GreatHearts
Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: 10 Precalculus
Teacher(s): Mr. Simmons

Weekly Plan:

Monday, May 11
☐ Story time!
☐ Check out the "Trig Cheat Sheet."
☐ Check answers from previous assignments.

Tuesday, May 12
☐ Complete problems 1-8 (A-G) from "Polar Plane."

Wednesday, May 13
☐ Check answers. (An answer key will be posted.)

Thursday, May 14
☐ Read "The Unit Circle."

Friday, May 15
☐ Attend office hours
☐ Catch up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____

Student Signature

____

Parent Signature

Monday, May 11

Happy Monday! I hope everyone's doing well.

1. Story time! If technologically feasible, please email me and let me know how you're doing. I love hearing from each and every one of you.

I heard from a few students (thanks so much for filling out the survey!), and one comment about the readings was that the information from each section wasn't all available in one place for reference while working problems. I hope the "Trig Cheat Sheet" included in this packet will help. It has all the formulas you're learning, and then some. (Don't feel like you need to go memorize the ones that the text I gave you doesn't cover.) I am also including answer keys for all previous problem sets (sorry I didn't have those earlier). From now on I'll post an answer key the day after each problem set. So:

2. At least glance at the cheat sheet. Better yet, print it out. Have it at the ready. Maybe frame it. Put it on your wall.
3. Check your answers to all previous problems using the answer keys included herein.

Tuesday, May 12

1. Complete Problems 1-8 from "Introduction to the Polar Plane" (pp. 144-46). For some of the problems, there are multiple exercises, labeled with capital letters. For each of those types of problems, please complete only A through G. (That means you're still doing every exercise labeled in small roman numerals.)

Wednesday, May 13

1. Check your answers from yesterday. (I'll post the answer key.)

Thursday, May 14

1. Read "The Unit Circle." Remember to read slowly! Try to work each of the example problems yourself before looking at the right answer.

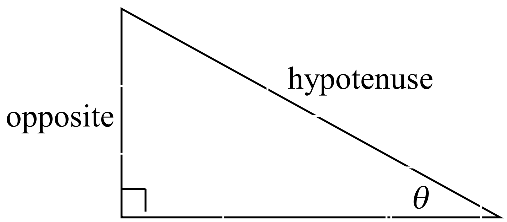
Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition

For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.



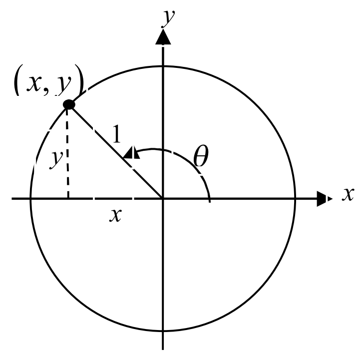
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition

For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle

$\cos\theta$, θ can be any angle

$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\csc\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

$\cot\theta$, $\theta \neq n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$

Range

The range is all possible values to get out of the function.

$-1 \leq \sin\theta \leq 1 \qquad \csc\theta \geq 1$ and $\csc\theta \leq -1$

$-1 \leq \cos\theta \leq 1 \qquad \sec\theta \geq 1$ and $\sec\theta \leq -1$

$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

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Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha + \beta) + \sin(\alpha - \beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha + \beta) - \sin(\alpha - \beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

