GreatHearts Irving

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

April 20 - 24, 2020

Course: 9 Geometry

Teacher(s): Mr. Mooney sean.mooney@greatheartsirving.org

Weekly Plan:

Monday, April 20

☐ Review “Answer Keys” IV.11 and IV.15

☐ Repeat constructions IV.11 and IV.15

Tuesday, April 21

☐ Read Book V Definitions 1-8

☐ Book V Definitions Questions #1-18

Wednesday, April 22

☐ Read V.Definitions 9-17

☐ Book V Definitions Questions #19-22

Thursday, April 23

☐ Read Book V Propositions (all)

☐ Answer Book V Propositions Questions

Friday, April 24

☐ Read VI.Defintions.1-3

☐ Read VI.1 and put into two-column

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____

Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____

Parent Signature

Dear Students,

Welcome to another week of Remote Geometry! I hope you are all doing well. I know that remote learning can be tough, especially when studying something as intellectually difficult as Euclid's *Elements*. I encourage you to persevere and to please email me or come to Office Hours if you have any questions.

We have now finished with Book IV, with its beautiful and triumphant constructions. This week, we plunge into the concepts of ratio and proportion, concepts which--I might argue--are the very heart and soul of all mathematics. These are difficult concepts to learn and will require a significant shift in the way we have been thinking.

I like to think of Books V and VI in terms of the following analogy. If we were going to travel to a foreign country (perhaps now is not the right time for that, but it's just an analogy!), we would first want to learn the language that is spoken there. If you learn the language, you will have a great time and easily be able to travel around and see all there is to see. If you *don't* learn the language, you will easily become lost and will not have a great time. Book V is the language, and Book VI the foreign country. In Book V, we learn to speak the language of ratio and proportion. In Book VI, we apply the language of ratio and proportion to figures like triangles. You may find Book V's definitions and propositions confusing and difficult at first, but I really want to encourage you to persevere and strive to master them--when we get to Book VI, you will be very glad that you did.

As you'll see, I have written explanations to go along with the definitions and propositions that we are studying in Book V. When we look at the propositions, you'll notice that our focus will only be on the *enunciations*--indeed, we will not look at the proofs at all. We will seek to understand how each enunciation is true and how it makes good sense, and then we will look at how it would be used. I've written practice problems for both the definitions and the enunciations, to help you get the hang of using them.

Then, finally, on Friday, we will begin to put our "language" skills to use as we begin our journey in the land of Book VI.

I wish you all the best of luck this week, and I hope you enjoy opening your mind to the fundamental ideas of ratio and proportion.

Let's get started!

Monday, April 20

Before we begin on Ratio and Proportion, let's first finish up and polish our knowledge and skills from last week. Today I would like you to:

1) Review the "Answer Keys" for the construction of IV.11 and IV.15
2) On the pages provided, repeat the constructions IV.11 and IV.15

Tuesday, April 21

Ratio and Proportion, here we come!

Today I would like you to:

3) Read V.Definitions.1-8 *in your textbook*. Spend about 3-5 minutes reading these, doing your best to understand. If there are things that you still do not understand after 3-5 minutes, that is fine, go ahead and move on.
4) Read Definitions 1-8 in the supplementary reading, to be found in this packet, under the title "Book V Definitions Reading."
5) Answer questions #1-18 in the section entitled "Book V Definitions Questions"

Wednesday, April 22

Today, we will finish up with Book V definitions. I would like you to:

1) Read V.Definitions.9-17 *in your textbook*. Spend about 3-5 minutes reading these, doing your best to understand. If there are things that you still do not understand after 3-5 minutes, that is fine, go ahead and move on.
2) Read Definitions 9-17 in the supplementary reading, to be found in this packet, under the title "Book V Definitions Reading."
3) Answer questions #19-22 in the section entitled "Book V Definitions Questions"

Thursday, April 23

Today, we begin Book V propositions! As I mentioned earlier, we will not be looking at any proofs of these propositions, focusing only on the enunciations. In fact, you do not even need to look at these propositions in your *Elements* (although of course you may if you are interested!).

I would like you to:

1) Read the pages entitled "Book V Propositions Reading" in this packet. This will cover V.7, V.9, V.11, V.12, V.15, V.16, V.18, and V.22. These are the only propositions we will study together in Book V.
2) Answer questions #1-22 in the section entitled "Book V Propositions Questions"

Friday, April 24

The time has come--today, we begin our journey into the foreign land of Book VI. The first sight to see is the four definitions of Book VI. And then we will head to the ancient (and eternal!) monument: proposition 1!

I would like you to:

1) Read VI.Definitions.1-4. Go ahead and read all of them, but focus your attention primarily on definitions 1 and 4. We will look more closely at what the others mean when they come up later on.
2) Read proposition VI.1 and write out the argument in two-column format.

That's all for this week! Good work! I wish you a very happy weekend, filled only with true, good, and beautiful things!

