7th Grade Textbook Packet
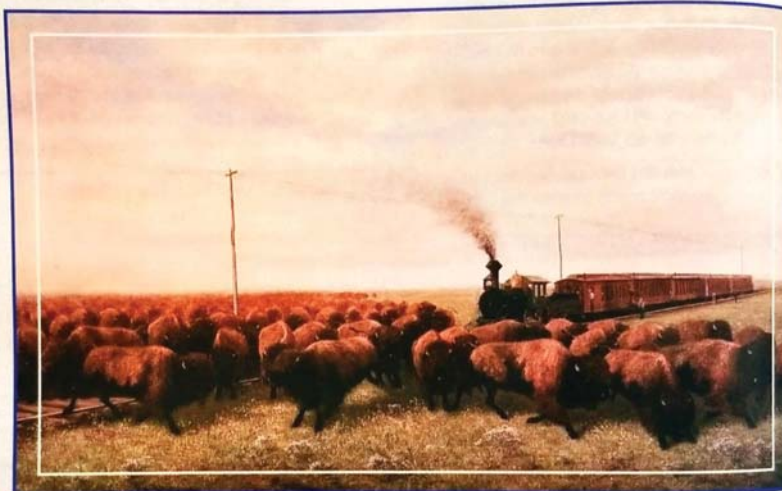## 4/6/2020-4/10/2020

Farming and Ranching in the Late 1800s

1 **The End of the Open Range in Texas**
2 **Railroads, Ranches, and Farms**
3 **From Family Farms to Commercial Farming**
4 **Agricultural Products of Texas**

The coming of the railroad had a dramatic impact on Texas farmers and ranchers. The railroad allowed them to ship their products to more distant markets. However, high rates charged by the railroads made life difficult for many farmers and ranchers.

EVENTS IN TEXAS

1870s
Market develops for cottonseed oil.

1872
About 100,000 immigrants settle in Texas.

1873
Texans form a branch of the Grange.

1874
Texas offers cheap public land to attract new settlers.

1878
Demonstration of barbed wire convinces Texans to buy the fencing material.

1870   1875   1880

WORLD AND UNITED STATES EVENTS

1873 ▲
United States enters a five-year economic depression.

▲ **1874**
National Grange calls for railroad regulation in the American Midwest.

▲ **1877**
Railroad strike cripples transportation in the United States.

The XIT Ranch

The XIT Ranch was the largest ranch in Texas. The state of Texas gave more than three million acres of land in the Panhandle to a group of Chicago investors. In exchange, the investors paid for the construction of the Texas Capitol in Austin.

Albers Equal-Area Projection

1880s
Drought hits Texas.

1883
Range war erupts.

1885
Kansas bars all Texas cattle.

1885–86
"Big Die-Up" destroys entire herds of cattle.

1890s
Railhead at Fort Worth becomes a collection point for Texas cattle.

1880   1885   1890

1881 ▲
Assassin kills President James A. Garfield.

1889 ▲
Montana, North Dakota, South Dakota, and Washington gain statehood.

# 1 The End of the Open Range in Texas

Reading Focus

• How did Texas land policies affect the open range?

• What events led to the range wars of the late 1800s?

• What changes did ranchers make in the way they raised cattle after the Big Die-Up?

Key Terms

open range
internal improvements
strike

Taking Notes

Copy this chart. As you read this section, fill in the chart with events that caused the end of the open range. Add as many "cause" boxes as you need.

> Cause:
> Population growth

> Effect: End of the open range
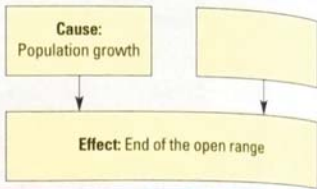
Main Idea Railroads, population growth, barbed wire, and other developments caused the end of open-range cattle ranching.

Experiencing Texas History

In the special activity that accompanies this chapter, you will explore how farming and ranching changed during the mid- and late 1800s.

Setting the Scene The 1880s were a decade of drastic change for Texas. The days of the cattle kingdoms ended as fences sprang up to enclose what had once been wide-open spaces. Cowboys mourned the loss of their traditional way of life in songs:

They say that heaven is a free range land,
Goodbye, goodbye, O fare you well;
But it's barbed wire for the devil's hat band;
And barbed wire blankets. . . .

The trail's a lane, the trail's a lane,
Dead is the branding fire.
The prairies wild are tame and mild,
All close corralled with wire.
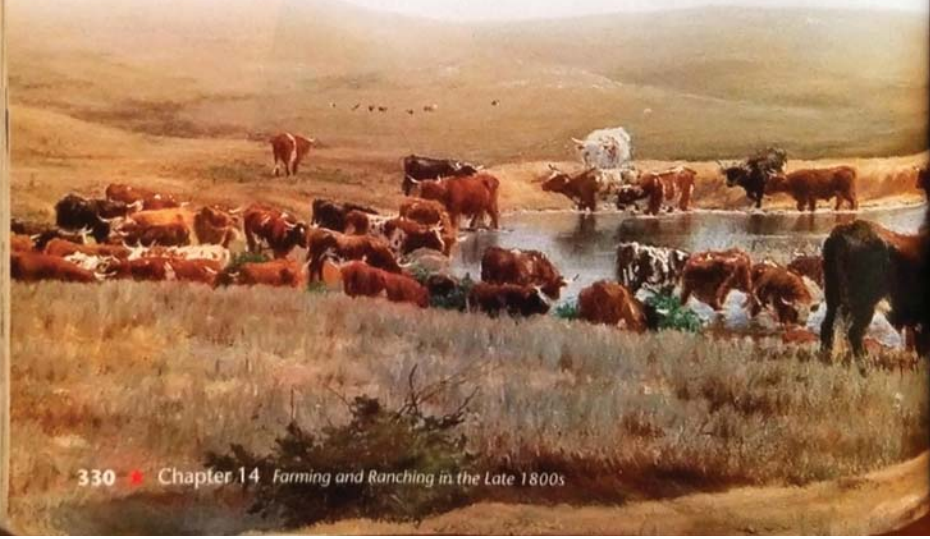
Ranching on the Open Range

As Native Americans were pushed out of West Texas, ranchers moved in. Early Texas ranchers realized that these lands, on which huge herds of buffalo had once roamed, could feed sheep and cattle, too.

Ranching in the west West Texas ranches differed at first from the earlier ranches in South Texas in one important way. South Texas ranchers, such as Richard King, owned the land on which their herds grazed. The first West Texas ranchers generally owned little or no land. Often, only the ranch house sat on private property. Some ranchers bought land along a river or stream in order to have a reliable source of water. Most livestock grazed on the **open range.** This was a vast area of undeveloped public land held by the state government for future sale.

Open-range ranching methods In the mid-1870s, there was so much public land in West Texas that most of the early ranchers saw no need to buy any of it. They simply found a good location and "squatted" there, without a legal right to the property. If someone bought the land, the squatter simply moved his ranch to another spot.

Of course, public land was no one's property. Because no fences enclosed the land, cattle from several ranches often intermingled. Ranchers determined the owner of an animal by its brand. Every spring the ranchers held a roundup on the open range. Calves born since the last roundup received the same brand as their mothers. Cowboys cut some steers from the herd and drove them to market to sell for their meat.

Ranching on the open range worked fairly well in those early days. Not even thousands of heads of cattle could crowd the vast West Texas plains. As a result, most ranchers had little interest in controlling the riverbanks and water holes. They viewed the grasslands as a resource that all could share, and none could own. In the next few years, all that would change.

As You Read

Use Prior Knowledge What invention would help close the open range?