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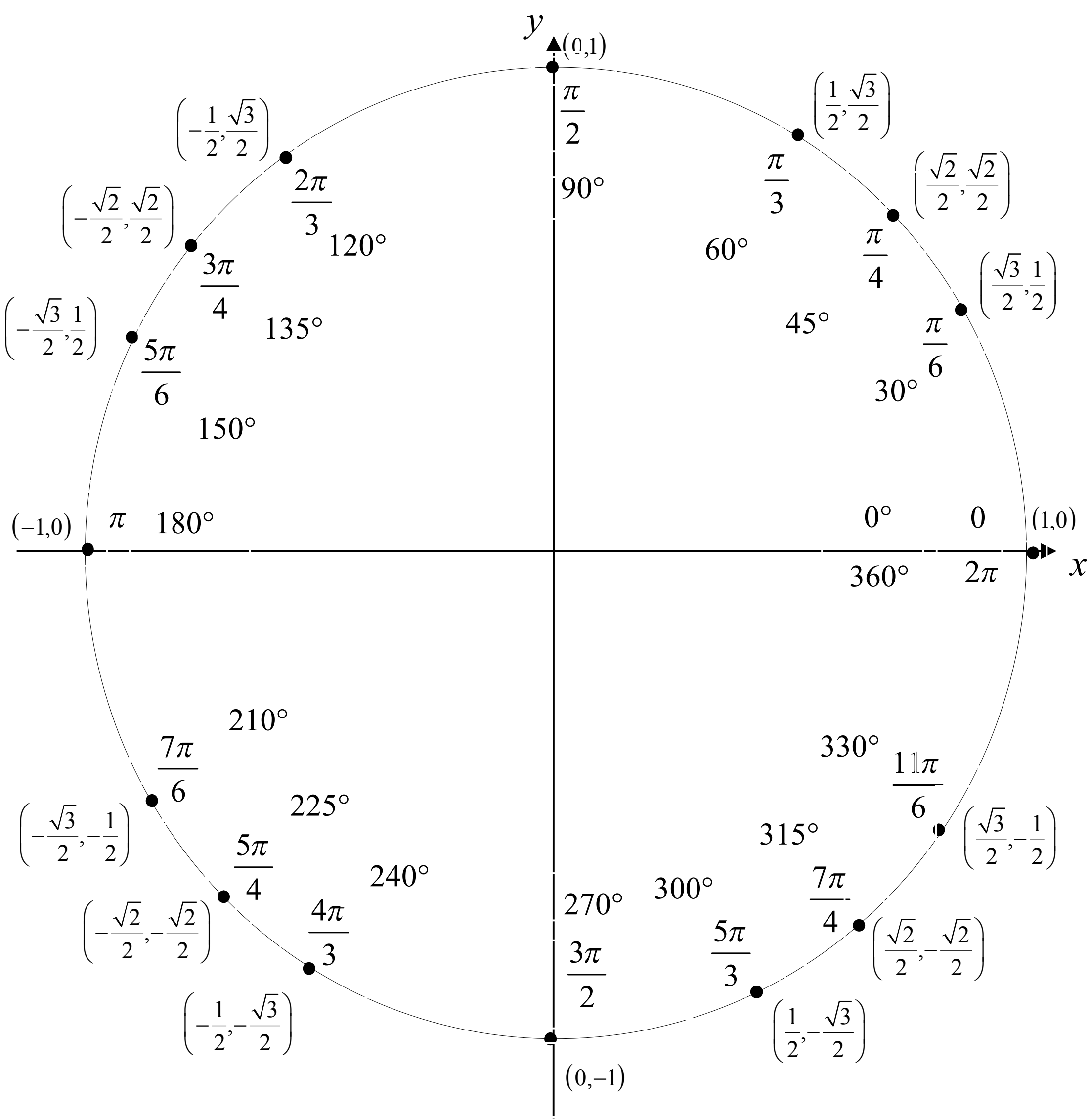
Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

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Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

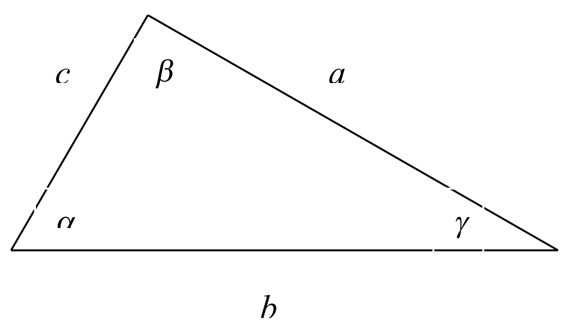
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

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4.1.1–5 odd – Answer Key

Precalculus
Mr. Simmons

1. (a)

$$\begin{aligned} 12^2 + 15^2 &= c^2 \\ 144 + 225 &= c^2 \\ 369 &= c^2 \\ c &= \sqrt{369} \\ &= 3\sqrt{41} \\ &\approx 19.21. \end{aligned}$$

   (b) $c = \sqrt{6.54} \approx 2.56$

   (c) $c = \sqrt{640} = 8\sqrt{10} \approx 25.30$

   (d) $b = \sqrt{204.75} \approx 14.31$

   (e) $a = \sqrt{27632} = 4\sqrt{1727} \approx 166.23$

   (f) $b = \sqrt{19.25} \approx 4.39$

   (g) $a = 3\sqrt{17} \approx 12.37$

   (h) $c = \sqrt{101} \approx 10.05$

3. No, not all triangles are possible. You cannot have a right triangle with legs of length 10 and 20 and a hypotenuse of 15, because $10^2 + 20^2 \neq 15^2$. Even outside of right triangles, not every triangle is possible. Try drawing a triangle with side lengths 5, 10, and 20.

5. (a) $c = 5$

   (b) $a = 5$

   (c) $a = 7, b = 24, c = 25$; $a = 8, b = 15, c = 17$; $a = 9, b = 40, c = 41$; $a = 11, b = 60, c = 61$; $a = 12, b = 35, c = 37$; $a = 13, b = 84, c = 85$ (and there are others).

4.2.1–5 – Answer Key

Precalculus
Mr. Simmons

1. (a) $\sin\alpha = 3/5$,
   $\cos\alpha = 4/5$,
   $\tan\alpha = 3/4$;
   $\sin\beta = 4/5$,
   $\cos\beta = 3/5$,
   $\tan\beta = 4/3$.