Sincerely,

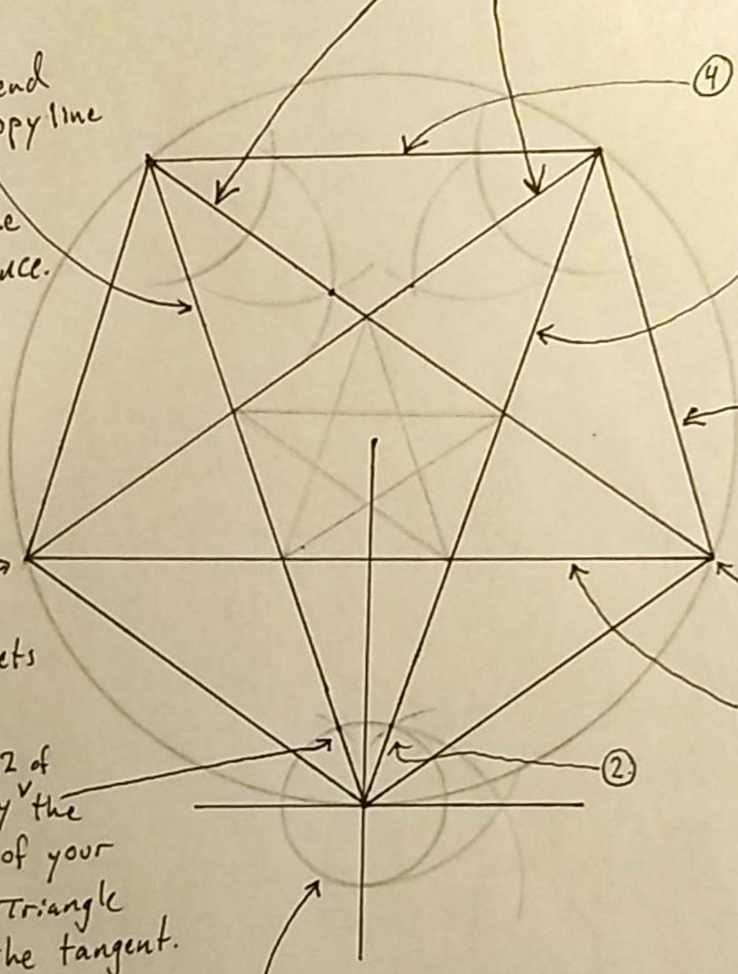
Mr. Mooney

IV. 11 Construction "Answer Key"

⑤ Bisect the two angles formed by joining the line in step 4.

③ Continue/extend your angle copy line until it reaches the circumference.

④ Join the endpoints with a straight line.



③

⑦ Join all 5 points on the circumference to form a pentagon.

⑥ Extend your angle bisection line until it meets circumference.

⑥

⑧ Join this line to form a second pentagon in the middle. The better your construction is, the better this smaller pentagon will look. (To repeat pattern, connect the 5 vertices of the smaller pentagon.)

② Copy 2 of the angles of your Golden Triangle onto the tangent. I chose to copy the two equal base angles.

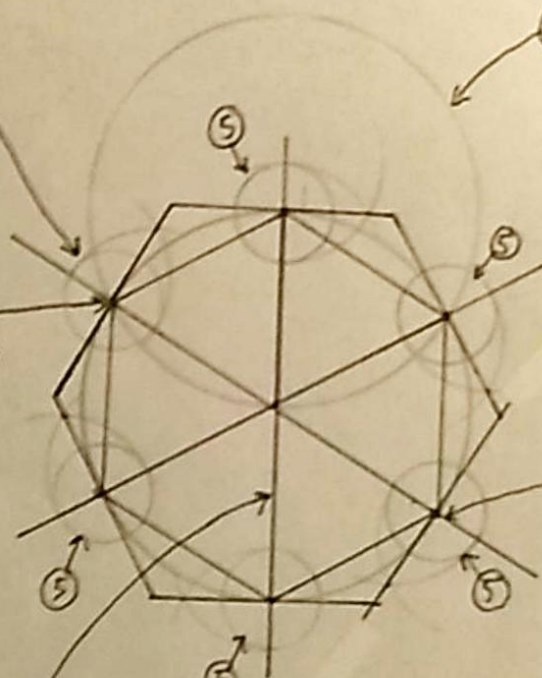
②

① Construct a tangent, used to inscribe a golden triangle into you circle.

IV. 15 Construction (+ Porism) "Answer Key"

⑤ Porism: Draw a perpendicular from each vertex, perpendicular to the diameter.

② Draw a circle around the endpoint of the diameter, with a radius equal to the radius of your original circle.



③ From the points where the two circles intersect, draw two new diameters.