Hardy Creatures

The hard hooves and long legs of Texas longhorns made them ideal animals for the trail. **Drawing Inferences** *Why would cowboys closely watch cattle at a watering hole?*

Changes on the Open Range

Several factors ended the open range in Texas. A key factor was the expansion of railroads in the state. Closely related to this were the state's land policies, population growth, and the spread of farming. It is important to understand how these factors changed ranching on the open range.

Land policies to encourage growth In the 1850s, the state government offered free public land to encourage **internal improvements.** These are changes made by people, such as the building of roads, which help boost a region's economy and population. Railroads received almost 90 percent of this land. The railroads, in turn, sold much of the land to recover the costs of laying track. Few railroads could stay in business merely by hauling goods and passengers from one end of the track to the other. To make money, a railroad needed people all along the line who would ride and ship products on its trains. To encourage settlement along the rail lines, Texas launched programs to sell more public land.

In 1874, Texas offered public land near railroad lines for $1.50 an acre. Within five years, the price fell to $1 an acre for land next to the tracks, and 50 cents an acre for all other land. Even at these prices, however, many people still could not afford to buy land. Texas created the State Land Board to sell land to poor settlers. Buyers could take up to 40 years to pay for the land.

Land and cattle companies The land policies forced open-range ranchers to change. Many who had once shared the land with their neighbors now felt that they had to own the land—before someone else bought it. To do this, many ranchers sought help outside Texas. Reports of high ranching profits had reached investors in the eastern United States and Great Britain. They provided money for a rancher to buy or lease land on the open range. The rancher then shared the profits with his financial partners.

Huge privately owned ranches soon developed. One was the JA Ranch. Texan Charles Goodnight managed the ranch, but John Adair of Great Britain owned most of it. In 1876, Goodnight ranched on a few thousand acres of open range. A decade later, the JA Ranch covered more than 700,000 acres of private property. Adair's money made this growth possible. Large ranches controlled by British investors came to dominate the Panhandle. The growth of these privately owned ranches pushed out most open-range ranchers.

Range wars Cattle ranchers had long complained that sheep, which ate the grass almost to the roots, destroyed the land for cattle grazing. Sheepherders often needed to drive their flocks over ranchers' property in order to move from one pasture to another. That practice had worked well when the state controlled the public lands. However, ranchers bought more and more of this land. As ranchers began fencing in their land in the 1880s, these conflicts worsened.

Light As Air and Cheap As Dirt
Barbed wire became a popular fencing material in the 1870s. It was promoted as being "light as air and cheap as dirt." **Linking Past and Present** *Why is barbed wire still in use today?*

AS YOU READ **Predict** How would the end of the open range change the way of life for cowboys and ranchers?

Some fences cut off public roads and water supplies. Angry small farmers fought back by cutting the fences. By 1883, a "range war" had developed over the issue of fences and fence cutting. Finally, the Legislature made it illegal to cut fences. People who fenced across public roads now had to provide gates and keep them in good repair.

An End and a Beginning

The end of the open range was one reason for the end of the great cattle drives. The expansion of railroads into Texas also made the cattle drives unnecessary. Ranchers now used the railroads to move their animals to market. Concern over a deadly disease, Texas fever, made drives even more difficult. Armed ranchers sometimes patrolled the trails, determined to prevent sick cattle from infecting their herds. In 1885, Kansans barred all Texas cattle from their state.

People in the East were now demanding better types of beef. Longhorns produced tough and stringy meat. Texas ranchers now had to compete with better breeds of stock being raised on ranches in Wyoming and Montana. The Texas beef market was headed into a long period of decline.

Changes for cowboys The end of the open range and cattle drives meant changes for cowboys, too. Fewer long trail drives and open-range roundups meant fewer jobs for cowboys. Thousands lost their jobs. Those who kept their jobs spent less time in the saddle and more on foot. Setting fence posts and stringing barbed wire made most cowboys feel like low-paid ranch hands.

In 1883, some Panhandle cowboys went on **strike.** A strike is a work stoppage to force an employer to meet certain demands. The strike failed. The Texas Rangers were called in to break the strike. Many cowboys lost their jobs. The failure of the strikers showed that the changes in ranching were permanent.

The Big Die-Up The final blow to the old way of ranching occurred in 1885 and 1886. Ranchers had built a series of "drift fences" that stretched from New Mexico to Indian Territory (present-day Oklahoma). These fences prevented cattle infected with Texas fever from "drifting" into the region and mingling with the healthy herds there. In 1885, a huge blizzard struck the plains. Cattle fled from the storm. The drift fences trapped them.

Thousands of cattle froze to death along the fence line. The following winter brought more blizzards and thousands more cattle deaths along the fences. The Big Die-Up, as ranchers called it, nearly wiped out the herds in the Panhandle. Many cattlemen lost everything as summer droughts followed the severe winters. Some British land and cattle companies sold their ranch lands and left the business in Texas.

Fence Cutters
During the range wars, armed men cut fences so that they could move their cattle to grazing areas.
Drawing Inferences *Why did fence cutters arm themselves?*

Killer Blizzard

Drift fences prevented cattle from escaping the blizzards that struck Texas in 1885 and 1886. **Identifying Cause and Effect** *What caused the death of the cattle?*

Modern ranching begins The Big Die-Up marked the start of the modern ranching industry in Texas. Ranchers realized that they could not leave cattle alone to graze on the range. Cattlemen who remained in business began to manage their herds. They determined how much pasture each group of cattle required. Then they fenced off that much pasture. This method enabled the ranchers to make more efficient use of the land.

The fences also enabled ranchers to improve the quality of their herds. Ranchers brought in new types of cattle to replace the lean, tough longhorns. They crossbred Angus, Herefords, and other breeds. Several new breeds of cattle resulted from these experiments. They produced more meat and could resist heat and Texas fever. From these beginnings came the scientific control that defines the Texas cattle industry today.

AS YOU READ **Summarize** How did the ranching industry change from open-range ranching to modern ranching?

AFTER YOU READ

★ Section 1 Assessment ★

Recall
1. **Define** (a) open range, (b) internal improvements.
2. **Identify** Explain the significance of the following: (a) strikes, (b) the Big Die-Up.

Comprehension
3. What role did the State Land Board play in Texas?
4. What changes did cowboys experience with the end of the open range?
5. How did the Big Die-Up affect the ranching industry?

Critical Thinking and Writing
6. **Exploring the Main Idea** How did railroads, population growth, and barbed wire lead to the end of open-range cattle ranching?
7. **Contrasting** How were early ranches in West Texas different from early ranches in South Texas?
8. **Summarizing** Summarize the effects that barbed wire had on West Texas.

ACTIVITY

Make a Poster

You are a member of the Texas State Land Board. Design a poster that encourages settlers to purchase land near railroad lines.

BEFORE YOU READ

2 Railroads, Ranches, and Farms

Reading Focus
- How did the railroads affect the cattle drives?
- How did the railroads change farming in Texas?
- What other effects did railroads have on the Texas economy?

Main Idea The extension of railroads into Texas changed the lives of ranchers, farmers, and other Texans.

Key Term
commercial agriculture

Taking Notes
Copy this chart. As you read this section, fill in the chart with changes brought on by the expansion of railroads into Texas. Add as many "effect" boxes as you need.

Cause: Railroads expand into Texas

Effect: End of cattle trails

Setting the Scene For years, the main cash crop of Abilene wasn't cotton or corn—it was bison bones. Hunters had destroyed the herds of bison in the area in the mid-1870s. They left the bones on the prairie to dry and bleach out in the hot Abilene sun. By the 1880s, after the founding of Abilene, a booming business in bison bones grew up. Manufacturers used elements from the bones for making fertilizers, buttons, and fine bone china. Bone brokers gathered the bones and piled them at the railroad station in Abilene. At one point, the pile was 30 feet wide and about five blocks long! All in all, about half a million tons of bison bones were shipped out of West Texas by rail.