   (b) $\sin\alpha = 1/\sqrt{82} \approx 0.11$,
   $\cos\alpha = 9/\sqrt{82} \approx 0.99$,
   $\tan\alpha = 1/9$;
   $\sin\beta = 9/\sqrt{82} \approx 0.99$,
   $\cos\beta = 1/\sqrt{82} \approx 0.11$,
   $\tan\beta = 9$.

   (c) $\sin\alpha = 2/\sqrt{5} \approx 0.89$,
   $\cos\alpha = 1/\sqrt{5} \approx 0.45$,
   $\tan\alpha = 2$;
   $\sin\beta = 1/\sqrt{5} \approx 0.45$,
   $\cos\beta = 2/\sqrt{5} \approx 0.89$,
   $\tan\beta = 1/2$.

   (d) $\sin\alpha = 2/4.5 \approx 0.44$,
   $\cos\alpha = 4/4.5 \approx 0.89$,
   $\tan\alpha = 1/2$;
   $\sin\beta = 4/4.5 \approx 0.89$,
   $\cos\beta = 2/4.5 \approx 0.44$,
   $\tan\beta = 2$.

2. (a) The "really nice angles" are 30°, 45°, and 60°.

   (b) [Drawing of any two right triangle with acute angles 30° and 60° or both 45°. Such triangles could have a variety of side lengths, two examples being side lenths 1-1-$\sqrt{2}$ and 3-4-5.]

   (c)

$\boldsymbol{\alpha}$	$\sin\boldsymbol{\alpha}$	$\cos\boldsymbol{\alpha}$	$\tan\boldsymbol{\alpha}$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

3. (a) It will decrease.
   (b) Zero.
   (c) Zero.
   (d) $\cos\alpha = 7/7.04 \approx 0.99$
   (e) The length of the hypotenuse will approach the length of the adjacent side.
   (f) One.

4. (a) An example of such a triangle is one with side lengths 1-10-$\sqrt{101}$. ($\sqrt{101} \approx 10.05$.) Here, α will be 84°.
   (b) $\sin 90° = 1$; $\cos 90° = 0$.

5. Adding to the earlier table and approximating when appropriate, we have the following table.

$\boldsymbol{\alpha}$	$\sin\boldsymbol{\alpha}$	$\cos\boldsymbol{\alpha}$	$\tan\boldsymbol{\alpha}$
0°	0	1	0
5°	~ 0.09	~ 0.996	~ 0.09
10°	~ 0.17	~ 0.98	~ 0.18
15°	~ 0.25	~ 0.97	~ 0.27
20°	~ 0.34	~ 0.94	~ 0.36
25°	~ 0.42	~ 0.91	~ 0.47
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{\sqrt{3}}{3} \approx 0.58$
35°	~ 0.57	~ 0.82	~ 0.70
40°	~ 0.64	~ 0.77	~ 0.84
45°	$\frac{\sqrt{2}}{2} \approx 0.71$	$\frac{\sqrt{2}}{2} \approx 0.71$	1
50°	~ 0.77	~ 0.64	~ 1.19
55°	~ 0.82	~ 0.57	~ 1.43
60°	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{1}{2} = 0.5$	$\sqrt{3} \approx 1.73$
65°	~ 0.91	~ 0.42	~ 2.14
70°	~ 0.94	~ 0.34	~ 2.75
75°	~ 0.97	~ 0.25	~ 3.73
80°	~ 0.98	~ 0.17	~ 5.67
85°	~ 0.996	~ 0.09	~ 11.43
90°	1	0	undefined