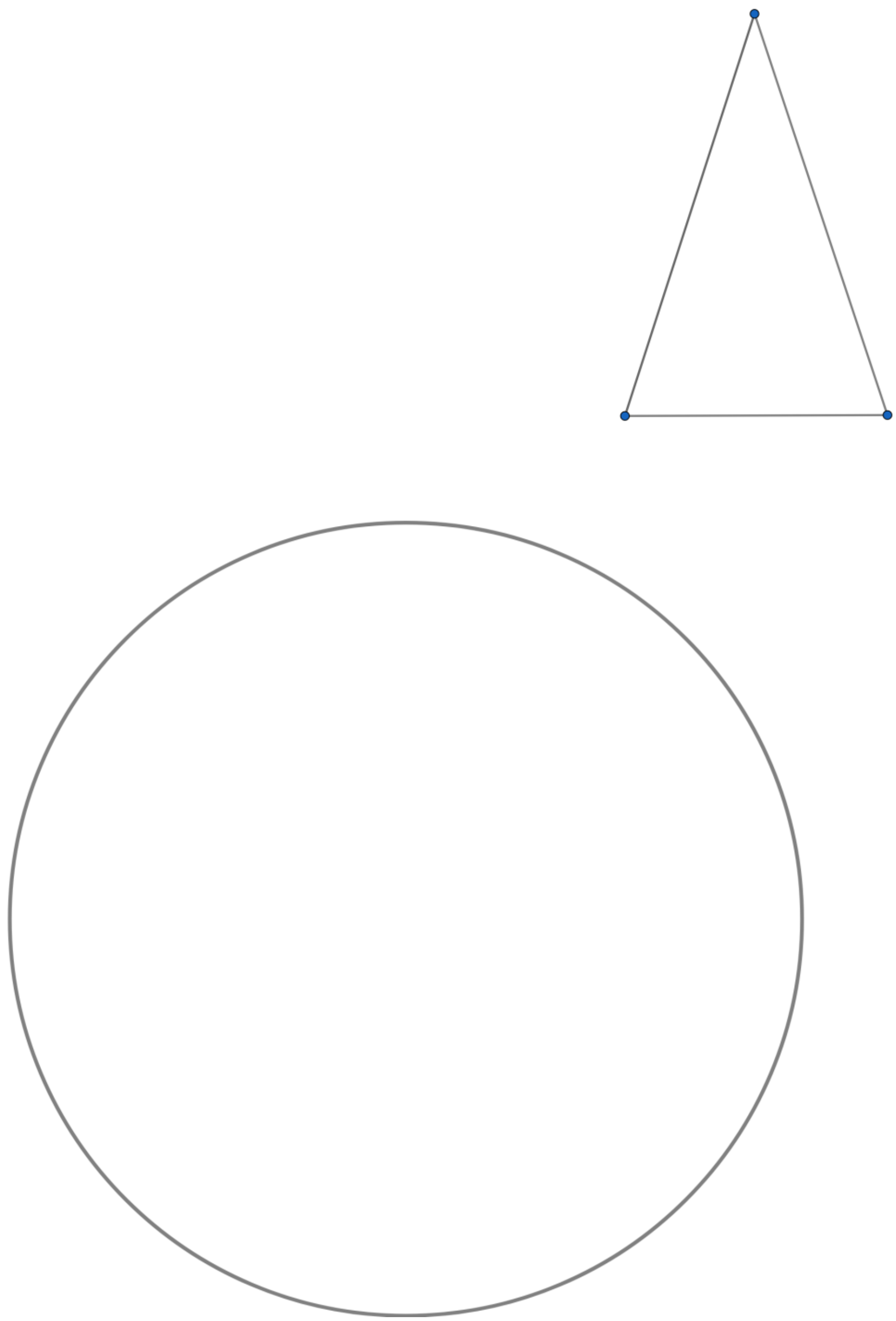
④ Now, you should have a total of 3 diameters, for a total of 6 endpoints on the circumference. These are the vertices of your hexagon. Join them.

① Draw a diameter in your given circle.

⑥ You now have two regular hexagons, one inscribed in the circle, and one circumscribed about it.

IV.11 and IV.15 Constructions (To be done with help of Answer Keys)

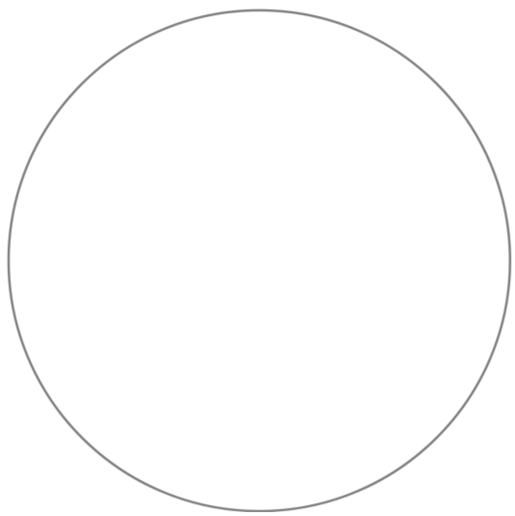
IV.11: Using the given Golden Triangle, inscribe a regular Pentagon in the given circle.



IV.11 and IV.15 Constructions (To be done with help of Answer Keys)

IV.15: In the given circle, inscribe a regular hexagon.

IV.15.Porism: On the same construction, circumscribe a regular hexagon about the original circle.



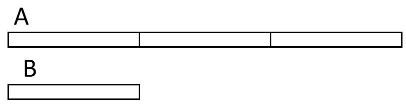
Book V Definitions Reading

1. PART

*"A <u>magnitude</u> is **part** of a magnitude, the <u>less of the greater</u>, when it <u>measures</u> the greater."*

a) <u>Magnitude</u>: any kind of quantity, e.g. lines, figures or areas, solids, numbers, etc.

b) <u>Measures</u>: Referring to the diagram below, we say that "B measures A three times." It is similar to the way sometimes we say a number "goes into" another number a certain number of times. For example, 3 goes into 12 four times.



Thus, if we say that magnitude B is **part** of magnitude A, we mean that B measures A. Sometimes, we will specify: B is the a **third part** of A—meaning that B measures A three times.

2. MULTIPLE

*"The greater is a **multiple** of the less when it is measured by the less."*

This definition is similar to the first, but from the other perspective. If speaking about B in relation to A, we say that B is part of A; but if we want to talk about A in relation to B, we say that A is a **multiple** of B.

For example, in the diagram below, C is a multiple of D. (Specifically, it looks like the *fourth* multiple)

C ________________________________

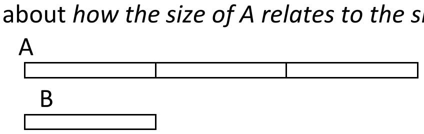
D _______

3. RATIO

*"A **ratio** is a sort of <u>relation</u> in <u>respect of size</u> between two magnitudes of the <u>same kind</u>."*

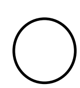
a) <u>Relation</u>: Ratio is a *relation*. That is, when thinking about ratio, we are considering *how something relates* to another thing.

b) <u>"in respect of size"</u>: Ratio is not just *any* relation, it is a relation of *size.* The ratio of A to B, for example, is about *how the size of A relates to the size of B*.



