GreatHearts
Irving

Remote Learning Packet

April 27 - May 1, 2020
Course: 9 Geometry
Teacher(s): Mr. Mooney sean.mooney@greatheartsirving.org

Weekly Plan:

Monday, April 27
☐ Review Book V Answer Keys
☐ Book V Definitions and Propositions Review
☐ Read Notation/Marking Guide

Tuesday, April 28
☐ Bell Work 1
☐ Review VI.1 Answer Key, and answer questions

Wednesday, April 29
☐ Bell Work 2
☐ VI.2 Two-Column and Questions

Thursday, April 30
☐ Bell Work 3
☐ VI.3 Two-Column and Questions

Friday, May 1
☐ Bell Work 3
☐ VI.4 in Two-Column

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____
Student Signature

____
Parent Signature

Dear Students,

Welcome to another week of Geometry! I hope you all are well, and that you are finding ways to make your time at home the very best that it can be. As a wise man once said, *"An inconvenience is only an adventure wrongly considered; an adventure is an inconvenience rightly considered."*[1] Though surely this remote learning could be called more than an inconvenience, I think we can rightly consider it an adventure.

Last week, we acquired the tools of ratio and proportion from Book V, which we will need for mastery of Book VI. These concepts are difficult, to be sure; if you struggled with them, you are in good company. I urge you to persevere nonetheless. Thinking in ratio and proportion requires a significant shift in the way you're used to thinking, so do not be surprised if it takes you time to get the hang of it.

At the end of last week, we put our Book V skills into action in the first proposition of Book VI. This proposition was very difficult, and I assume you probably will need to do a lot of correcting. For that reason, I am including an answer key to VI.1 in this packet. Learn from your mistakes!

This week, we will forge ahead to VI.2-4. While these proofs are difficult, I think you will be relieved to find that they are not as difficult as VI.1, especially since your understanding of ratio and proportion will continue to strengthen as you go along. I will also be including some partial answer keys to these so that you have some immediate feedback and confirmation that you are on the right track.

To help you solidify and strengthen your ratio and proportion skills, I am also including "Bell Work" each day as a quick review of the basics. Please note that, in the interest of saving paper, I combined the Bell Work with some other questions that I want you to answer that day. Pay attention to my daily instructions though, for the order in which you should complete them (usually it is Bell Work, then something else, then back to the other questions.)

I think that is all I need to say. I miss you all very much, but I take comfort in knowing that you are continuing steadfastly to persevere on the path to wisdom and Truth. Keep fighting the good fight.

Sincerely,

Mr. Mooney

[1] G.K. Chesterton was the wise man who said it.

Monday, April 27

Today, let's focus simply on polishing our work from last week. I would like you to:

1) Review the Answer Keys for the questions on Book V Definitions and Propositions, which I have included in this packet. Learning from your mistakes here will help you, not only on the review worksheets today, but also on everything else we do in Book VI.
2) Complete the Book V Definitions Review and the Book V Propositions Review, to be found directly after the answer keys in this packet. I've made sure to label them correctly this time :)
3) Read the two pages entitled "A Few Words about Notation and Marking." This, I hope, will be very helpful to you in all of the work you do in Book VI.

Tuesday, April 28

Today, we will revisit the glorious and difficult VI.1. I would like you to:

1) Complete Bell Work 1.
2) Review the VI.1 Two-Column Answer Key, included in this packet.
3) Answer "VI.1 Questions," which can be found on the same page as your Bell Work.

Wednesday, April 29

Today, we move forward in Book VI to Proposition 2--one of my very favorites! I would like you to:

1) Complete Bell Work 2
2) Read VI.2 and write it out in two-column notes. The answer key for the converse (part 2) is included. Please do not look at it until after you have completed it on your own. Then you may go back and make corrections as needed.
3) Answer the "VI.2 Questions," which can be found on the same page as your Bell Work.

Thursday, April 30

Today, I would like you to:

1) Complete Bell Work 3.
2) Read VI.3 and write it out in two-column notes. The answer key for the converse (part 2) is included. Please do not look at it until after you have completed it on your own. Then you may go back and make corrections as needed.
3) Answer "VI.3 Questions," which can be found on the same page as your Bell Work.