4.3.1–14,16 – Answer Key

Precalculus
Mr. Simmons

1. (a) $\sin\alpha = 12/13$,
$\cos\alpha = 5/13$,
$\tan\alpha = 12/5$,
$\csc\alpha = 13/12$,
$\sec\alpha = 13/5$,
$\cot\alpha = 5/12$.
   (b) $\sin\alpha = 4/5$,
$\cos\alpha = 3/5$,
$\tan\alpha = 4/3$,
$\csc\alpha = 5/4$,
$\sec\alpha = 5/3$,
$\cot\alpha = 3/4$.
   (c) $\sin\alpha = 24/25$,
$\cos\alpha = 7/25$,
$\tan\alpha = 24/7$,
$\csc\alpha = 25/24$,
$\sec\alpha = 25/7$,
$\cot\alpha = 7/24$.
   (d) $\sin\alpha = 1/4$,
$\cos\alpha = \sqrt{17}/4$,
$\tan\alpha = 1/\sqrt{17}$,
$\csc\alpha = 4$,
$\sec\alpha = 4/\sqrt{17}$,
$\cot\alpha = \sqrt{17}$.
   (e) $\sin\alpha = \sqrt{69}/13$,
$\cos\alpha = 10/13$,
$\tan\alpha = \sqrt{69}/10$,
$\csc\alpha = 13/\sqrt{69}$,
$\sec\alpha = 13/10$,
$\cot\alpha = 10/\sqrt{69}$.
   (f) $\sin\alpha = 5/\sqrt{61}$,
$\cos\alpha = 6/\sqrt{61}$,
$\tan\alpha = 5/6$,
$\csc\alpha = \sqrt{61}/5$,
$\sec\alpha = \sqrt{61}/5$,
$\cot\alpha = 6/5$.
   (g) $\sin\alpha = 2/7$,
$\cos\alpha = 3\sqrt{5}/7$,
$\tan\alpha = 2/\left(3\sqrt{5}\right)$,
$\csc\alpha = 7/2$,
$\sec\alpha = 7/\left(3\sqrt{5}\right)$,
$\cot\alpha = 3\sqrt{5}/2$.
   (h) $\sin\alpha = \sqrt{51}/10$,
$\cos\alpha = 7/10$,
$\tan\alpha = \sqrt{51}/7$,
$\csc\alpha = 10/\sqrt{51}$,
$\sec\alpha = 10/7$,
$\cot\alpha = 7/\sqrt{51}$.

2. The triangle in Example 1c has opposite leg length a and hypotenuse length 1. If we call the adjacent leg length b, we have $a^2 + b^2 = 1^2$, so $b = \sqrt{1-a^2}$. So the six trig ratios are as follows:
$\sin\alpha = a$,
$\cos\alpha = \sqrt{1-a^2}$,
$\tan\alpha = a/\sqrt{1-a^2}$,
$\csc\alpha = 1/a$,
$\sec\alpha = 1/\sqrt{1-a^2}$,
$\cot\alpha = \sqrt{1-a^2}/a$.

3. (a) If $\tan\alpha = d$, then we can consider the triangle with leg lengths d and 1, with hypotenuse $\sqrt{d^2+1}$. Then have the following trig ratios:
$\sin\alpha = d/\sqrt{d^2+1}$,
$\cos\alpha = 1/\sqrt{d^2+1}$,
$\tan\alpha = d$,
$\csc\alpha = \sqrt{d^2+1}/d$,
$\sec\alpha = \sqrt{d^2+1}$,
$\cot\alpha = 1/d$.

4. (a) $\sin\alpha = a/c$,
$\cos\alpha = b/c$,

$\tan\alpha = a/b$,
$\csc\alpha = c/a$
$\sec\alpha = c/b$
$\cot\alpha = b/a$.

5. (a) $\sin 30° = \cos 60°$
   (b) $\cos 10° = \sin 80°$
   (c) $\cot 7° = \tan 83°$
   (d) $\sec 64° = \csc 26°$
   (e) $\cos 31° = \sin 59°$
   (f) $\tan 14° = \cot 76°$
   (g) $\csc 47° = \sec 43°$
   (h) $\sin 25° = \cos 65°$
   (i) $\tan(\beta+\gamma) = \cot(90° - \beta - \gamma)$
   (j) $\sin\beta = \cos(90° - \beta)$

6. $\sin(90° - \theta) = \cos\theta$,
$\cos(90° - \theta) = \sin\theta$,
$\csc(90° - \theta) = \sec\theta$,
$\sec(90° - \theta) = \csc\theta$,
$\tan(90° - \theta) = \cot\theta$,
$\cot(90° - \theta) = \tan\theta$.

7. $\csc\theta = 1/\sin\theta$,
$\sin\theta = 1/\csc\theta$,
$\sec\theta = 1/\cos\theta$,
$\cos\theta = 1/\sec\theta$,
$\cot\theta = 1/\tan\theta$,
$\tan\theta = 1/\cot\theta$.

8. (a) $\sin^2 30° = (1/2)^2 = 1/4$
   (b) $\cos^2 30° = \left(\sqrt{3}/2\right)^2 = 3/4$
   (c) $\tan^2 30° = \left(\sqrt{3}/3\right)^2 = 1/3$
   (d) $\cos^2 45° = \left(\sqrt{2}/2\right)^2 = 1/2$
   (e) $\tan^2 45° = 1^2 = 1$
   (f) $\sin^2 60° = \left(\sqrt{3}/2\right)^2 = 3/4$
   (g) $\cos^2 60° = (1/2)^2 = 1/4$
   (h) $\tan^2 60° = \sqrt{3}^2 = 3$
   (i) $\sin 30°^2 = \sin 900° =$ undefined (until we have the unit-circle definition of sine)
   (j) $\cos^2\alpha = (\cos\alpha)^2$

9.