How *does* magnitude A's size relate to magnitude B's? The expression of this relation—that is, this ratio—is $A : B$.[1]

c) <u>Same Kind</u>: Lastly, there can only be a ratio between magnitudes of the same kind. For example,

A ______________________ B 

There can be no ratio $A : B$ here, because A is a *length* and B is a *figure/area*, and those are *different kinds* of magnitudes.

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[1] In the example above, since you can actually tell that B measures A three times, we could also say that $A : B$ as $3 : 1$ . . . but this is getting a little bit ahead.

Book V Definitions Reading

4. "HAVE A RATIO"

*"Magnitudes are said to **have a ratio** to one another which are capable, when multiplied, or exceeding one another."*

a) Multiplied: A line is said to be "multiplied" when its length is set out multiple times.

For example, if I multiply the line A three times, I get:

A ___________ → 3A _________________________________

This, of course, also applies to the use of "multiplication" that you are more familiar with: the multiplication *of number*.

For example, if I multiply the number 3 four times, I get…

b) Exceeding: If one magnitude exceeds another magnitude, it surpasses the other in size.

This definition is something like a test to see what magnitudes can be said to have a ratio with one another. Can A and 3A have a ratio? Yes, says definition four, because if A is multiplied enough (four times) it will eventually exceed 3A. Can the number five have a ratio to infinity (∞)? No, it cannot. The number five, no matter how many times you multiply it, will never exceed infinity.

5. SAME RATIO

*"Magnitudes are said to be **in the same ratio**, the first to the second and the third to the fourth, when, if any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth, the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order."*

Can one person's height be the *same* as another person's height? Of course! Can one length be the *same* as another? Sure! But can one relation be the *same* as another relation? This last question is more difficult to answer. Definition Five tells us what it would mean to say that two magnitudes relate to each other *in the same way* as two other magnitudes relate to each other—that is, that they have the *same ratio*.
Here is an example of what it could look like if the ratio of two lengths A and B was the *same* as the ratio between two other lengths, C and D?

A _______________ C ___
B ______________________ D _____

Now, my diagram is not perfect, but do you see what is meant here? The ratio of $A\!:\!B$—that is, the way A's size relates to B's size—is the *same* as the ratio of $C\!:\!D$—that is, it is the same as the way that C's size compares to D's size.

Let us now make sense of the difficult definition, which aims to put this idea into strict, logical, and theoretically verifiable[2] terms.

Euclid is talking about four magnitudes: namely, the first, second, third, and fourth.

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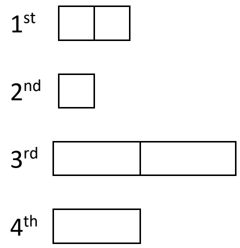
[2] Remember, we are not judging things by appearance, or with a ruler. We need some other strictly theoretical way of determining whether or not two ratios are the same.

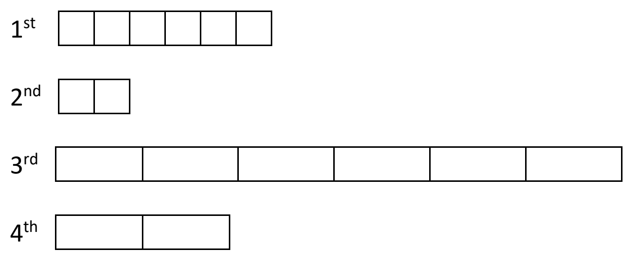
Book V Definitions Reading



Can you tell from my diagram that the ratio 1^{st} : 2^{nd} *appears* to be the same as the ratio 3^{rd} : 4^{th} ? Now, though it may *appear* that way, we have to check it using this definition of same ratio. Euclid says that the first will have the same ratio to the second as the third has to the fourth *"when…*

"If any equimultiples whatever be taken of the first and third, and any equimultiples whatever of the second and fourth…"

The test lies in taking "equimultiples" (that is, the *same* multiple) of the first and third magnitudes, and then doing the same for the second and fourth. Let's try it out:



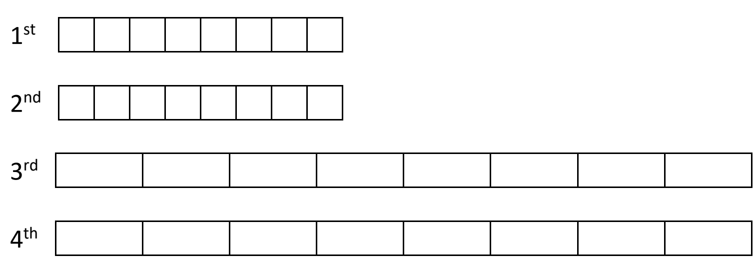
Can you see what multiple was taken of the first and the third? It is the *third* multiple of each of them (and since the same multiple was taken of both, they are called *equimultiples*). You can see that equimultiples have been taken of the second and fourth as well (specifically, the second multiple of each).
Euclid says that if you do this, taking equimultiples as we just did above, and …

> "*the former equimultiples alike exceed, are alike equal to, or alike fall short of, the latter equimultiples respectively taken in corresponding order"*

then, that means they have the *same ratio*.

Notice that there are three parts: alike exceed, alike equal, or alike fall short of. Looking back up at the diagram above, we see that the former equimultiples (that is, the first and third), *alike exceed*—they have passed the first "test," so to speak.

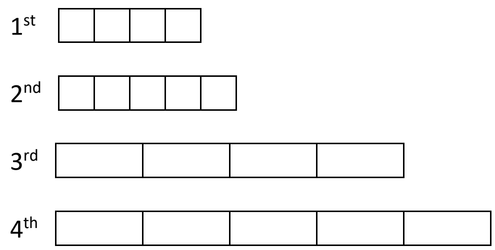
If we take different equimultiples, can we get them to pass the other two tests: alike equal and alike fall short of?