Friday, May 1

You made it! Happy Friday, and happy first of May! You also made it to VI.4 which, as you'll soon see, is a very wonderful proposition. I would like to:

1) Complete Bell Work 4.
2) Read VI.4 Note (on the same page as the Bell Work).
3) Read VI.4 and write it out in two-column notes.

ANSWER KEY: BOOK V DEFINITIONS QUESTIONS (DAY 1)

1. part; measures
2. b, d, e
3. B is the *third* part of A
4. Answers may vary: lines, figures, areas, solids, numbers, etc.
5. Diagrams may vary:

AB is a part of CD	A------B C------ǀ------D
EF is a multiple of GH	E------------------ǀ------------------ǀ------------------F G------------------H
PQ measures RS three times	P------Q R------ǀ------ǀ------S
AB is the fourth part of XY	A------B X------ǀ------ǀ------ǀ------Y
CD is the second multiple of QR	C------------------ǀ------------------D Q------------------R

6. A ratio is a sort of relation in respect of size between two magnitudes of the same kind.
7. relation
8. They are being related in respect of *size*.
9. Check: Two finite lines; two angles in a triangle; two finite areas
   X: A finite line and an infinite line; The number 5 and the number 0; A line and a circle

10. Answers may vary: It means the magnitudes involved compare to each other in the same way. It is kind of like an analogy: my height compares to a 2nd grader's height *in the same way* that the height of Mt. Everest compares to the height of Mt. Whitney (roughly double).
11. d
12. It means the same multiple of two or more different magnitudes. For example 10 and 15 are equimultiples of 2 and 3 respectively (that is, they are the *same multiple*—i.e. the fifth multiple—of each magnitude, 2 and 3).
13. a,b,e (Note: although f might appear correct, notice that the ratios are in the wrong order. *Order matters in ratios and proportions!* If we flipped one of the ratios around it would be true.)
14. third, first, second, first, first, first
15. It fails the "alike equal" test (notice that the multiples of the first and second are equal, but the multiples of the 3rd and 4th are *not*). This means that the ratios are *not* the same, so this is a *false proportion*.
16. As A is to B, so C is to D (Or, As A is to B, so is C to D)
17. $AB : CD :: EF : GH$
18. 2:3 is greater than 3:5

ANSWER KEY: BOOK V DEFINITIONS QUESTIONS (DAY 2)

19.

A ___

B___|___|___|___

C ____________|____________|____________|____________

(Since A seems to be the fourth part of B, make C such that B is the fourth part of C)

20. Alternate $A:C :: B:D$
Inverse $B:A :: D:C$
Componendo $A+B:B :: C+D:D$
Separando $(A-B):B :: (C-D):D$
Convertendo $B:(A-B) :: D:(C-D)$

21. Original $A:B :: C:D$
Inverse $B:A :: D:C$
Alternate $B:D :: A:C$
Componendo $B+D:D :: A+C:C$
Separando $B:D :: A:C$
Alternate $B:A :: D:C$

22. $AB:EF :: GH:MN$

ANSWER KEY: BOOK V PROPOSITIONS QUESTIONS

1. $AB:BC :: AC:BC$ (The equal lines AB and AC have the same ratio to the same thing, BC)
2. $AB = AC$ (This is true by V.9, which says if two magnitudes AB and AC have the same ratio to the same thing BC, then they will be equal.)
3. $AB:DE :: BC:EF$
4. $BC:AB :: EF:DE$
5. $DF:EF :: AC:BC$
6. $BC:AB :: EF:DE$
7. Answers may vary: $A:B :: 2A:2B$, $A:B :: 3A:3B$, $A:B :: 100A:100B$
8. Answers may vary: For example, $A:100A :: B:100B$
9. $AB:BC :: GH:HI$ (Notice that both of these ratios are the same with the same ratio $DE:EF$, so we can use V.11, the "common notion one of proportions")
10. alternately $GH:DE :: HI:EF$

    inversely $HI:GH :: EF:DE$

    componendo $(GH + HI) :: HI :: (DE + EF):EF$
11. $A:B :: (A + C + E) :: (B + D + F)$
    (Instead of $A:B$, you can actually use any of the three ratios in the proportion and it would still be true)
12. $A:(A + C + E) :: B:(B + D + F)$
13. $AB:AC :: DE:DF$
14. $AB:DE :: AC:DF$
15. $A:B :: E:F$ (by V.11)
16. $2:3 :: (2 + 4 + 8):(3 + 6 + 12)$
    (We see here, calculating each sum, that we get $2:3 :: 14:21$, which we know to be true $\frac{2}{3} = \frac{14}{21}$
17. $1:3 :: 2:6$
18. $(1 + 2):2 :: (3 + 6):6$
    (We see here calculating each sum, that we get $3:2 :: 9:6$, which we know to be true ($\frac{3}{2} = \frac{9}{6}$).
19. $3:9 :: 2:6$
    (This, we see is also true—both ratios are the same as $1:2$)
20. Answers may vary:

    $2:3 :: 4:6$ (taking the second multiples of each)

    $2:3 :: 10:15$ (taking the fifth multiples of each)

    $2:3 :: 24:36$ (taking the twelfth multiples of each)
21. $2:12 :: 4:24$
    (We see that the *ex aequali* proportion is true, both ratios being the same as a $1:6$ ratio.)
22. $2:4 :: 12:24$

Book V Definitions Review

1. Looking at the diagram to the right, which of the following statements is true (circle all that apply):

   a) A is part of B
   b) B is part of A
   c) A measures B
   d) B measures A
   e) A is a multiple of B
   f) B is a multiple of A

   A ___

   B ___|___|___|___|___

2. In particular, B is the ______ multiple of A.

3. Draw diagrams to the right to match the statements in the left column. Use line segments for all of your magnitudes, and make sure you are neat enough to make your meaning clear.

EF is the fourth part of KL	
AB is the second multiple of CD	
MN is the third part of PQ	

4. Write out the definition of *ratio* word for word:

   ____

   ____

5. A ratio is a type of _______________________.

6. When two magnitudes are related in a ratio, in what respect are they being related?

   ____

7. Which of the following are said to *have a ratio* to one another? Put a check next to those that *can*, and an X next to those that cannot.

   _____ Two finite lines

   _____ Three finite lines

Book V Definitions Review

_____ The number 100 and ∞ (infinity)

_____ Two triangles

_____ A triangle and a line

_____ A point and a circle

8. Looking at the magnitudes below, which of the ratios *appear* to be the same?

A __________ C _________________________ E ________

B ___ D __________________ F _______________________

a) A:B and C:D
b) C:D and E:F
c) A:B and E:F
d) B:A and E:F

9. Write the proportion that indicates that they (from #8 directly above) are the same:

____

10. Write the following proportion out in words: $AB:CD :: EF:GH$

____

11. Write the following proportion in each of the following forms:

$$PQ:RS :: CD:EF$$

Alternate: ____

Inverse: ____

Componendo: ____

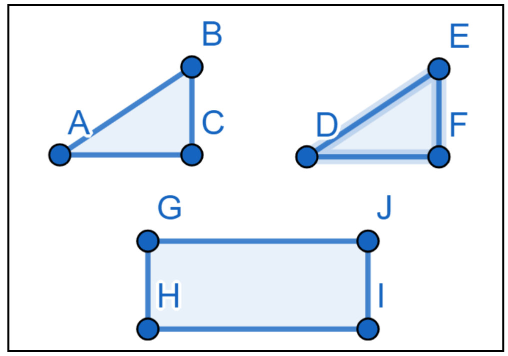
12. Given the following two proportions, what new proportion would arise *ex aequali*?