α	$\sin^2\alpha$	$\cos^2\alpha$	$\tan^2\alpha$
$0°$	0	1	0
$5°$	~ 0.01	~ 0.99	~ 0.01
$10°$	~ 0.03	~ 0.93	~ 0.03
$15°$	~ 0.07	~ 0.88	~ 0.07
$20°$	~ 0.12	~ 0.82	~ 0.13
$25°$	~ 0.18	~ 0.67	~ 0.22
$30°$	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	$\frac{1}{3} \approx 0.33$
$35°$	~ 0.33	~ 0.59	~ 0.49
$40°$	~ 0.41	~ 0.59	~ 0.70
$45°$	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$	1
$50°$	~ 0.59	~ 0.41	~ 1.42
$55°$	~ 0.67	~ 0.33	~ 2.04
$60°$	$\frac{3}{4} = 0.75$	$\frac{1}{4} = 0.25$	3
$65°$	~ 0.82	~ 0.18	~ 4.60
$70°$	~ 0.88	~ 0.12	~ 7.55
$75°$	~ 0.93	~ 0.07	~ 13.93
$80°$	~ 0.97	~ 0.03	~ 32.16
$85°$	~ 0.99	~ 0.01	~ 130.65
$90°$	1	0	undefined

10. (a) 1; 1; no maximum
   (b) 0; 0; 0
   (c) The sin and $\sin^2$ functions have the same shape, but $\sin^2$ has only positive values and is shorter. It also has hills/valleys twice as frequently. (We say that its "period" is half the length.)

11. $\sin^2\theta + \cos^2\theta = 1$

12. These expressions are all of the form $\sin^2\theta + \cos^2\theta$, so they all equal one.

13. Since $\sin^2\theta + \cos^2\theta = 1$, we have

$$\sin^2\theta = 1 - \cos^2\theta$$

and

$$\cos^2\theta = 1 - \sin^2\theta.$$

14. (a)

$$\begin{aligned}\tan\alpha\cdot\csc\alpha &= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{1}{\sin\alpha}\\ &= \frac{1}{\cos\alpha}\\ &= \sec\alpha.\end{aligned}$$

(b)

$$\begin{aligned}(\sin\alpha+\cos\alpha)^2 &= \sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha\\ &= 2\sin\alpha\cos\alpha+1.\end{aligned}$$

16. (a) True. Although this is not the cofunction identity (that would be $\sin\alpha = \cos(90^\circ-\alpha)$), the statement here is still true because

$$\begin{aligned}\sin\alpha &= \cos(90^\circ-\alpha)\\ &= \cos(-[90^\circ-\alpha])\\ &= \cos(\alpha-90^\circ).\end{aligned}$$

This fact is, however, dependent on the unit-circle definition of cosine, so, strictly speaking, since cosine is undefined for negative angle measures in right-triangle trigonometry, "false" would also be an acceptable answer, as long as you understood the reason.

(b) False. Counterexample: $\alpha=\beta=30^\circ$.

(c) False. Counterexample: $\alpha=5^\circ$

(d) False. Squares are never negative.

5.1.1–14 – Answer Key

Precalculus
Mr. Simmons

1. (a) 2
   (b) $1/3 \approx 0.33$
   (c) $20/3 \approx 6.67$
   (d) $64/3 \approx 21.33$
2. (a) 10
   (b) 9
   (c) $5/4 = 1.25$
   (d) 5
3. (a) 5
   (b) 1
   (c) $10/3 \approx 3.33$
   (d) 12
4. See 1, 2, and 3 sketched on pp. 129–30. Come to office hours if you want to see the rest sketched.
5. (a) $10\pi/9$
   (b) $5\pi/9$
   (c) $5\pi/18$
   (d) $270/\pi°$
   (e) $15°$
   (f) 4π
   (g) $36°$
   (h) $180°$
6. (a) $\pi/6$, $\pi/4$, $\pi/3$, $\pi/4$,$\pi/2$,$3\pi/2$,2π.
   (b) $\pi/2$
   (c) $120°$, $135°$, $150°$, $210°$, $225°$, $240°$, $300°$, $315°$, $330°$.
   (d) $2\pi/3$, $3\pi/4$, $5\pi/6$, $7\pi/6$, $5\pi/4$, $4\pi/3$, $5\pi/3$, $7\pi/4$, $11\pi/6$.
7. (a) $1/2$
   (b) The more pieces you divide a whole into, by necessity the smaller the pieces will be (if they are of equal size—if they are unequally sized, still the average size will be smaller).
   (c) $\pi/3$
   (d) 2
   (e) π
   (f) $15\pi/4$
8. A half rotation would be π, which, written as $6\pi/6$, is obviously greater than $5\pi/6$.
9. Quadrant IV. One way to tell quickly is that $11\pi/6$ is very nearly 2π (which fact is made obvious by writing 2π as $12\pi/6$).
10. $5\pi/3 = 300° < 315° = 7\pi/4$
11. $$\frac{5\pi}{3} = \frac{20\pi}{12} < \frac{21\pi}{12} = \frac{7\pi}{4}$$
12. (a) 24π in
    (b) 12π in
    (c) 120π in
    (d) 3 (radians)
13. (a) 10 in
    (b) 10π (radians)
14. (a) $60/(13\pi)$ ft ≈ 1.47 ft $= 17.64$ in
    (b) $80/3$ ft ≈ 26.67 ft $= 320.04$ in