The railroads made the bison bone business possible. They also changed the business of farming and ranching. Now farmers and ranchers could ship their products farther than ever before.

A Busy Rail Yard
Railroads enabled farmers to ship their goods to distant markets. This photograph shows Houston's Grand Central Depot as it looked in 1894. **Drawing Conclusions** *Are the trains in this image arriving or departing? How can you tell?*

Railroads Change Ranching

For years, cowboys herded their cattle along the trails that led from Texas to the rail links in Kansas. The building of rail lines in Texas, however, changed the cattle business. Now cattle could be shipped by rail straight out of Texas.

The first railroads The first rail line in Texas opened in 1851. Over the next 10 years, tracks slowly spread across Texas. Nearly all of the early rail lines linked Houston to nearby towns. Ranchers, however, lived in rural areas, so the early trains were of no use to them. Ranchers needed rail lines to move cattle from the Texas plains to other states, where hungry customers would pay cash for beef.

Railroads link Texas to the rest of the nation Railroad building in Texas had ceased during the Civil War. After the war, construction resumed. Yet progress was slow. Then in 1873, the Texas rail system linked to the nationwide rail network. That same year, some Texas ranchers began shipping their cattle by rail. The linking of Texas to the rest of the nation changed the ranching business in Texas.

From trails to rails Although Texas ranchers now had access to rail lines, it was still cheaper to drive the cattle over trails. Yet the quarantines and barbed wire you read about in Section 1 convinced ranchers to use the railroads to move their livestock to market. By the 1890s, the railhead at Fort Worth had become a collection point for Texas cattle.

At first, Fort Worth served as a place to gather cattle for transport. Ranchers herded the animals onto trains and shipped them elsewhere for slaughter. This changed in 1901. That year, Swift and Company and Armour and Company built meatpacking plants in Fort Worth. Within about a decade, Fort Worth had established itself as one of the nation's largest meatpacking centers. Although the days of the open range were over, the cattle business was still vital to Texas.

Railroads Change Farming

The railroads changed the way Texans farmed. With access to railroads, a farmer who grew more than his family needed could ship the surplus to market. In this way, the rail lines led to **commercial agriculture** in the state. Commercial agriculture is the growing of crops for sale in order to make a profit. Farmers could also use the railroads to bring in seeds, plows, and other items they needed.
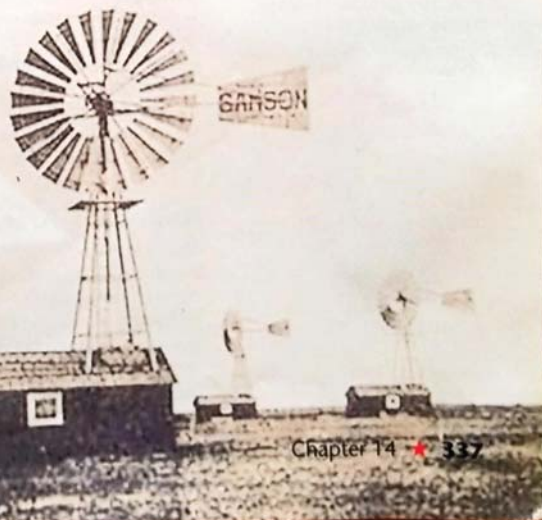
AS YOU READ **Draw Inferences** Why would beef fetch a much lower price in Texas than it would outside the state?

Windmills
Windmills enabled farmers to pump water from beneath the ground.
Drawing Inferences Why would there be more windmills in areas that received little rainfall?

Causes of the Closing of the Open Range

As industry and the population of Texas grew, the open range gradually closed.

1. **Comprehension** Name the two factors relating directly to business and industry that led to the closing of the frontier.

2. **Critical Thinking**
Making Predictions What do you think will happen to land in Texas once the open range is closed?

Economics

| Expansion of railroads | State's land policies | Population growth | Spread of farming as a business |

End of the Open Range

Farmers ride the rails to West Texas Railroads also led people to begin farming the land in West Texas. The railroad companies wanted people to buy the land the railroads owned there. They also hoped these new settlers would become rail customers.

Growing crops in West Texas required a different method of farming. This was because there was less rainfall than in the eastern part of the state. Farmers tried to conserve water. They also tried to irrigate their land and use well water. However, stored water evaporated quickly in the West Texas heat.

Different crops and different tools Farmers found that corn and other crops that grew well in East Texas did not do as well in the western part of the state. As a result, they grew other crops. The railroads had shown Texas farmers the power of machines. More farmers began to use machines to help plant and harvest their crops. Some farmers even used steam-powered tractors. Machines helped farmers increase their harvest.

New machines and railroad rates were expensive. Indeed, farmers were producing more crops. Yet to earn a profit, those crops had to fetch a certain price in the markets. When crop prices fell in the late 1800s, many farmers went into debt. Some had to give up their land and farm on land owned by others.

AS YOU READ **Ask Questions** Before the arrival of railroads, why did it not benefit small farmers to grow more than they needed for themselves and their families?

Railroads Affect the Economy

Most Texans understood that the lack of a railroad could mean economic doom for their town. As a result, they did what they could to convince railroads to lay track in their region. Towns donated sites for railroad stations, storehouses, and holding pens for livestock. Some people paid money to railroad companies to get rail access. Counties and cities borrowed money to provide railroads with cash and other incentives to build tracks.

Many Texas cities and towns owe their success to the business created by railroads. Abilene, Amarillo, and Midland prospered as railroad towns in the 1880s. Railroads in El Paso and Laredo helped these towns to grow. Growth of border cities tied Texas more closely to the economy of Mexico, even as the railroads forged stronger links to the economy of the United States.

Railroads and trade Railroads also changed old patterns of trade in Texas. Before railroads, most trade had been local. After the railroads, Texans dealt with distant suppliers and buyers. These dealings involved people they did not know, and over whom they had no control. This was a change. Many Texans were used to face-to-face contact. Railroads put Texans in contact with more distant markets, quickly and cheaply. Overall, these new business dealings were a big boost to the state's economy.

Railroads spur business Railroads helped the Texas economy in many other ways in the late 1800s. Railroad companies needed lumber to build rail ties, bridges, stations, holding pens, and freight platforms. They also needed timber to build homes for their workers. These needs spurred the growth of the lumber industry.

The people who worked on the railroads also spurred business. People founded new towns or moved old towns in order to be closer to the rails. In addition, railroads needed coal to power their trains. As a result, Erath County and Eagle Pass on the Mexican border began mining coal.

Drawbacks of railroads Large companies from outside Texas gained control of most Texas railroads in the late 1800s. The Southern Pacific purchased some Texas lines, while others became part of the Missouri Pacific or the Atchison, Topeka, and Santa Fe Railroad. In taking over the Texas rail lines, companies gained great influence over much of the state's economy.

AS YOU READ **Summarize** How did the railroads improve life for some Texans but not for others?

AFTER YOU READ ★ **Section 2 Assessment** ★

Recall
1. **Define** (a) commercial agriculture.
2. **Identify** Explain the significance of the following:
   (a) Swift and Company,
   (b) farm machinery.

Comprehension
3. How did railroads change ranching?
4. What did Texans do to encourage railroad companies to build in their state?

5. How did railroads boost the Texas economy?

Critical Thinking and Writing
6. **Exploring the Main Idea** What changes did railroads bring to ranching and farming?
7. **Comparing** Compare aspects of the cattle industry before and after the growth of railroads.
8. **Identifying Cause and Effect** What happened to a town if it could not attract a railroad to it?