Book V Definitions Reading

Here we take the *fourth* multiple of the first and third, and the *eighth* multiple of the second and fourth, and we see that the first and third are *alike equal to* the second and fourth respectively.

And lastly,



The last condition—*alike fall short of*—is met when we take the second multiple of the first and third, and the fifth multiple of the second and fourth.

What does this all mean? It means that the ratios are the same! The ratio of 1st : 2nd is the same as the ratio of 3rd : 4th. This all may perhaps seem a bit underwhelming to you right now, but as you will see, the idea of *same ratio* is absolutely the most important and foundational idea of Books V and VI and will thus occupy our thoughts for the remainder of the year.

6. PROPORTIONAL

*"Let magnitudes with have the same ratio be called **proportional**."*

Thus, if the ratio of $A\!:\!B$ is the *same* as the ratio of $C\!:\!D$,

A _______________ C ___
B _______________________ D _____

we would say that the *ratios are the same,* and the *magnitudes are proportional*. That is, "proportional" is an adjective that describes the magnitudes when they are in the same ratio.

A **proportion** is a statement that the magnitudes are proportional (that is, that they have the same ratio). The proportion in this case would be written:

$$A\!:\!B \;::\; C\!:\!D$$

This would be read "*As* A *is to* B, *so* C *is to* D."

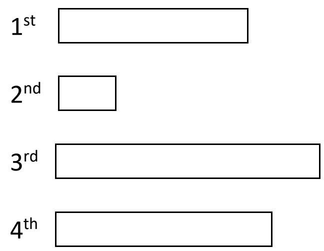
7. HAVE A GREATER RATIO

*"When, of the equimultiples, the multiple of the first magnitude exceeds the multiple of the second, but the multiple of the third does not exceed the multiple of the fourth, then the first is said to **have a greater ratio** to the second than the third has to the fourth."*

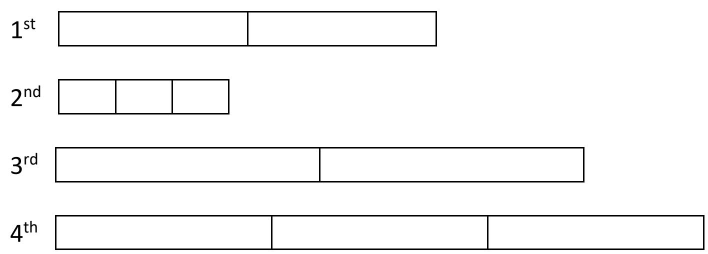
This definition carries on from definition five. That is, what if our magnitudes had *not* passed the tests for same ratio? What if, when equimultiples were taken, the first and third did not exceed, equal, or fall short of the second and fourth respectively? If the ratios are not the same, then one of them is greater. Definition seven serves to tell us how to know which ratio is greater.

Book V Definitions Reading

Consider the following four magnitudes.



Let's take the second multiple of the first and third magnitudes, and the third multiple of the second and fourth magnitudes.



Notice that the 1st and 3rd magnitudes are multiplied twice and the 2nd and 4th magnitudes are multiplied three times. When we do this, the 1st exceeds the 2nd, but the 3rd falls short of the fourth. This means, according to this definition seven, that the first has to the second as greater ratio than the third has to the fourth.

We can write it this way: $1\text{st} : 2\text{nd} \;>\; 3\text{rd} : 4\text{th}$

8. PROPORTION IN THREE TERMS

*"A **proportion in three terms** is the least possible."*

So far, the proportions that we have looked at involve four terms. For example, in the proportion

$$AB:CD \;::\; EF:GH$$

we have four terms (AB, CD, EF, and GH). How could there be a proportion in three terms only?

Let's consider three magnitudes PQ, RS, and TU:

P ______Q

R ____________________S

T ____U

I have attempted to draw these so that it appears that the ratio of PQ to RS is the same as the ratio of RS to TU (PQ is roughly a third the size of RS, and RS is roughly a third of TU). That is:

$$PQ:RS \;::\; RS:TU$$

Euclid says that such a proportion is the "least possible," apparently to say that it is less likely to find a proportion in three terms than in four.

Book V Definitions Reading

9. DUPLICATE RATIO

*"When three magnitudes are proportional, the first is said to have to the third the **duplicate ratio** of that which it has to the second."*

Consider the example of three proportional magnitudes, from definition eight above:

P ______Q

R ___________________S

T ____U

The first (PQ) is said to have to the third (TU) the *duplicate ratio* of that which it (PQ) has to the second (RS).

Let's say that PQ is exactly one third of RS (that is, they have a 1 : 3 ratio). Since they are proportional, RS is therefore exactly one third of TU.

According to this definition, PQ has to TU the duplicate ratio of that which it (PQ) has to RS. We know that PQ has to RS a ratio of 1 : 3. Since RS and TU *also* have a 1 : 3 ratio, can you figure out what the ratio of PQ to TU would be? … That's right! It would have to be a 1 : 9 ratio!

In algebra, the duplicate ratio is thought of as the ratio "squared." It would look something like this: $(1:3)^2$. And $(1:3)^2 = 1:9$.

You could think of it this way too: if PQ is one third of RS, then PQ is *a third of a third* of TU. (And "a third of a third" is the duplicate ratio, which is equal to one ninth.)

10. TRIPLICATE RATIO

*"When four magnitudes are [continuously] proportional, the first is said to have to the fourth the **triplicate ratio** of that which has to the second, and so on continually, whatever be the proportion."*

Triplicate ratio is just like duplicate ratio. Consider *four* proportional magnitudes:

A ___B

C ______D

E ____________F

G ________________________H

We would write the proportion thus: $AB:CD :: CD:EF :: EF:GH$.
In this case, AB has to GH the triplicate ratio of that which it has to CD.