§2 **Exercises**

1.) Plot the following points. You may use the same Polar Plane if you wish.
   - (A) $A(2, 30°)$
   - (B) $B(1, 45°)$
   - (C) $C(3, 300°)$
   - (D) $D(2, 150°)$
   - (E) $E(4, 315°)$
   - (F) $F(2, 210°)$
   - (G) $G(0.5, 60°)$
   - (H) $H(3.5, 135°)$
   - (I) $I(20, 180°)$
   - (J) $J(1, 260°)$
   - (K) $K(3.5, 0°)$
   - (L) $L\left(\frac{7}{2}, 350°\right)$

2.) Before you start plotting points using radians, create a Polar Plane just like we did in this unit, except instead of using $30°, 45°, \ldots$ use the corresponding radian measures. Make sure each angle is labeled.

3.) Now that you have a Polar Plane with radians, plot the following points.
   - (A) $A\left(2, \frac{\pi}{2}\right)$
   - (B) $B\left(3, \frac{\pi}{3}\right)$
   - (C) $C\left(4, \frac{2\pi}{3}\right)$
   - (D) $D\left(1, \frac{5\pi}{4}\right)$
   - (E) $E(4,1)$
   - (F) $F\left(3, \frac{11\pi}{6}\right)$
   - (G) $G(2, \pi)$
   - (H) $H\left(2, \frac{3\pi}{4}\right)$
   - (I) $I\left(2, \frac{\pi}{6}\right)$
   - (J) $J\left(300, \frac{3\pi}{2}\right)$
   - (K) $K\left(5, \frac{5\pi}{3}\right)$
   - (L) $L\left(2, \frac{7\pi}{4}\right)$
   - (M) $M\left(3, \frac{7\pi}{6}\right)$

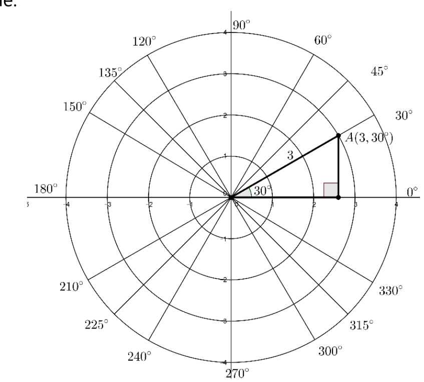
4.) Now let's work with negative angles.
   - (A) What does a negative angle represent, or tell you to do?
   - (B) Now create a Polar Plane just like you did in 2.), except this time label each angle as $-30°, -45°, \ldots$ as we started to do in the reading. Make sure each angle is labeled.
   - (C) Create another Polar Plane, except this time use negative radian measures. Again, make sure each angle is labeled.

5.) Plot the following points.
   - (A) $A(2, -45°)$
   - (B) $B\left(3, -\frac{2\pi}{3}\right)$
   - (C) $C(1, -180°)$
   - (D) $D\left(2, -\frac{7\pi}{4}\right)$
   - (E) $E(3, -300°)$
   - (F) $F\left(4, -\frac{5\pi}{6}\right)$
   - (G) $G(5, -25°)$
   - (H) $H(2, -2)$
   - (I) $I(3, -100°)$
   - (J) $J(4, -2\pi)$
   - (K) $K(3, -135°)$
   - (L) $L\left(2, -\frac{3\pi}{4}\right)$

6.) Now let's work with coterminal angles. This is incredibly important for our work in trigonometry, and was one of the main reasons we chose to work with the Polar Plane before working with the Unit Circle. Use the Polar Planes you created from previous exercises to help you.

   - (A) Give two different coterminal angles (in degrees) of the given angle measure.
     - i. $30°$.
     - ii. $45°$
     - iii. $90°$
     - iv. $135°$
     - v. $150°$
     - vi. $180°$
     - vii. $200°$
     - viii. $10°$
     - ix. $0°$
     - x. $330°$
     - xi. $240°$
     - xii. $120°$
   - (B) Give two different coterminal angles (in radians) of the given angle measure.
     - i. $\frac{\pi}{6}$
     - ii. $\frac{\pi}{4}$
     - iii. $\frac{\pi}{3}$
     - iv. $\frac{\pi}{2}$
     - v. $\frac{2\pi}{3}$
     - vi. $\frac{3\pi}{4}$
     - vii. $\frac{5\pi}{6}$
     - viii. π
     - ix. $\frac{7\pi}{6}$
     - x. $\frac{5\pi}{4}$
     - xi. $\frac{4\pi}{3}$
     - xii. $\frac{3\pi}{2}$
     - xiii. $\frac{5\pi}{3}$
     - xiv. $\frac{7\pi}{4}$
     - xv. $\frac{11\pi}{6}$
     - xvi. 2π

7.) Let us now explore how to convert a Polar point into a normal rectangular point. Consider the Figure below, of $A(3,30°)$. We will create a triangle using this point and the Pole.