ACTIVITY

Listing Pros and Cons
Suppose you are a farmer in Texas in the late 1800s. Make a list of the positive effects of the railroads. Then list the negative effects of railroads. Overall, do you think that the railroads have brought greater benefits or greater drawbacks to your life? Why?

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3 From Family Farms to Commercial Farming

BEFORE YOU READ

Reading Focus
• What encouraged people to move to Texas?
• What factors made it hard for tenant farmers to prosper?
• What hardships affected Texas farmers in the late 1800s?

Key Terms
tenant farmer
interest
irrigation
depressions

Taking Notes
Copy this chart. As you read this section, fill in the chart with the effects of hard times for Texas farmers. Add as many boxes as you need.

West Texas farmers' reactions to hard times

Farmers go out of business
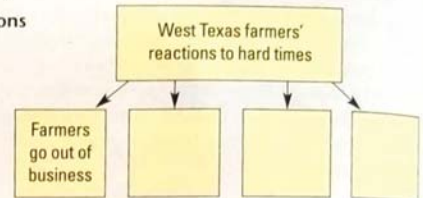
Main Idea Many Texans who had once owned their own farms became tenant farmers. The high cost of land and low price of cotton forced many landowners into tenancy.

Setting the Scene The family ate breakfast in the dark so they could be in the field by dawn. They each carried a long sack that could hold 25 pounds of cotton. Dragging their sacks behind them, mother, father, and children worked down the rows. They stopped only to empty their sacks into the cotton wagon, and to eat lunch under the hot July sun. By afternoon the backs of the taller family members ached from bending over the plants. Yet the work was hardest on the hands. In the mornings the cotton was wet with dew. "It softened your fingers so the sharp point on the cotton burrs pricked your fingers until they might bleed," one Texan remembered of his childhood on the farm. "In a little while the sun would come up and drive the dew away. Then the burr points would get sharper, but you kept on picking." Work ended when the sun set. The next day the routine began again.

Lots for Sale
Notices such as this drew buyers to Texas towns. **Comparing** *How does this notice compare to modern real estate advertisements?*

EAST Del Rio
GRAND AND IMPORTANT
AUCTION SALE
OF
TOWN LOTS
Del Rio, Val Verde County, Texas,
NEW ENCAMPMENT ON
APRIL 30TH
Thursday, May 1st, 1890.
ADVANTAGES AND RESOURCES.
EAST DEL RIO
TERMS OF SALE.
JOSEPH FRIEDLANDER, MESSRS. LEON & H. BLUM,
or CAPT. J. A. H. HOSACK,
SAN ANTONIO, TEXAS.

Farming and Growth

Life for the average Texas farm family was not much different in 1870 than it had been before the Civil War. The plantation system and slavery were gone, of course. Thousands of small farms remained in eastern and central Texas. They still raised cotton and corn, as well as a few cattle, sheep, hogs, and chickens. Corn provided food for people and livestock. Farmers sold the cotton for cash, which was often in short supply. Farmers needed cash not only for daily use, but to pay their taxes as well.

Each fall, farmers slaughtered some of their cattle and hogs. They sold the hides and preserved the meat for food. Farmers still plowed their land with oxen or mules and planted seeds by hand. This type of agriculture, called subsistence farming, produced little more than a family farm needed to survive. Husbands still made the household furniture and most of the tools used in the home and fields. Wives still produced the family's clothing, soap, candles, and bedding. Farm men and women came together to hold house-raisings and quilting parties, just as they had done for some 50 years.

Population growth By 1870, Texas was still mostly rural. Galveston, with just 14,000 people, was the state's largest city. San Antonio was slightly smaller. Houston and Austin were the only other major cities. Dallas, Fort Worth, and El Paso were all still small towns on the frontier. Some counties, such as Washington, had no large towns at all. Still the state's population was growing. Much of this growth was due to immigration.

Newcomers were encouraged to come to Texas. The state needed workers to replace the freed slaves, as well as farmers to settle the western parts of the state. To meet these needs, Texas turned to the southern states. Newspapers printed articles that said a farmer needed "no money to secure a good farm in almost any part of Texas." All that was needed to succeed were a "good character, industrious habits, and one or two boys."

More people and more farms In 1872 alone, about 100,000 people came to Texas. More people moved into the state. By 1890, some 2.2 million people lived in Texas. Most of the newcomers were from the South, mainly from Georgia and Tennessee. Many were poor and moved to Texas to escape their debts. In East Texas some arranged with local landowners to farm the land. Others headed farther west, buying public land from the state. Still others bought land from the railroads. Most new settlers were farmers.

Changes in Texas Farming

Texas plantations had long been involved in growing crops for cash. Until the Civil War, however, most rural Texans practiced subsistence agriculture. Once the railroads came, farmers could move their crops to distant markets. This gave even small farmers a good reason to grow as many cash crops as possible.

To many farmers, cotton was an ideal crop. Demand for it existed worldwide. In addition, it withstood drought better than many other crops. Growing cotton also suited the farming experience of the newcomers arriving from the South.

Tenant farming When slavery ended, few plantation owners had the money to hire workers. As a result, the number of **tenant farmers** grew. A tenant farmer is a person who rents a plot of land from its owner and pays for its use with a share of the crop. Tenant farmers provided their own mules, tools, and other necessities. They typically kept two thirds of their crop and paid the rent for the land with one third of the harvest.

Tenant farmers differed from sharecroppers. Tenant farmers owned tools and work animals. They paid rent for the land and legally owned the crops grown on that land. In contrast, sharecroppers owned nothing. They were hired hands who received wages for their labor with a portion of the harvest. Sharecroppers did not rent the land, and they did not legally own the crops grown on that land.

Although land was cheap, buying food and seeds to start a farm required money. As a result, many freedmen became tenant farmers. Freedmen were newly freed slaves. Many remained as sharecroppers on the very plantations where they had worked as slaves.

AS YOU READ **Use Prior Knowledge** Why were many southerners so poor in the years following the Civil War?

Sam Ryon Sharecropper Farm

Sam Ryon was an African American sharecropper in Texas. He worked on a cotton farm in the 1890s. Today, his farmhouse is part of the George Ranch Historical Park. Visitors can tour the park in Fort Bend County. Ryon's farmhouse looks much as it did in the 1800s.

Take It to the NET

Virtual Field Trip For an interactive look at Sam Ryon's farm, visit the *Lone Star* section of **www.PHSchool.com**.

Most tenant farmers viewed their condition as temporary. They wished to save money and then buy their own land. Yet it was hard to succeed at tenant farming. One reason was that landowners gave tenant farmers only small plots of land. This prevented the farmers from raising enough crops to make a profit. Most tenant farmers were deeply in debt.

The cycle of debt Tenant farmers often could not produce enough food to feed their families. This was because they almost always planted cotton. Tenant farmers had to buy food from merchants or landowners. If a family had no money, the merchant or landowner would let the family buy what it needed on credit. The farmer paid for these purchases after the harvest, when he sold his share of the crop.

This arrangement was risky for all concerned. No one could predict the size of the crop in advance. A drought or a sudden flood might make the crop smaller, or even wipe it out. Also, no one knew what the price of cotton would be at harvest time. If the price was low, the farmer's share of the crop might not bring enough money to pay his bill. To buy on credit, the farmer had to pay **interest**. Interest is additional money paid by a borrower to a lender for the use of the money borrowed.

Because of these risks, interest rates were usually high. Once their bills and interest were paid, few tenant farmers had much money left after the harvest. Many even found that they owed more than they received for their share of the crop. These families started the next season already in debt from the last one. This made it even more difficult to get free of the cycle of debt and the tenant farming system.

AS YOU READ **Monitor Your Reading** Why was it difficult to prosper as a tenant farmer?