$$AB:CD :: GH:KL$$

$$\underline{CD:EF :: KL:MN}$$

∴ _______________________

Book V Propositions Review

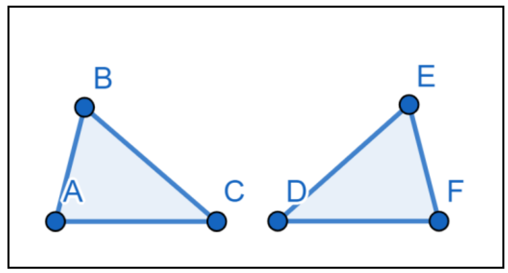


1. If ΔABC and ΔDEF are equal, and there is some third area ▭GHIJ

   then (by V.7) ________________________________

2. Referring to the same diagram,
   If $\Delta ABC : \square GHIJ :: \Delta DEF : \square GHIJ$

   then (by V.9) _________________________________

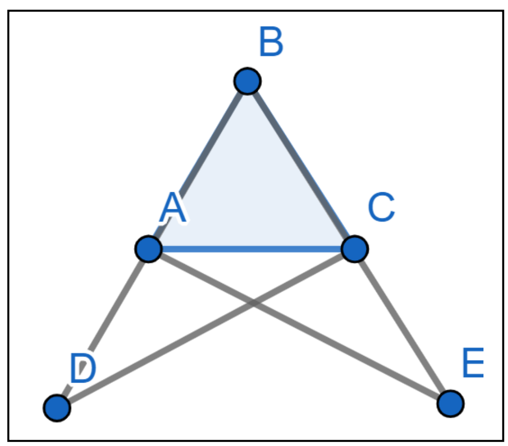


3. If in triangle ABC and DEF,
   $AB : BC :: EF : DE$,

   then alternately _______________________________

4. Referring to the same diagram
   If $AC : BC :: DF : EF$,

   then inversely ________________________________



5. If $\Delta DAC = \Delta ECA$, and ΔABC is a third area,

   then __________________________

6. If $BA : AD :: BC : CE$,

   then alternately ____________________________

   and componendo ______________________________

7. If $\Delta BDC : \Delta ABC :: \Delta ABE : \Delta ABC$,

   then (by V.9) ____________________________________

8. Take the proportion in your answer to #7 above alternately: _____________________________

A Few Words about Notation and Marking:

I want to take a moment to clarify some key notations that we will be using for the rest of the year. The sooner you get a handle on these, the easier it will be to read and understand Euclid, and to write out his proofs in two-column format.

How to Speak and Write Proportions
A proportion is simply a statement that two ratios are the same. So in the example below, the ratio $A{:}B$ is stated to be the same as the ratio $C{:}D$.

$$A{:}B :: C{:}D$$

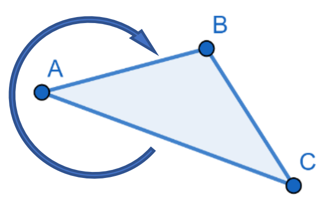
To read this proportion out loud, we say "**As** A **is to** B, **so** C **is to** D." Notice that we always start with the word "As" in front of the first ratio, and "so" in front of the second ratio. The colon between the magnitudes in a ratio is always read "is to."

Conversely, if we read a ratio in Euclid, such as "as the base BC is to the base CD, so is the triangle ABC to the triangle ACD" (taken from the prove statement of VI.1), we can simply follow the reverse process, and write

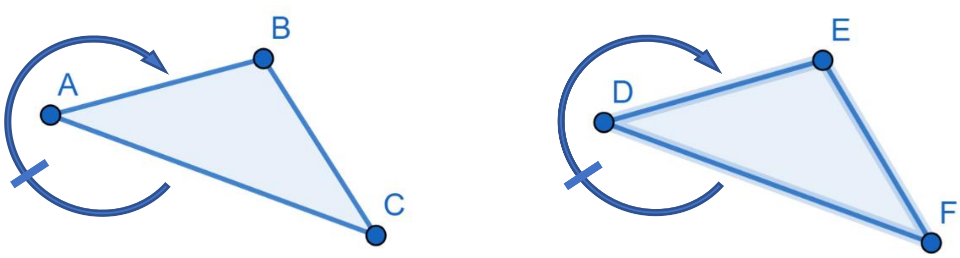
$$BC{:}CD :: \Delta ABC{:}\Delta ACD$$

How to Mark Ratios and Proportions on Diagrams
Throughout the school year, I emphasized to you the importance of marking your diagram. *We want to do our thinking on the diagram*. Nowhere is this more important than in Book VI. It is difficult enough to think in ratios and proportions; if you do not mark them, you will much more easily become lost.

We will mark ratios with arrows. The direction of the arrow will indicate the direction of the ratio. For example, the arrow below is drawn *from* AC, going *to* AB. It thus represents the ratio $AC{:}AB$.



A proportion, as you recall, is a statement that *two ratios* are the *same.* Thus, our notation for proportions will be *two* ratio arrows, marked the *same* with tick marks. For example,



By marking both of these ratio arrows with one tick mark each, I am indication that the ratio of AC to AB is the same as the ratio of DF to DE. That is, $AC{:}AB :: DF{:}DE$.