Notice that if we find the lengths of the two legs of the created right triangle, then we will have found the x and y distance, and thus, the x- and y-coordinate of our point? Converting a Polar point to a rectangular one, then, amounts to finding lengths of a triangle.

But we know how to do this!

(A) Find the length of each leg of the triangle shown above.
(B) What, then, are the rectangular coordinates of the given Polar point?

8.) Using the same method above (and drawing a picture), convert the following Polar points into rectangular points.
(A) $A(1,60°)$
(B) $B(1,45°)$
(C) $C(1,90°)$
(D) $D\left(2,\frac{\pi}{6}\right)$
(E) $E\left(1,\frac{\pi}{3}\right)$
(F) Did any of your previous results seem familiar to what you've already learned? How so?
(G) Can you generalize the process of converting Polar coordinates into rectangular coordinates? It seems like there's some treasure hidden in this Exercise…

§3 The unit circle

We now embark on perhaps the most important section in Trigonometry. It is imperative that you learn and master the techniques in this section, as it will make much of Trigonometry (and, subsequently, Calculus) much easier. In this section we provide a tool to help visualize and efficiently evaluate most of the Trig functions you'll run into. Of course, as has been the case in many of these Trig sections, you must be adept at the previous lessons as well, including quickly evaluating $\sin 30°$, for example.[i]

Up to this point, we've worked with only a few Trig functions, such as $\cos 30°$. And, as we mentioned, there are many Trig functions that we simply can't evaluate with any sort of efficiency. In this section we'll learn how to evaluate a handful more – an infinite amount, actually! – quickly and efficiently.

Recall that the main reason we're able to evaluate a Trig expression like $\tan 45°$ is because we can create a special right triangle (in this case, a $45° - 45° - 90°$ triangle) which we can find the length of the sides for very quickly. Then we just have to write the

[i] Although, frankly, at this point, this warning should not be necessary.

corresponding ratio.[ii] Without these special right triangles, we would have to resort to approximation methods, which aren't very interesting to study.

But let us bring back our Polar Plane from the previous section in Figure 66. Then let us see if we can't create any other special right triangles.

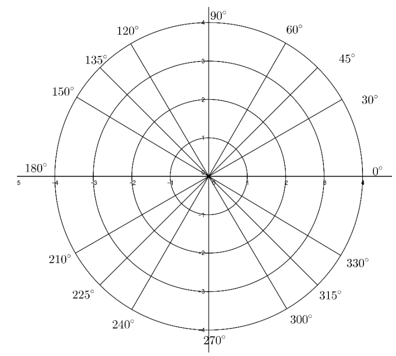


Figure 66

Can you create any special right triangles using this picture? Try making a point on one of the angles.

It turns out that not only can we make a few, but we can make many of them! We consider one example below.

Example 1

Given the point $A(2, 120°)$, create some special right triangle.

We first plot the given point. We show this in Figure 67.

[ii] We must also bring up the fact that the size of the triangle we choose does not matter.

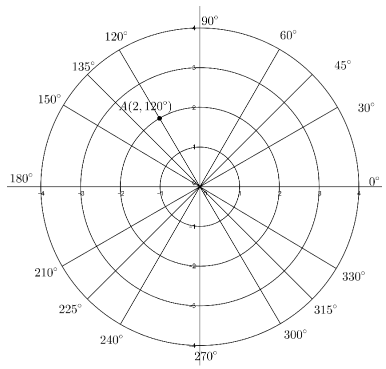


Figure 67

Two possible right triangle present themselves, and we show them both in Figure 68a and b.

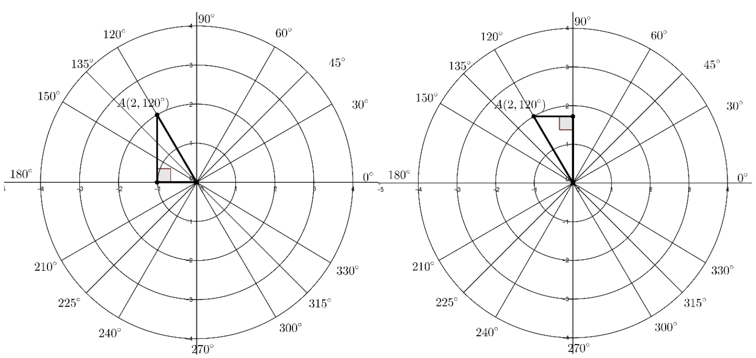


Figure 68a and b

To draw these right triangles, we just dropped a straight line down from the point the x-axis (in Figure a) and then a straight line to the right from the point to the y-axis. Right triangles are great, and this allows us to use what we've learned in previous sections. But it would be even better if they were special right triangles, right?