The economics of farming For tenant farmers to succeed, the price of cotton had to be high and the amount of cotton produced had to be great. These two conditions rarely occurred together, because of the principle of supply and demand. When demand is high and supply is low, prices go up. For example, if buyers want more cotton than is available, the price of cotton will rise. On the other hand, if the supply of cotton goes up and demand stays the same or goes down, the price falls.

More farmers in the state meant bigger cotton harvests. Texas cotton production jumped from about 350,000 bales in 1869 to more than 3.5 million bales in 1900. Yet the supply grew faster than the demand. Soon, the price of cotton fell. Cotton that had sold for 16.5 cents per pound in 1869 brought only 5.7 cents per pound in 1898.

Farming New Lands

The railroads wanted to increase settlement in West Texas. They hoped new settlers would ship and receive products by train. The rail lines also wanted to sell their land to new settlers. In this way, they hoped to recover the cost of laying the tracks.

The state still sold public land. As ranches failed in the mid-1880s, many ranchers sold their land. To meet this competition, railroads offered land from their land grants at very low prices. Sometimes farmers were able to buy railroad land for just a few cents an acre.

Farming West Texas Much of West Texas could not support farming without **irrigation.** Irrigation supplies water to land by artificial means. Most railroad companies knew this. Yet they wanted to sell their land. The railroads did their best to lure farmers to places that were unsuitable for farming. They hid the truth about West Texas from outsiders. Texas newspapers rarely wrote about dry conditions in the state. Even geography books described the region as "less humid" than other areas.

A period of greater than average rainfall convinced farmers that they could prosper on the western plains. Yet settlers had not been there long enough to know that years of drought often followed such rains. A stream of immigrants poured into the land.

Hard times for farmers From the 1870s through the 1890s the United States experienced three economic **depressions.** A depression is a period during which business activity and prices drop and many workers lose their jobs. Demand for many cash crops fell. Crop prices fell, too. Many farmers had borrowed money to buy their land and tools. Across the nation, thousands of farms failed.

Field Hands

After slavery ended, many former slaves took jobs as hired field hands. This 1907 photograph shows a group of workers in a cotton field near Dallas. **Drawing Inferences** *Why might a former slave continue to work on cotton plantations?*

Advocates of Agriculture

The Grange worked to improve the lives of farmers. This mural for the Grange emphasizes the value of farm work. **Analyzing Images** *What activities does this mural depict?*

AS YOU READ **Summarize** What challenges did farmers face in the years after the Civil War?

By 1885, Texas farmers were starting to recover. Then, the wet years ended in West Texas. Two years of severe drought followed. More depression and droughts hit the state in the 1890s. Low crop prices drove many West Texas farmers out of business. Some sold their farms to wealthy men and then stayed on as hired hands. Others became tenant farmers. Still others headed for East Texas where they joined other tenant farmers on plantations or took jobs in towns. By 1900, half of all farmers in Texas were tenant farmers.

Farmers help themselves Farmers throughout the nation experienced the same hardships as those in Texas. In 1867, farmers in Minnesota founded the National Grange to help farmers and their families. Six years later, Texans formed a branch of the Grange.

The National Grange helped farmers by sharing knowledge about the latest farming techniques. It also set up stores where farmers could buy goods at lower prices than those charged by other merchants. These stores failed, however, when the depression and droughts of the 1880s left many farmers unable to pay their bills.

Farmers met with one another, listened to speakers, and used the library at Grange halls. Grangers demanded better schools and free textbooks. In 1876 they helped create an agricultural college in Texas. This was the Texas Agricultural and Mechanical College (now Texas A&M University). Grangers also called for the regulation of railroads.

Membership in the Texas Grange declined in the 1880s. Hard times caused many farmers to join a group called the Farmers' Alliance. This group was small at first, in part because it called for drastic changes in the state. Within a few years, though, it had about 100,000 members.

AFTER YOU READ ★ ★ **Section 3 Assessment** ★ ★

Recall

1. Define (a) tenant farmer, (b) interest, (c) irrigation, (d) depressions.

2. Identify Explain the significance of the following: (a) railroads, (b) cycle of debt.

Comprehension

3. How did population growth affect Texas farms?

4. Why did railroad companies want to sell land to settlers?

5. What were the goals and accomplishments of the National Grange?

Critical Thinking and Writing

6. Exploring the Main Idea What conditions led some Texans to become tenant farmers?

7. Making Generalizations Based on the section, how would you generalize the life of a tenant farmer?

ACTIVITY

Writing a Journal
Suppose you are a farmer from East Texas who has moved to West Texas with your family. Write a journal entry describing farm life in the west. Describe the land and your experiences farming.

3-7 Negative Integers as Exponents

You know by the rule of exponents that you learned for multiplying powers of the same base that

$$10^1 \times 10^2 = 10^{1+2} = 10^3.$$

Since we want to apply the same rule to negative exponents, we must have

$$10^1 \times 10^{-1} = 10^{1+(-1)} = 10^0 = 1$$

$$10^2 \times 10^{-2} = 10^{2+(-2)} = 10^0 = 1$$

and so on. We know that

$$10^1 \times \frac{1}{10} = 10 \times 0.1 = 1 \text{ and } 10^2 \times \frac{1}{10^2} = 100 \times 0.01 = 1,$$

so 10^{-1} should equal $\frac{1}{10}$ and 10^{-2} should equal $\frac{1}{10^2}$. This example suggests the following general rule.

Rule

For all numbers $a(a \neq 0)$, m, and n,

$$a^{-m} = \frac{1}{a^m}$$

EXAMPLE Write the expression without exponents.

    a. 5^{-2} **b.** $(-3)^{-2}$ **c.** $(-4)^{-1}(-4)^{-2}$

Solution   **a.** $5^{-2} = \frac{1}{5^2} = \frac{1}{5 \times 5} = \frac{1}{25}$

            b. $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{(-3)(-3)} = \frac{1}{9}$

            c. $(-4)^{-1} \times (-4)^{-2} = (-4)^{-1+(-2)} = (-4)^{-3}$

                    $= \frac{1}{(-4)^3} = \frac{1}{(-4)(-4)(-4)} = \frac{1}{-64}$

Class Exercises

Use the rules for exponents to state the expression without exponents.

    1. 3^{-4} **2.** $(-6)^{-2}$ **3.** $10^4 \times 10^{-4}$ **4.** $3^5 \times 3^{-7}$ **5.** $(-2)^3(-2)^{-1}$

Self-Test B

Find the product.

1. $4.2(-11.3)$ 2. $-6.7(20.4)$ 3. $7.5(-4.2)(-12)$ **[3-5]**

Find the quotient.

4. $121 \div (-11)$ 5. $-68.2 \div 2.2$ 6. $-0.56 \div (-0.07)$ **[3-6]**

Write the expression without exponents.

7. 4^{-2} 8. $(-6)^{-3}$ 9. $7^5 \times 7^{-8}$ 10. $(-9)^{-2} \times (-9)^0$ **[3-7]**

Self-Test answers and Extra Practice are at the back of the book.

▮▮▮▮ NONROUTINE PROBLEM SOLVING: *Greatest Integer Function*

We use the symbol $[x]$ (read *the greatest integer in x*) to represent the greatest integer less than or equal to x.
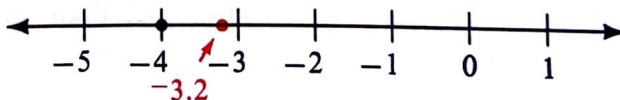
EXAMPLE   a. $[5.4]$ b. $[-3.2]$

Solution   a. There is no integer equal to 5.4, so we must find the greatest integer that is less than 5.4.
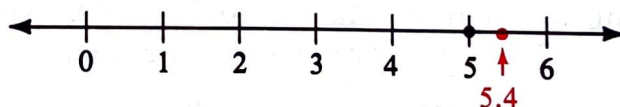
As shown on the number line, the greatest integer that is less than 5.4 is 5. Thus *the greatest integer in 5.4 is 5.*

b. There is no integer equal to -3.2, so we must find the greatest integer that is less than -3.2.