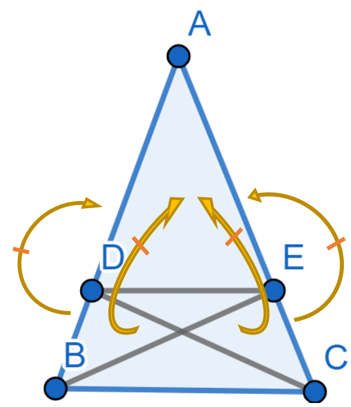
Why It Is So Important
In VI.2, you will see different areas and sides that are proportional, and you will want to mark them on the diagram. As you will see, the following claims will be made:

$BDE : ADE :: CDE : ADE$

and $BDE : ADE :: BD : DA$

and $CDE : ADE :: CE : EA$,

Let's mark them all on the diagram!



1) $BDE : ADE :: CDE : ADE$
Notice that my ratio arrows for these two ratios are pointing to and from the *insides* of the triangles. By pointing to the *inside* of the shape, I am indicating that it is a ratio of their *areas*.

2) $BDE : ADE :: BD : DA$,
You'll notice in marking this proportion that one of the ratios($BDE : ADE$) was already marked from the first proportion. You do <u>not</u> need to make a new arrow for that one (nor should you), and actually, since $BD : DA$ is stated to be the *same* as $BDE : ADE$, you should still only mark one tick mark.

3) $CDE : ADE :: CE : EA$
Similarly, with your third proportion, the ratio $CDE : ADE$ is already marked, so just write in one arrow for $CE : EA$. Again, since $CE : EA$ is the same as $CDE : ADE$, keep just one tick mark.

And behold! Just from looking at the diagram, you can see clearly see four ratios that are the same, (they all have one tick mark) making the V.11 step—ratios that are the same with the same ratio are the same with one another—very easy to see!

$$\therefore BD : DA :: CE : EA$$

So, as I said, marking ratios and proportions on the diagram makes our lives much easier. It is a much clearer way to think about things. Just imagine if I gave you the following argument and asked you if you understood:

$BDE : ADE :: CDE : ADE$ **[V.7]**

but $BDE : ADE :: BD : DA$ **[V.I.1]**

and $CDE : ADE :: CE : EA$, **[VI.1]**

$\therefore BD : DA :: CE : EA$ **[V.11]**

It's the exact same argument as above, of course, but how much more difficult (if not impossible) it is to follow without looking at the diagram and marking it up with ratio arrows.

I rest my case. *Mark your proportions on the diagram!*

Bell Work 1 and VI.1 Questions

Bell Work 1:

1. Define Ratio (V.Def.3):

   ____

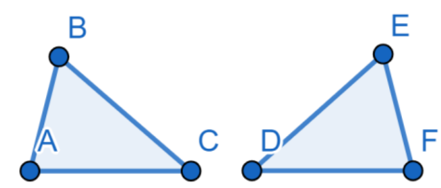
2. Mark the following proportion on the diagram:

   $AB: BC :: EF: ED$

   Take the proportion alternately:

   ____

   Now mark the alternate proportion on the diagram.



VI.1 Questions

1. Draw the diagram in the box on the right, including *only the given parts* (i.e. do not include anything that was constructed). Using ratio arrows, mark on the diagram what VI.1 sets out to prove.

2. In the second paragraph, Euclid says *"…let <u>any number</u> of straight lines BG, GH be made equal to the base BC, and any number of straight lines DK, KL equal to the base CD."* In both cases, he makes two. Would it have worked if he had, for example, made five equal lines equal to BC and eleven lines equal to CD? Explain.

3. The entire proof of VI.1 is one big set-up to use V.Def.5 (the definition of same ratio). One of the key moments is establishing "any equimultiples whatever of the first and third" and "any equimultiples whatever of the second and fourth." How does the proof accomplish this?

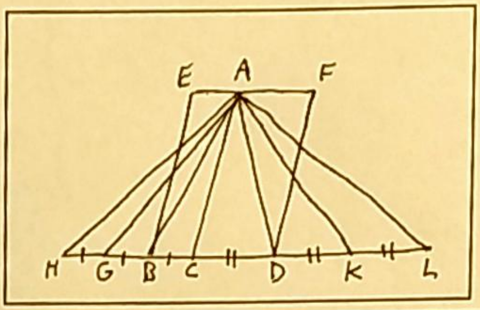
4. The second key moment is establishing that, no matter what, the "former equimultiples *alike exceed, are alike equal to, or alike fall short of* the latter equimultiples respectively." How does the proof show that this will certainly always be the case (thereby establishing same ratio)?