That is, if AB is ½ of CD, then it is a ½ of a ½ of a ½ of GH. And a ½ of a ½ of a ½ is the *triplicate ratio*, and we can see that it equal $\frac{1}{8}$.
Thus, we can also express the triplicate ratio as $(1:2)^3$, which we know is equal to $1:8$

<u>**Book V Definitions Reading**</u>

11. CORRESPONDING MAGNITUDES

*"The term **corresponding magnitudes** is used of antecedents in relation to antecedents, and of consequents in relation to consequents."*

This definition is simply giving us some terms with which to talk about the parts of proportions. Let's take a look at an example proportion:

$$A : B :: C : D$$

The terms A and C, since they are the first terms in each ratio, are called the *antecedents*. The terms B and D, since they are the second terms in each ratio, are called the *consequents*.

When antecedents or consequents are taken *together,* they are called *corresponding magnitudes*. In this example, A and C are corresponding magnitudes, and B and D are corresponding magnitudes.

12. ALTERNATE RATIO

*"**Alternate ratio** means taking the antecedent in relation to the antecedent and the consequent in relation to the consequent."*

This is the first of several definitions that describe different forms of proportions. The importance of these forms will not become clear until later, but it is important that we take the time to learn them now. The first of these is the *alternate* form.

If you have this original proportion:

$$A : B :: C : D$$

Then the alternate form would be:

$$A : C :: B : D$$

13. INVERSE RATIO

*"**Inverse ratio** means taking the consequent as antecedent in relation to the antecedent as consequent."*

Thus, if you have this original proportion:

$$A : B :: C : D$$

Then the alternate form would be:

$$B : A :: D : C$$

14. COMPOSITION OF A RATIO

*"**Composition of a ratio** means taking the antecedent together with the consequent as one in relation to the consequent by itself."*

Thus, if you have this original proportion:

$$A : B :: C : D$$

Then the composed (or "componendo") form would be:

$$(A + B) : B :: (C + D) : D$$

Book V Definitions Reading

15. SEPARATION OF A RATIO

*"**Separation of a ratio** means taking the excess by which the antecedent exceeds the consequent in relation to the consequent by itself."*

Separation is like composition, except with subtraction.

Thus, if you have this original proportion:

$$A:B :: C:D$$

Then the separated (or "separando") form would be:

$$(A-B):B :: (C-D):D$$

16. CONVERSION OF A RATIO

*"**Conversion of a ratio** means taking the antecedent in relation to the excess by which antecedent exceeds the consequent."*

The conversion of a ratio is the inverse of the separando form.
Thus, if you have this original proportion:

$$A:B :: C:D$$

Then the converted (or "convertendo") form would be:

$$A:(A-B) :: C:(C-D)$$

17. EX AEQUALI

*"A ratio **ex aequali** arises when, there being several magnitudes and another set equal to them in multitude which taken to and two are in the same proportion, as the first is to the last among the first magnitudes, so is the first to the last among the second magnitudes."*

This definition is difficult to understand, especially without a diagram to refer to. We will see example of it later on; for now, let's simply look at how to recognize when a proportion has *arisen ex aequali* from two other proportions. It looks like this:

Original two proportions

$$A:B :: D:E$$
$$B:C :: E:F$$

Proportion *arising ex aequali*

$$\therefore A:C :: D:F$$

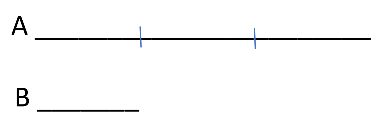
Can you follow the pattern here? Notice, in the ratios on the left-hand side of each of the original two proportions, B appears in both, but it does not appear in the left-hand ratio of the *ex aequali* proportion. Then, the antecedent of the first (A) and the consequent of the second (C) end up in a ratio in the *ex aequali* proportion.

Similarly, on the right side, E appears in both original proportions, but not in the one *ex aequali*. And the antecedent of the first (D) and the consequent of the second (F) are in a ratio in the *ex aequali* proportion.

Book V Definitions Questions

1. Definition 1: "A magnitude is ________ of a magnitude, the less of the greater, when it __________________ the greater."

2. Looking at the diagram to the right, which of the following statements is true (circle all that apply):
   a) A is part of B
   b) B is part of A
   c) A measures B
   d) B measures A
   e) A is a multiple of B
   f) B is a multiple of A



3. In particular, which multiple/part is B of A?

   ____

4. List three different kinds of magnitudes: ____________, _____________, and ______________

5. Draw diagrams to the right to match the statements in the left column. Use line segments for all of your magnitudes, and make sure you are neat enough to make your meaning clear.

AB is a part of CD	
EF is a multiple of GH	
PQ measures RS three times	
AB is the fourth part of XY	
CD is the second multiple of QR	

6. Write out the definition of *ratio* word for word:

   ____

   ____

Book V Definitions Questions

7. A ratio is a type of ______________________.

8. When two magnitudes are related in a ratio, in what respect are they being related?

   ____

9. Which of the following are said to *have a ratio* to one another, according to definition four. Put a check next to those that *can*, and an X next to those that cannot.