As shown on the number line, the greatest integer that is less than ⁻3.2 is ⁻4. Thus *the greatest integer in ⁻3.2 is ⁻4.*

Find the value of each of the following.

1. $[6.2]$ 2. $[1.23]$ 3. $[3]$ 4. $[45]$

5. $[-12]$ 6. $[-1]$ 7. $[-4.89]$ 8. $[-0.36]$

4-3 Least Common Denominator

In calculations and comparisons, we work with more than one fraction. It is sometimes necessary to replace fractions with equivalent fractions so that all have the same denominator, called a *common denominator*. For example, in the addition $\frac{1}{6} + \frac{3}{4}$ we may write $\frac{1}{6}$ as $\frac{2}{12}$ and $\frac{3}{4}$ as $\frac{9}{12}$, using 12 as a common denominator. We may also use 24, 36, 48, or any other multiple of both denominators as a common denominator.

The **least common denominator** (LCD) of two or more fractions is the *least common multiple* (LCM) of their denominators. The LCD of $\frac{1}{6}$ and $\frac{3}{4}$ is 12, since 12 is the LCM of 6 and 4.

EXAMPLE 1   Write equivalent fractions with the LCD: $\frac{1}{24}, \frac{5}{48}, \frac{11}{120}$.

Solution   The LCD is the least common multiple of 24, 48, and 120. Use prime factorization to find the LCM.

Prime factorization of 24: $2 \times 2 \times 2 \times 3$, or $2^3 \times 3$
Prime factorization of 48: $2 \times 2 \times 2 \times 2 \times 3$, or $2^4 \times 3$
Prime factorization of 120: $2 \times 2 \times 2 \times 3 \times 5$, or $2^3 \times 3 \times 5$

The LCM is the product of the highest powers of each factor. The LCM $= 2^4 \times 3 \times 5 = 240$, so the LCD $= 240$.

$$\frac{1}{24} = \frac{1 \times 10}{24 \times 10} = \frac{10}{240} \qquad \frac{5}{48} = \frac{5 \times 5}{48 \times 5} = \frac{25}{240} \qquad \frac{11}{120} = \frac{11 \times 2}{120 \times 2} = \frac{22}{240}$$

EXAMPLE 2   Replace __?__ with $<$, $>$, or $=$ to make a true statement.

   a. $\frac{5}{6} \underline{?} \frac{6}{7}$ **b.** $-\frac{5}{8} \underline{?} -\frac{9}{14}$

Solution   First rewrite each pair of fractions as equivalent fractions with the LCD. Then compare the fractions.

a. The LCD is the LCM of 6 and 7, or 42.

$$\frac{5}{6} = \frac{5 \times 7}{6 \times 7} = \frac{35}{42} \qquad\qquad \frac{6}{7} = \frac{6 \times 6}{7 \times 6} = \frac{36}{42}$$

$$\frac{35}{42} < \frac{36}{42}, \text{ so } \frac{5}{6} < \frac{6}{7}.$$

b. The LCD is the LCM of 8 and 14, or 56.

$$-\frac{5}{8} = -\frac{5 \times 7}{8 \times 7} = -\frac{35}{56} \qquad\qquad -\frac{9}{14} = -\frac{9 \times 4}{14 \times 4} = -\frac{36}{56}$$

$$-\frac{35}{56} > -\frac{36}{56}, \text{ so } -\frac{5}{8} > -\frac{9}{14}.$$

When fractions have variables in their denominators, we may obtain a common denominator by finding a common multiple of the denominators.

EXAMPLE 3 Write equivalent fractions with a common denominator: $\frac{2}{a}, \frac{3}{b}$.

Solution $\quad \frac{2}{a} = \frac{2 \times b}{a \times b} = \frac{2b}{ab} \qquad \frac{3}{b} = \frac{3 \times a}{b \times a} = \frac{3a}{ab}$

Class Exercises

State the LCM of the pair of numbers.

1. 6, 18   **2.** 11, 4   **3.** 10, 8   **4.** 15, 12   **5.** 32, 48

State the LCD of the pair of fractions.

6. $\frac{1}{2}, \frac{3}{4}$ **7.** $\frac{5}{6}, \frac{1}{2}$ **8.** $\frac{3}{4}, -\frac{1}{3}$ **9.** $-\frac{2}{9}, \frac{1}{6}$ **10.** $\frac{1}{16}, \frac{5}{12}$

Written Exercises

Write the fractions as equivalent fractions with the least common denominator (LCD).

A   **1.** $\frac{1}{3}, \frac{1}{12}$ **2.** $\frac{1}{4}, \frac{1}{12}$ **3.** $\frac{3}{4}, \frac{5}{8}$ **4.** $\frac{3}{8}, \frac{3}{16}$

     5. $\frac{2}{9}, -\frac{1}{27}$ **6.** $-\frac{1}{7}, \frac{1}{49}$ **7.** $\frac{10}{21}, \frac{2}{49}$ **8.** $\frac{7}{18}, \frac{5}{36}$

     9. $-\frac{2}{3}, \frac{7}{30}$ **10.** $\frac{5}{12}, -\frac{6}{11}$ **11.** $\frac{4}{75}, \frac{7}{100}$ **12.** $\frac{9}{56}, \frac{4}{63}$

     13. $-\frac{3}{7}, -\frac{7}{112}$ **14.** $-\frac{4}{17}, -\frac{9}{16}$ **15.** $\frac{5}{42}, \frac{5}{49}$ **16.** $\frac{5}{84}, \frac{7}{12}$

B   **17.** $\frac{7}{8}, \frac{5}{16}, \frac{21}{40}$ **18.** $\frac{1}{4}, \frac{1}{9}, \frac{1}{5}$ **19.** $\frac{3}{7}, \frac{7}{4}, \frac{4}{9}$ **20.** $\frac{11}{18}, \frac{1}{54}, \frac{2}{27}$

     21. $\frac{1}{65}, \frac{3}{5}, \frac{9}{26}$ **22.** $-\frac{7}{8}, \frac{9}{28}, \frac{2}{49}$ **23.** $-\frac{5}{6}, \frac{2}{9}, -\frac{7}{8}$ **24.** $\frac{11}{12}, \frac{1}{72}, -\frac{7}{8}$

     25. $\frac{a}{3}, \frac{b}{6}$ **26.** $\frac{m}{25}, \frac{n}{15}$ **27.** $\frac{h}{25}, \frac{h}{100}, \frac{h}{125}$ **28.** $\frac{a}{2}, \frac{b}{3}, \frac{c}{4}$

Write the fractions as equivalent fractions with a common denominator.

29. $\frac{1}{c}, \frac{2}{3c}$ **30.** $\frac{1}{x}, \frac{1}{y}$ **31.** $\frac{3}{x}, \frac{1}{y}, \frac{5}{z}$ **32.** $\frac{-1}{r}, \frac{2}{rs}, \frac{r}{s}$

4. Carl is 4 ft tall. If he grew $1\frac{1}{8}$ in. during the past year, and $\frac{3}{4}$ in. the year before, how tall was he one year ago?

5. On Monday, Kim jogged $1\frac{1}{2}$ mi in $\frac{1}{4}$ h. On Wednesday she jogged $2\frac{1}{3}$ mi in $\frac{1}{3}$ h. How much farther did Kim jog on Wednesday than on Monday?