Answer Key

VI.1 : Triangles and parallelograms which are under the same height are to one another as their bases.

Given:

△ABC, △ACD, ▭EC, ▭CF, all under same height



To Prove:

BC : CD :: △ABC : △ACD
and BC : CD :: ▭EC : ▭CF

Statements	Reasons
1. Extend BD → H, L	1. Post. 2
2. Cut BG, GH = BC	2. I.3
3. Cut DK, KL = CD	3. I.3
4. Join AG, AH, AK, AL	4. Post. 1
5. CB = BG = GH	5. step 2
6. ∴ △ABC = △AGB = △AHG	6. I.38 (being "under same height" is logically equivalent to "same parallels")
7. ∴ HC & △AHC are equimultiples of BC & △ABC	7. V. Def. 2
8.	8.
9. ∴ LC & △ALC are equimultiples of CD & △ACD	9. Similar argument (steps 5–7)
10.	10.
11. If HC = CL, then △AHC = △ACL	11. I.38
12. If HC > CL, then △AHC > △ACL	12. I.38
13. If HC < CL, then △AHC < △ACL	13. I.38
☆ 14. ∴ BC : CD :: △ABC : △ACD	14. V. Def. 5
15. ▭EC = 2 × △ABC	15. I.41
16. ▭FC = 2 × △ACD	16. I.41
17. ∴ △ABC : △ACD :: ▭EC : ▭FC	17. V.15
☆ 18. ∴ BC : CD :: ▭EC : ▭FC	18. V.11 (steps 14, 17)
19.	19.
20.	20.

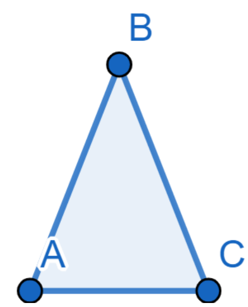
Bell Work 2 and VI.2 Questions

Bell Work 2:

1. Define Proportional (V.Def.6):

____

2. Mark the following proportion on the diagram:



$AB:BC :: AC:BC$

What, therefore, must be true by V.9?

____

3. Take $AB:BC :: AC:BC$ alternately: ____________________

4. If $EF:GH :: WX:YZ$ and $KL:MN :: WX:YZ$, then ______________________________

VI.2 Questions

1. Draw the diagram in the box on the right, including *only the given parts* (i.e. do not include anything that was constructed). Mark the given and prove of part one (which will be the prove and given of part two).

2. In the last proposition, VI.1, there are two triangles and they are under the same height. Find the first use of VI.1 in this proposition and explain: what are the two triangles, and what are the same height they are under? (Hint: You might have to tilt your head to the side, or maybe erase some extraneous lines, in order to see the VI.1 in this diagram.)

3. Explain how V.11—the so-called "common notion one of proportions"—is used in this proposition.

____

____

____

____

_______:

Given:

To Prove:

Statements	Reasons
1. ______________________	1.
2. ______________________	2.
3. ______________________	3.
4. ______________________	4.
5. ______________________	5.
6. ______________________	6.
7. ______________________	7.
8. ______________________	8.
9. ______________________	9.
10. ______________________	10.
11. ______________________	11.
12. ______________________	12.
13. ______________________	13.
14. ______________________	14.
15. ______________________	15.
16. ______________________	16.
17. ______________________	17.
18. ______________________	18.
19. ______________________	19.
20. ______________________	20.

Answer Key

Part 2

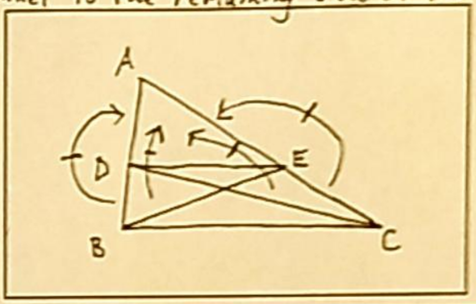
VI.2 (converse): If the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.

