides.

$$40 - 15 = 15 + \frac{5}{3}n - 15$$

$$25 = \frac{5}{3}n$$

Multiply both sides by $\frac{3}{5}$.

$$\frac{3}{5} \times 25 = \frac{3}{5} \times \frac{5}{3}n$$

$$\frac{3}{\cancel{5}_{1}} \times \cancel{25}^{5} = n$$

$$15 = n$$

The solution is 15.

INTRODUCTION TO CHEMISTRY

In terms of the four causes, chemistry is the study of substances insofar as they are material causes. Chemistry studies the properties of materials, the different kinds of elemental materials that exist, and the ways that materials combine to form compounds and mixtures. Though chemistry is considered its own branch of science today, this was not always the case. The study of material bodies was grouped with that of motion, the cosmos, and the Earth in particular. In our study of chemistry, we will begin by learning a few key terms commonly used to name the properties of matter. For the first half of our study, we will turn to the Pre-Socratic natural philosophers, some of the earliest thinkers to explore the nature of matter. After this, we will trace the question of how many elemental materials exist – from the classical theory of the four elements all the way to the doorstep of Mendeleev's periodic table. Questions surrounding the nature of the atom naturally intertwine with the study of the elements. We will carefully examine the history of atomic theory from Democritus and Lucretius to the Bohr model of the atom. In exploring the historical progression of these thinkers, we will also come to learn the important foundations of chemical analysis and synthesis.

Note that the word "matter" has the same root as *mater, matris, f.*, which means "mother" in Latin. The matter of a body, then, is in some sense the source or the origin of the body. Just as a mother gives birth to her baby, so also the matter of thing makes possible its existence as a body. Without matter, a body could not be a body. At the same time, it is vital to remember that a body is not a body without both *matter* and *form* together. What we often call "matter" is still *formed* matter, even if our minds are oriented towards its material. Thus, chemistry studies material bodies with an interest in the underlying materials that make bodies bodies, by abstracting from the formal differences that distinguish living from non-living material bodies.

SOME PROPERTIES OF MATTER

Volume	The amount of space a body occupies in three dimensions. A product of its length, width, and depth.
Weight	The heaviness or lightness of a body; the downward tendency exhibited by a body.
Temperature	The degree of hotness or coldness in a body.
Mass	The quantity or amount of matter in a body.
Density	The degree of compactness in a body. The amount of mass per unit of volume.
Inertia	The tendency of a body to remain in its state of resting or moving, or to resist a change in its state.

OBSERVABLE AND MEASURABLE PROPERTIES

It is crucial to remember that properties do not have to be measurable in order to be real properties of bodies. Remember from your study of measurement: measuring involves our attempt to *quantify* the qualities and properties of a thing. Often this means expressing continuous quantities in terms of discrete units. Though the ancient thinkers with whom we begin this section did not take up measurement and experiment as their primary methods, their observations and conclusions are still worthy of our sustained attention and reflection. Chemistry, as it developed, tended to focus more and more on the measurable properties of matter in order to determine the laws of chemical combinations and the essential and differentiating features of materials. The turn from more philosophical, speculative inquiry to an experimental, more mathematical approach will be evident in the progression of readings.

THE PRE-SOCRATIC PHILOSOPHERS

The Pre-Socratics (620 B.C. to 440 B.C.) were a group of thinkers united by their common attempt to understand the nature of the world (*kosmos*) and the ultimate principles of nature. "Principle" in this context means the cause, source, or foundation (*archê*) of a thing, or the reasons that explain the origin and nature of a thing. The Pre-Socratics were therefore in search of material and efficient causes of the whole universe. These questions were common among the Pre-Socratics:

1. What are the first or ultimate principles of the cosmos? Are they material or of some other kind?

2. How many principles are necessary to explain everything in the cosmos?

3. From what material does everything come?

4. How are generation (coming into being) and destruction (passing out of being) possible? How do we explain the appearance of change in the universe?