   _____ Two finite lines

   _____ A finite line and an infinite line

   _____ The number 5 and the number 0

   _____ Two angles in a triangle

   _____ Two finite areas

   _____ A line and a circle

10. Explain in your own words what it means to say that two ratios are the "same.

____

____

11. Looking at the magnitudes below, which of the ratios *appear* to be the same?

A _________ C _______________________ E _______
B ___ D ________________ F _____________________

   a) A:B and C:D
   b) C:D and E:F
   c) A:B and E:F
   d) B:A and E:F

12. What is meant by equimultiples?

____

____

Book V Definitions Questions

13. Which of the following proportions are true? Circle all that apply.

    a) $1:2 :: 4:8$

    b) $1:3 :: 2:6$

    c) $1:3 :: 5:7$

    d) $2:3 :: 4:5$

    e) $2:10 :: 3:15$

    f) $4:6 :: 3:2$

14. Let's test out Euclid's definition of same ratio on a true proportion: $1:2 :: 3:6$.

Original	Alike Exceed	Alike equal	Alike fall short of
1 ○	○○○	○○	○
2 ○○	○○	○○	○○
3 ○○○	○○○○○○○○○	○○○○○○	○○○
6 ○○○○○○	○○○○○○	○○○○○○	○○○○○○

In the second column, the **equimultiples** of the first and third magnitudes alike exceed the **equimultiples** of the second and fourth magnitudes.

What multiple is taken of the first and third? The __________________ multiple.

What multiple is taken of the second and fourth? The __________________ multiple.

In the third column, the **equimultiples** of the first and third magnitudes alike equal the **equimultiples** of the second and fourth magnitudes.

What multiple is taken of the first and third? The __________________ multiple.

What multiple is taken of the second and fourth? The __________________ multiple.

In the second column, the **equimultiples** of the first and third magnitudes alike fall short of the **equimultiples** of the second and fourth magnitudes.

What multiple is taken of the first and third? The __________________ multiple.

What multiple is taken of the second and fourth? The __________________ multiple.

Book V Definitions Questions

15. Now, test out Euclid’s definition of same ratio on a false proportion: $1:2 :: 3:4$.

Original	Alike Exceed	Alike equal	Alike fall short of
1 ○	x3 ○○○	x2 ○○	x1 ○
2 ○○	x1 ○○	x1 ○○	x2 ○○○○
3 ○○○	x3 ○○○○○○○○○	x2 ○○○○○○	x1 ○○○
4 ○○○○	x1 ○○○○	x1 ○○○○	x2 ○○○○○○○○

Which of the three tests does this proportion fail?

____

What does that mean about this proportion?

____

16. Write the following proportion out in words: $A:B :: C:D$

____

17. Write the following out in proportion notation: As AB is to CD, so is EF to GH.

____

18. Which of the following ratios is greater, $2:3$ or $3:5$? ________________

19. Consider the following proportion in three terms: $A:B :: B:C$. Look at A and B, given below, and draw in C to be the appropriate length to make the proportion true.

A ___

B ____________

C

Book V Definitions Questions

20. Write the following proportion in each of the following forms:

$$A : B :: C : D$$

Alternate: ______________________________________

Inverse: ______________________________________

Componendo: ______________________________________

Separando: ______________________________________

Convertendo: ______________________________________

21. Now, put into each form the proportion that immediately preceded it. That is, when you are taking the alternate, it is not the alternate of the original proportion, but the alternate of the inverse proportion that you just wrote. And so on.

Original: $A : B :: C : D$

Inverse: ______________________________________

Alternate: ______________________________________

Componendo: ______________________________________

Separando: ______________________________________

Alternate: ______________________________________

22. Given the following two proportions, what new proportion would arise *ex aequali*?

$$AB : CD :: GH : KL$$

$$\underline{CD : EF :: KL : MN}$$

$$\therefore \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

Book V Propositions Reading

V.7: *"Equal magnitudes have to the same the same ratio, as also has the same to equal magnitudes."*

The truth of this proposition is very straightforward and easy to see. If you have two equal magnitudes, they will have the same ratio to the same third magnitude.

A ___________

C ____________________________

B ___________

In if-then form, it looks like this:

IF: A=B, and there is some third magnitude C

THEN: $A : C :: B : C$

The last part—*"as also has the same to equal magnitudes"*—means that the inverse form of the proportion is also true.

The obvious truth of this proposition is clear if you think about it. If you and I are the same height, obviously we will compare to some third person's height in the same way.

V.9: *"Magnitudes which have the same ratio to the same are equal to one another; and magnitudes to which the same has the same ratio are equal."*

This proposition is the converse of V.7. It says:

IF: $A : C :: B : C$

THEN: $A = B$

Again, think about it: if you and I both compare to some third person's height *in the same way*, then of course we would have to be the same height.

V.11: *"Ratios which are the same with the same ratio are also the same with one another."*

Do you hear it? This is sort of like the common notion one of *ratios*. It looks like this:

IF: $A : B :: C : D$
and $C : D :: E : F$

THEN: $A : B :: E : F$

To make this clear and obvious, imagine a particular numerical ratio for A:B, say 1:3. If A:B is a 1:3 ratio, and it is the same with C:D, then C:D is also in a 1:3 ratio. But if $C : D :: E : F$, then E:F is also a 1:3 ratio. And that, of course, means that the ratio of A:B is the same as the ratio of E:F (both 1:3).

,

Book V Propositions Reading

V.12: *"If any number of magnitudes be proportional, as one of the antecedents is to one of the consequents, so will all the antecedents be to all the consequents."*

This truth may surprise you. It says:

IF: $AB: CD :: EF: GH :: KL: MN$

THEN: $AB: CD :: (AB + EF + KL): (CD + GH + MN)$

Do you see how this works? As one of the antecedents is to one of the consequents (e.g. AB:CD, or any of the three ratios in the proportion), so will all the antecedents *taken together* be to all the consequents *taken together* (AB+EF+KL : CD+GH+MN).