Given:

△ABC, BD : DA :: CE : EA
DE joined

To Prove:

DE // BC



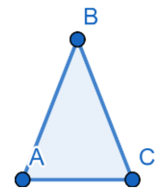
Statements	Reasons
1. Same Construction (Join BE, CD)	1. Post. 1
2. BD : DA :: CE : EA	2. Given
3. But BD : DA :: △BDE : △ADE	3. VI.1
4. And CE : EA :: △CDE : △ADE	4. VI.1
5. ∴ △BDE : △ADE :: △CDE : △ADE	5. V.11
6. ∴ △BDE = △CDE	6. V.9
7. ∴ DE // BC	7. I.39 (same base DE)
8.	8.
9.	9.
10.	10.
11.	11.
12.	12.
13.	13.
14.	14.
15.	15.
16.	16.
17.	17.
18.	18.
19.	19.
20.	20.

Bell Work 3 and VI.3 Questions

Bell Work 3:

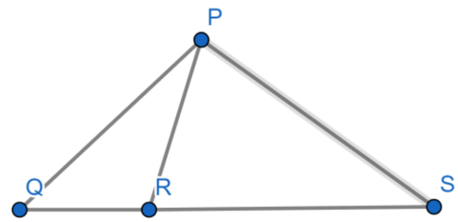
1. If AB = BC,

   then (by V.7) ________________________________



2. If ΔPQR and ΔPRS are under the same height,

   then ____________________________________

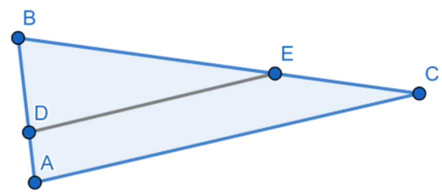


3. Can you take that proportion (from #2) alternately? Why not?

   ____

4. If in ΔABC, DE is parallel to BC,

   then ____________________________



5. Mark your answer on the diagram.

6. Take that proportion (from #4) alternately: ________________________________

VI.3 Questions

1. Draw the diagram in the box on the right, including *only the given parts* (i.e. do not include anything that was constructed). Mark the given and prove of part one (which will be the prove and given of part two).

2. Explain how VI.2 is used in the first part (i.e. not the converse) of this proposition. Which triangle is being considered, and what is the parallel line?

3. Explain how, again in part one, substitution is used to get from that VI.2 step (in question #2) to the conclusion.

____

____

____

____

_______:

Given:

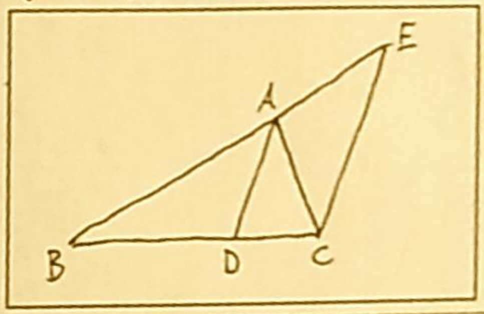
To Prove:

Statements	**Reasons**
1. ______________________________________	1.
2. ______________________________________	2.
3. ______________________________________	3.
4. ______________________________________	4.
5. ______________________________________	5.
6. ______________________________________	6.
7. ______________________________________	7.
8. ______________________________________	8.
9. ______________________________________	9.
10. ______________________________________	10.
11. ______________________________________	11.
12. ______________________________________	12.
13. ______________________________________	13.
14. ______________________________________	14.
15. ______________________________________	15.
16. ______________________________________	16.
17. ______________________________________	17.
18. ______________________________________	18.
19. ______________________________________	19.
20. ______________________________________	20.

Answer Key

Part 2

VI.3: ...and, if the segments of the base have the same ratio as the remaining sides of the triangle, the straight line joined from the vertex to the point of section will bisect the angle of the triangle.