B 6. Last year, total rainfall for April and May was $7\frac{1}{4}$ in. This year 3 in. of rain fell in April and $2\frac{5}{8}$ in. fell in May. How much less rain fell this year than last year during April and May?

C 7. A 512-page book has pages 7 in. wide by 9 in. high. The printed area measures $5\frac{3}{8}$ in. by $7\frac{3}{4}$ in. The left margin is $\frac{5}{16}$ in. and the top margin is $\frac{9}{16}$ in. How wide are the margins at the right and the bottom of each page?

Self-Test A

Complete.

1. $5 \times \underline{} = \frac{5}{8}$ 2. $3 \times \underline{} = -1$ [4–1]

3. $7 \div 9 = \underline{}$ 4. $-\frac{2}{3} = \frac{-2}{3} = \underline{}$

Write as a proper fraction in lowest terms or as a mixed number in simple form.

5. $\frac{16}{64}$ 6. $-\frac{72}{30}$ 7. $\frac{32}{42}$ 8. $\frac{-71}{48}$ 9. $\frac{68}{16}$ [4–2]

Write as an improper fraction.

10. $2\frac{1}{5}$ 11. $-3\frac{2}{3}$ 12. $6\frac{4}{15}$ 13. $1\frac{7}{12}$ 14. $-8\frac{5}{8}$

Write as equivalent fractions with the least common denominator.

15. $\frac{7}{9}, \frac{7}{8}$ 16. $-\frac{10}{49}, \frac{2}{21}$ 17. $\frac{3}{50}, \frac{6}{225}$ 18. $-\frac{8}{15}, -\frac{1}{30}$ [4–3]

Add or subtract. Write the answer as a proper fraction in lowest terms or as a mixed number in simple form.

19. $\frac{1}{3} + \frac{1}{4}$ 20. $\frac{1}{5} - \frac{1}{3}$ 21. $\frac{2}{15} + \left(-\frac{5}{6}\right)$ [4–4]

22. $16\frac{5}{8} - \frac{3}{4}$ 23. $17\frac{1}{3} + 5\frac{1}{9}$ 24. $-6\frac{3}{8} + 2\frac{1}{6}$

Self-Test answers and Extra Practice are at the back of the book.

4-7 Fractions and Decimals

Any fraction can be represented as a decimal. You may recall that a fraction such as $\frac{3}{4}$ can be easily written as an equivalent fraction whose denominator is a power of 10, and then as a decimal. To represent $\frac{3}{4}$ as a decimal, we first write it as an equivalent fraction with denominator 100.

$$\frac{3}{4} = \frac{3 \times 25}{4 \times 25} = \frac{75}{100} = 0.75$$

For most fractions, however, we use the fact that $\frac{a}{b} = a \div b$ and divide numerator by denominator.

EXAMPLE 1   Write as a decimal: **a.** $-\dfrac{5}{16}$ **b.** $\dfrac{24}{55}$

Solution   **a.** First find $5 \div 16$.

```
      0.3125
16)5.0000
    4 8
      20
      16
      40
      32
      80
      80
       0
```

Therefore, $-\frac{5}{16} = -0.3125$.

The decimal -0.3125 is called a **terminating decimal** because the final remainder is 0 and the division ends.

b. Find $24 \div 55$.

```
      0.43636
55)24.00000
   22 0
    2 00
    1 65
      350
      330
      200
      165
      350
      330
       20
```

Therefore, $\frac{24}{55} = 0.43636\ldots$.

The digits 36 continue to repeat without end. The decimal $0.43636\ldots$ is called a **repeating decimal.** We often write $0.43636\ldots$ as $0.4\overline{36}$, with a bar over the block of digits that repeats.

To say that $\frac{24}{55} = 0.43636\ldots$ means that the successive decimals 0.436, 0.4363, 0.43636, and so on, will come closer and closer to the value $\frac{24}{55}$.

We can predict when a fraction will result in a terminating decimal because the fraction in lowest terms has a denominator with no prime factors other than 2 and 5. Thus, the fraction $\frac{24}{55}$ does not result in a terminating decimal because its denominator has 11 as a prime factor.

When working with a mixed number, such as $-1\frac{5}{16}$ or $1\frac{24}{25}$, we may consider the mixed number as a sum of a whole number and a fraction, or we may rewrite the mixed number as an improper fraction and then divide.

If a and b are integers and $b \neq 0$, the quotient $a \div b$ is either a terminating decimal or a repeating decimal. The reason for this is that, for any divisor, the number of possible remainders at each step of the division is limited to the whole numbers less than the divisor. Sooner or later, either the remainder is 0 and the division ends, as in part (a) of Example 1, or one of the remainders reappears in the division as in part (b) of Example 1. Then the same block of digits will reappear in the quotient.

Property

Every rational number can be represented by either a terminating decimal or a repeating decimal.

You already know how to write a terminating decimal as a fraction. Rewrite the decimal as a fraction whose denominator is a power of 10.

EXAMPLE 2   Write -0.625 as a fraction in lowest terms.

Solution $-0.625 = -\dfrac{625}{1000} = -\dfrac{625 \div 125}{1000 \div 125} = -\dfrac{5}{8}$

The next example shows a method for writing a repeating decimal as a fraction.

EXAMPLE 3   Write $-1.\overline{21}$ as a fraction in lowest terms.

Solution   Let $n = 1.\overline{21}$.

Multiply both sides of the equation by a power of 10 determined by the number of digits in the block of repeating digits. Since there are

2 digits that repeat in the number $1.\overline{21}$, we multiply by 10^2, or 100.

$$100n = 121.\overline{21}$$

Subtract: $\qquad n = \quad 1.\overline{21}$

$$99n = 120$$

$$n = \frac{120}{99} = \frac{40}{33}$$

Thus, $-1.\overline{21} = -\frac{40}{33}$, or $-1\frac{7}{33}$.

Property

Every terminating or repeating decimal represents a rational number.

Some decimals, such as those below, neither terminate nor repeat.

$\qquad$ 0.01001000100001 . . . $\qquad\qquad$ 1.234567891011121314 . . .

The two decimals shown follow patterns, but they are not repeating patterns. The decimal on the right is made up of consecutive whole numbers beginning with 1.

$\qquad$ Decimals that neither terminate nor repeat represent **irrational numbers.** Together, the rational numbers and the irrational numbers make up the set of **real numbers.** The number line that you have studied is sometimes called the **real number line.** For every point on the line, there is exactly one real number and for every real number there is exactly one point on the number line.

Class Exercises

Tell whether the decimal for the fraction is terminating or repeating. If the decimal is terminating, state the decimal.

1. $\frac{1}{4}$ $\qquad$ **2.** $\frac{5}{6}$ $\qquad$ **3.** $2\frac{2}{5}$ $\qquad$ **4.** $-\frac{9}{10}$ $\qquad$ **5.** $-1\frac{1}{2}$ $\qquad$ **6.** $\frac{13}{30}$

State as a fraction in which the numerator is an integer and the denominator is a power of 10.

7. 0.13 $\qquad$ **8.** -0.9 $\qquad$ **9.** 1.4 $\qquad$ **10.** -0.007 $\qquad$ **11.** 3.03 $\qquad$ **12.** -5.001

Self-Test B

Multiply or divide. Write the answer as a proper fraction in lowest terms or as a mixed number in simple form.