As you read and reflect on the thought of Thales, Anaximander, Anaximenes, Pythagoras, Heraclitus, Parmenides, Anaxagoras, Empedocles, and Democritus, it will be helpful to refer back to these questions and compare the different answers they propose.

Thales

"Of those who first pursued philosophy, the majority believed that the only principles of all things are principles in the form of matter [*hulē*]. For that of which all existing things are composed and that out of which they originally come into being and that into which they finally **perish**, the substance persisting but changing in its attributes, this they state is the element and principle of things that are… For there must be one or more than one nature out of which the rest come to be, while it is preserved." (Aristotle, *Metaphysics* I.3, 983b6-18)

perish – die

"However, not all agree about the number and form of such a principle, but Thales, the founder of this kind of philosophy, declares it to be water. (This is why he indicated that the earth rests on water.) Maybe he got this idea from seeing that the nourishment of all things is moist, and that the hot itself comes to be from this and lives on this (the principle of all things is that from which they come to be)—getting this idea from this consideration and also because the seeds of all things have a moist nature; and water is the principle of the nature of moist things." (*Metaphysics* I.3, 983b18-27)

Anaximander

"He [Anaximander] says that the first principle is neither water nor any other of the things called elements, but some other nature which is **indefinite**, out of which come to be all the heavens and the worlds in them. The things that are perish into the things out of which they come to be, according to necessity, for they pay penalty and retribution to each other for their injustice in accordance with the ordering of time, as he says in rather poetical language." (Simplicius, *Commentary on Aristotle's Physics*, 24.13-21)

indefinite – not specific, undetermined, formless

underlying nature – first principle, element

boundless – without a limit, limitless

definite –

rarity – how spread out the parts are

density – how close the parts are

Anaximenes

"Anaximenes… like Anaximander, declares that the **underlying nature** is one and **boundless**, but not indeterminate as Anaximander held, but **definite**, saying that it is air. It differs in **rarity** and **density** according to the substances [it becomes]. Becoming finer it comes to be fire; being condensed it comes to be wind, then cloud, and when still further condensed it becomes water, then earth, then stones, and the rest come to be out of these. He too makes motion eternal and says that

CHEMISTRY ETYMOLOGIES

SCIENCE Derivative:	Greek/Latin Root(s)	Translite-ration of root(s)	Relevant Translation of Root(s)	Section in Manual
matter, material	mater, matris, f.		mother; origin, source	Q3: Introduction to Chemistry
corporeal	corpus, corporis, n.		body	Q3: Introduction to Chemistry
volume	volumen, volumenis, n.		book, roll ("volume" developed from the bulk of a book)	Q3: Introduction to Chemistry
density	densus, -a, -um		thick, crowded; cloudy	Q3: Introduction to Chemistry
inertia	in + ers, from ars, artis		unskilled, inactive, (not doing work)	Q3: Introduction to Chemistry
mass	massa, massae, f.		lump, kneaded dough; load, bulk	Q3: Introduction to Chemistry
principle	principium, principii, n.		beginning, origin; foundation, element	Q3: The Pre-Socratics
mutable	muto, mutare, mutavi, mutatus		to move, change	Q3: The Pre-Socratics
cosmos	κοσμος	kosmos	order, beauty, the beauty resulting from order, ornament, world, universe, ordered whole, the order and beauty of the world	Q3: The Pre-Socratics
harmony	αρμονια	harmonia	agreement, concord of sounds	Q3: The Pre-Socratics
element	elementem, elementi, n.		rudiment, first principle, matter in its most basic form	Q3: The Classical Elements
compound/ composite	com + pono, ponere		to put together	Q3: The Classical Elements
atom	α + τομος	a + tomos	not cut, indivisible, uncuttable	Q3: The Atom
particle	pars, partis, f. with a diminutive French ending		little part	Q3: The Atom
solid	solidus, -a, -um		firm, whole, entire	Q3: The Constitution of Matter
liquid	liquor, liqui		to flow	Q3: The Constitution of Matter