Given:

△ABC, BA:AC :: BD:DC, AD joined

To Prove:

∡BAC is bisected by AD

Statements	Reasons
1. Same Construction (CE∥AD, BA→E)	1. (I.31, Post. 2)
2. BD:DC :: BA:AC	2. Given
3. AD∥CE	3. step 1 (construction)
4. ∴ BD:DC :: BA:AE	4. VI.2 (△BCE, AD∥CE)
5. ∴ BA:AC :: BA:AE	5. V.11
6. ∴ AC = AE	6. V.9
7. ∴ ∡AEC = ∡ACE	7. I.5
8. But ∡AEC = ∡BAD	8. I.29 (Ext. & Opp. int. ∠'s)
9. And ∡ACE = ∡CAD	9. I.29 (alternate ∠'s)
10. ∴ ∡BAD = ∡CAD	10. C.N.1 (steps 7, 8, 9)
11. ∴ ∡BAC is bisected by AD	11. definition of bisection, step 10
12.	12.
13.	13.
14.	14.
15.	15.
16.	16.
17.	17.
18.	18.
19.	19.
20.	20.

Bell Work 4 and Note on IV.4

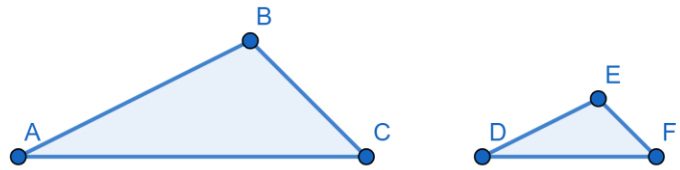
Bell Work 4:

1. Define "Similar Rectilineal Figures" (VI.Def.1): ____

   ____

2. If $AB:AC :: DE:DF$
   and $AC:BC :: DF:FE$

   then *ex aequali* ________________

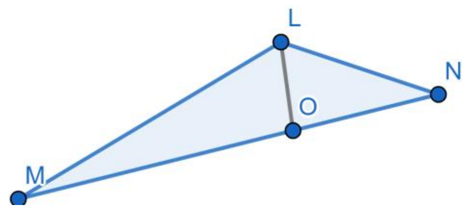


3. Mark all three proportions (the two given ones, and the conclusion) on the diagram. Be sure to use different tick marks for different proportions!

4. If in ΔLMN, LO bisects angle MLN,

   then _________________________________

5. Mark the proportion (from #4) on the diagram.



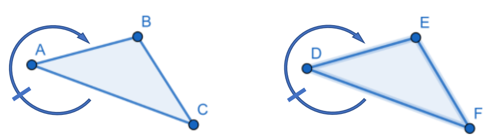
A Note on VI.4: AAA Similarity

This proposition is our very first *Similarity* proposition. It proves that two triangles, under certain conditions, are *similar*. Similar figures are figures that have the same *shape*.

As VI.Def.1 says, in order for two figures to be *similar*, they must have 1) all angles equal respectively, or "severally", and 2) all the sides about the equal angles proportional.

We will nickname VI.4 "Angle-Angle-Angle Similarity" or "AAA Similarity"[1], because it is given that all three angles of one triangle are respectively equal to all three angles in another triangle. Thus, the given itself fulfills the first condition of similarity.

The second condition of similarity is proportional sides about equal angles. Assuming BAC and EDF to be equal angles, this is what it looks like for "sides about the equal angles" to be proportional:



Furthermore, assuming angles ABC=DEF and BCA=EFD, $BA:BC :: DE:EF$ and $BC:CA :: EF:DF$.

If all three pairs of sides are thus proportional, and all three pairs of angles are equal, then the triangles are similar by VI.Def.1.

VI.4 shows that, in triangles, whenever the angles are all equal, the sides about those angles will be proportional (thus making the triangles similar). The symbol for similarity is ~

When you read VI.4, you'll notice that Euclid makes no mention of similarity. Since this is what is implied, however, let's go ahead and make $\Delta ABC \sim \Delta CDE$ our prove statement.

---

[1] It is frequently called AA similarity, since if two triangles have two angles equal to two angles respectively, the third angles will also be equal by an application of I.32.

____
____
____
____

______:

Given:

To Prove:

Statements	Reasons
1. ______________________________	1.
2. ______________________________	2.
3. ______________________________	3.
4. ______________________________	4.
5. ______________________________	5.
6. ______________________________	6.
7. ______________________________	7.
8. ______________________________	8.
9. ______________________________	9.
10. ______________________________	10.
11. ______________________________	11.
12. ______________________________	12.
13. ______________________________	13.
14. ______________________________	14.
15. ______________________________	15.
16. ______________________________	16.
17. ______________________________	17.
18. ______________________________	18.
19. ______________________________	19.
20. ______________________________	20.