1. $\frac{3}{4} \times 5$

2. $\frac{1}{8} \times \left(-\frac{1}{3}\right)$

3. $\frac{28}{35} \times \frac{21}{14}$ [4-5]

4. $8\frac{3}{4} \times \frac{3}{16}$

5. $-3\frac{1}{8} \times \left(-4\frac{4}{5}\right)$

6. $-2\frac{4}{7} \times 3\frac{1}{6}$

7. $\frac{5}{8} \div \frac{10}{24}$

8. $-\frac{11}{16} \div \frac{44}{8}$

9. $-\frac{18}{5} \div \left(-\frac{9}{35}\right)$ [4-6]

10. $1\frac{1}{4} \div 25$

11. $4\frac{1}{3} \div \left(-\frac{26}{27}\right)$

12. $-4\frac{2}{7} \div \left(-2\frac{1}{14}\right)$

Write as a decimal. Use a bar to show repeating digits.

13. $\frac{5}{8}$

14. $\frac{2}{11}$

15. $-\frac{1}{80}$

16. $\frac{7}{6}$ [4-7]

Write as a proper fraction in lowest terms or as a mixed number in simple form.

17. 0.875

18. $1.\overline{6}$

19. -2.213

20. $0.2\overline{3}$

Self-Test answers and Extra Practice are at the back of the book.

■■■ COMPUTER INVESTIGATION: *Decimal Approximation*

The following program will print a decimal approximation for any positive proper fraction. The first approximation is rounded to the number of digits that the computer usually displays. The computer will then print an approximation to any desired number of digits.

```
10   PRINT "INPUT NUMERATOR, THEN DENOMINATOR"
20   INPUT N,D
30   IF N > D THEN 10
40   PRINT N;"/";D;" = ";N / D
50   PRINT "HOW MANY DIGITS DO YOU WANT ?";
60   INPUT K
70   PRINT "OR .";
80   FOR I = 1 TO K
90   LET P = 10 * N / D
100  PRINT  INT (P);
110  LET N = 10 * N - D *  INT (P)
120  NEXT I
130  END
```

Use the program to discover the repeating pattern for these fractions.

1. $\frac{1}{7}$ 2. $\frac{2}{7}$ 3. $\frac{10}{11}$ 4. $\frac{5}{13}$ 5. $\frac{76}{99}$ 6. $\frac{12}{37}$ 7. $\frac{17}{33}$ 8. $\frac{134}{333}$

5-2 Equivalent Equations

On page 9, you learned that a *solution* of an equation involving a variable is a value of the variable that makes the equation true. Because the equation

$$x + 5 = 18$$

is true when $x = 13$, 13 is a solution of the equation. An equation may have no solution, one solution, or more than one solution.

The set of numbers that a variable may represent is called the *replacement set*. When the replacement set is small, we may substitute the values for the variable to solve the equation. When the replacement set is larger, for example the rational numbers, substitution may not be practical. In such cases, we use the properties that we have learned to change, or **transform,** the equation into a simpler equation that has the same solution. Two equations that have the same solution are called **equivalent equations.**

One transformation that we may use to obtain an equivalent equation is the following.

> Simplify numerical expressions and variable expressions.

The example that follows shows how to use this transformation to obtain equivalent equations. You will learn how to solve equations in the lessons that follow.

EXAMPLE   Simplify the expressions on both sides of the equation to obtain an equivalent equation.

   a. $y - 8 + 3 = 24 \div 6$ **b.** $38 = 2n + 6 + 3n$ **c.** $2(a + 3) = 18$

Solution   **a.** Perform the operations on each side of the equation to simplify the numerical expressions.
$$y - 8 + 3 = 24 \div 6$$
$$y - 5 = 4$$

b. Combine like terms to simplify the variable expressions.
$$38 = 2n + 6 + 3n$$
$$38 = 2n + 3n + 6$$
$$38 = 5n + 6$$

c. Use the distributive property to simplify the left side.
$$2(a + 3) = 18$$
$$2a + 2(3) = 18$$
$$2a + 6 = 18$$

Throughout the rest of the chapter, if no replacement set is given for an equation, assume that the solution can be any number.

Class Exercises

Simplify the expressions on both sides of the equation to obtain an equivalent equation.

1. $m - 9 + 5 = 32 \div 8$

2. $4 \times 12 = 3 + h - 10$

3. $13 + 8 + k = 11 \times 5$

4. $45 \div 9 = t - 7 - 8$

5. $42 = 3n + 2 + 5n$

6. $8v - 10 + 2v = 30$

7. $7q + 15 - 2q = 25$

8. $35 = y - 7 + 5y$

9. $3(x + 2) = 18$

10. $20 = 5(a - 3)$

Written Exercises

Simplify the expressions on both sides of the equation to obtain an equivalent equation.

A

1. $b + 10 - 3 = 44 \div 2$

2. $5 \times 23 = 8 + k + 7$

3. $24 \times 3 = 8 + 12 + u$

4. $c - 11 + 4 = 54 \div 9$

5. $26 \div 13 = -7 - 5 + v$

6. $8 \times 12 = e + 14 - 2$

7. $4 + 9 + f - 3 = 24 \div 2 \times 3$

8. $6 \times 8 \div 3 = t + 12 - 9 + 6$

9. $33 = 4n - 7 + 6n$

10. $12a + 8 - 3a = 35$

11. $13q - 2q - 2 = 20$

12. $14 = 7d - 2 + d$

13. $15 = 3z + 5 + 2z$

14. $-8v + 2v + 3 = 21$

15. $4x + 8x - 2x + 7 = -13$

16. $18 = 2f - 5f + 9 + 6f$

17. $4(j + 2) = 36$

18. $42 = (g + 3)7$

19. $20 = (e - 5)2$

20. $-8(m - 1) = 24$

Equations and Inequalities **147**

Solve.

34. $15 - 6(x + 2) = 1$ **35.** $50 = 2(5r + 2) - 1$ **36.** $26 = -5(4 + 8y) - 6$

37. $3a = 2.5(8 + a)$ **38.** $0.3(p + 4) = 2.7p$ **39.** $x(4.8 - 2) = 4.8x - 2$

40. $4q + 19 = \frac{5}{6}q$ **41.** $\frac{3}{5}c = 2c - 7$ **42.** $\frac{2}{3}a = -15 - a$

C **43.** $\frac{1}{2}b + \frac{3}{5} = 2b - \frac{9}{10}$ **44.** $\frac{3}{4}f - \frac{1}{2} = 2f + \frac{1}{2}$

45. $-\frac{1}{6}\left(t - \frac{1}{3}t\right) = -t + 36$ **46.** $-z - 9 = \frac{3}{4}\left(2z + \frac{4}{3}\right)$

47. $\frac{2}{3}\left(m - \frac{1}{2}\right) + 3 = \frac{m}{3} - 7$ **48.** $\frac{1}{2}(-4x - 5) = 7x - \frac{1}{2}$

Write an equation for the word sentence and solve for y.

49. Nine divided by six is four times the difference of a number y subtracted from four.

50. Five times the sum of a number y and 11 is seventeen minus three.

51. Five eighteenths subtracted from one third of a number y is twice the number y.

52. One half the sum of five and a number y is one quarter y.

Self-Test A

Use one of the properties of equality to form a true sentence.

1. If $35 + p = 47$, then $p = \underline{\ ?\ }$. **2.** If $-6a = 48$, then $a = \underline{\ ?\ }$. [5-1]

Simplify the expressions on both sides of the equation to obtain an equivalent equation.

3. $17 = 3x - 9 + 5x - 8$ **4.** $24 \div 3 = 5(m - 9)$ [5-2]

Use transformations to solve the equation.

5. $x + 19 = 24$ **6.** $36 = y - 11$ **7.** $a + 14 = -9 - 3$ [5-3]

8. $-8q = 56$ **9.** $\frac{m}{18} = 9$ **10.** $\frac{1}{4}d = 8$ [5-4]

11. $27 = 2y - 13$ **12.** $-2(7 + 2b) = 52$ **13.** $6n - 9 - 3n = n - 17$ [5-5]

Self-Test answers and Extra Practice are at the back of the book.