V.15: *"Parts have the same ratio as the same multiples of them taken in corresponding order."*

This means that if two magnitudes have a ratio, then equimultiples of them will have the same ratio. That is:

IF: $A: B$

Then: $A: B :: CD: EF$

A __ C _______ D
B ________ E ________________________ F

If CD and EF are equimultiples of A and B respectively (say CD is the third multiple of A, and EF is the third multiple of B), then CD and EF are in the same ratio as A and B (i.e. $A: B :: CD: EF$)

It will always be true that $A: B :: 3A: 3B$, or $A: B :: 5A: 5B$. As long as **equi**multiples are taken, they will be in the same ratio as the original magnitudes.

V.16: *"If four magnitudes be proportional, they will also be proportional alternately."*

This proposition simply states that the alternate form of a true proportion will always be true.

IF: $A: B :: C: D$

THEN: $A: C :: B: D$

V.18: *"If magnitudes be proportional* separando*, they will also be proportional* componendo.*"*

This proposition simply states that the *componendo* form of a true proportion will always be true.

IF: $A: B :: C: D$

THEN: $A + B: B :: C + D: D$

Book V Propositions Reading

V.22: *"If there be any number of magnitudes whatever, and others equal to them in multitude, which taken two and two together are in the same ratio, they will also be in the same ratio* ex aequali*."*

This proposition simply states that proportions that arise *ex aequali* from true proportions will always be true.

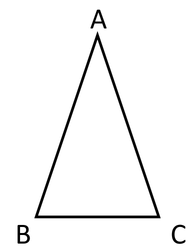
IF: $\left\{\begin{array}{l} A:B :: D:E \\ B:C :: E:F \end{array}\right.$

THEN: $\{ \therefore A:C :: D:F$

Book V Definitions Questions

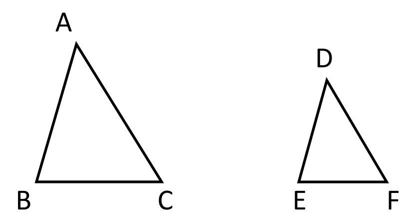
1. If AB and AC are equal,

   then (by V.7) __________________________



2. If $AB : BC :: AC : BC$

   then _________________________________



3. If in triangle ABC and DEF,
   $AB : BC :: DE : EF$,

   then alternately ______________________________

4. (Referring to the same diagram) If $AB : BC :: DE : EF$,

   then inversely _______________________________

5. (Referring to the same diagram) If $DF : AC :: EF : BC$,

   then alternately _____________________________

6. (Referring to the same diagram) If $BC : EF :: AB : DE$

   then alternately______________________________

7. If A and B have a ratio (A:B),

   then by V.15, write three true proportions:

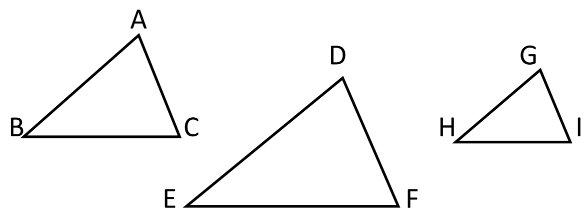
   __________________________, __________________________, and __________________________

8. Take the alternate form of one of your proportions in number 7: __________________________

Book V Definitions Questions

9. If $AB:BC :: DE:EF$
   and $GH:HI :: DE:EF$

   then ________________________



10. If $GH:HI :: DE:EF$,

    then alternately __________________________

    and inversely _________________________

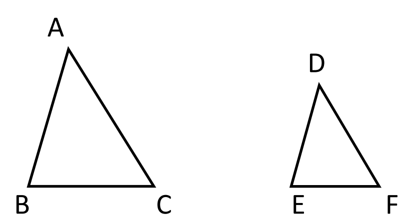
    and componendo ____________________________

11. If $A:B :: C:D :: E:F$

    then by V.12 _________________________________

12. Take the proportion in your answer to #11 above alternately: ___________________________

13. If in triangle ABC and DEF,
    $AB:BC :: DE:EF$, and
    $BC:AC :: EF:DF$
    then *ex aequali* _____________________________



14. Now take the *ex aequali* proportion alternately ____________________________

15. If $A:B :: C:D$, and $C:D :: E:F$,

    then ____________________________

Book V Definitions Questions

As you know, in Geometry, we do not deal much with number. Just to see that these truths hold true for *all magnitudes* (which includes number), let's try a few of them out with numbers. Your prior knowledge of fractions should confirm that all your conclusions are true.

16. If $2:3 :: 4:6 :: 8:12$

    then by V.12 ____

17. If $1:2 :: 3:6$

    then alternately ___________________________

18. If $1:2 :: 5:10$

    then componendo ________________________

19. Take the proportion in your answer to #18 alternately _________________________

20. If the number 2 and the number 3 have a ratio (2:3),

    then by V.15, list three true proportions:

    ____

    ____

    ____

21. If $2:3 :: 4:6$ and
    $3:12 :: 6:24$

    then *ex aequali* _________________________________

22. Take your *ex aequali* proportion (from #21 above) alternately: _________________________

____
____
____
____

______:

Given:

To Prove:

Statements	Reasons
1. ________________________	1.
2. ________________________	2.
3. ________________________	3.
4. ________________________	4.
5. ________________________	5.
6. ________________________	6.
7. ________________________	7.
8. ________________________	8.
9. ________________________	9.
10. ________________________	10.
11. ________________________	11.
12. ________________________	12.
13. ________________________	13.
14. ________________________	14.
15. ________________________	15.
16. ________________________	16.
17. ________________________	17.
18. ________________________	18.
19. ________________________	19.
20. ________________________	20.