But that's exactly what each of them are! And it is made even more evident by drawing them on our Polar Plane like we did. In Figure 68a, for example, you can see that we have a $30° - 60° - 90°$ triangle, which we draw separately in Figure 69.

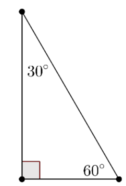


Figure 69

This is significant, and we'll demonstrate why as we proceed through this section. Hold on to this thought for a moment because we need to establish something else before making our greater point.

One of the Exercises from the previous section saw you converting Polar coordinates into rectangular coordinates. This was done by forming a right triangle (not unlike our previous Example) and then using your memorized Trig functions to find the missing lengths of the right triangle (which corresponded to the x- and y-values of the point[iii]). Let us reconsider that with some more formality and see if we can't uncover some truth.

Example 2a

Convert the Polar coordinate $(2, 45°)$ to rectangular coordinates.

Although you did this problem in the previous section, let us formalize this process. We first plot the given point, and then create a right triangle, as shown in Figure 70.

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iii Huh? Did you miss something? There's something very interesting going on here; can you feel it?

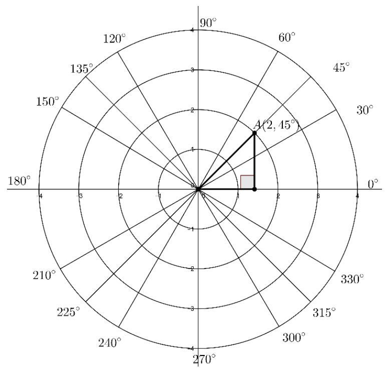


Figure 70

Now we can redraw that triangle by itself, adding in what we know, viz. the radius. We show this in Figure 71.

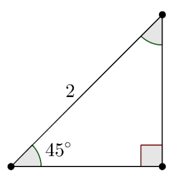


Figure 71

Recall that when we graph something on the coordinate plane, we go right some distance, and then up some distance. In the case of our picture, do you recognize that the origin is the point to the left of the right angle? Then if we travel right to the right angle, and then up, wouldn't we have just went through the process of plotting a point on the rectangular plane?[iv] Thus, if we find the length of each leg, we'll have our x- and y-coordinates, right?

So that's what we'll do. And this is quite easy, since this is an isosceles right triangle, we just divide the hypotenuse by $\sqrt{2}$, and find that each leg has a length of

[iv] That is confusing in words. Try doing what I wrote to help you see that you're just following the same procedure you've used perhaps thousands of times to plot a point. Or ask your teacher to demonstrate.

$$\frac{2}{\sqrt{2}}.$$

Let us rationalize this number so we can perhaps recognize it after we're done with it:

$$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}.$$

So each leg measures $\sqrt{2}$ in length, and this tells us that the point $A(2,45°)$ can also be written as $A(\sqrt{2},\sqrt{2})$.

This is neat, but more importantly that number $\sqrt{2}$ reminds us of a number that popped up quite a few times in the previous unit: $\frac{\sqrt{2}}{2}$. In fact, our result was twice that of $\frac{\sqrt{2}}{2}$. Why is this interesting? Because what is $\sin 45°$? And what is $\cos 45°$? And is this a coincidence??

Example 2b

Convert the Polar coordinate $B(3,30°)$ into rectangular coordinates.

Let's do one more test before trying to generalize and formalize our results. Following our previous procedure, then, we end up with Figure 72.

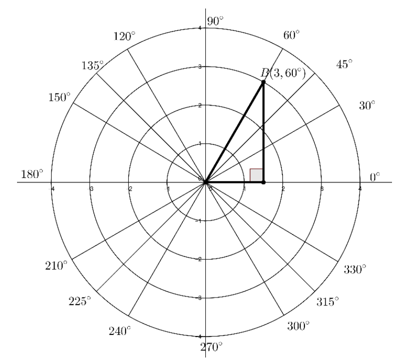


Figure 72

From here, we once again draw a right triangle, and, once again, in Figure 73 we see a special right triangle, don't we?

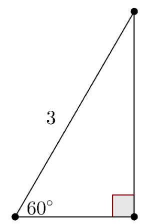


Figure 73

Again, what we're looking for are the legs of this right triangle, as that will tell us the x- and y-coordinates of the point we're looking for. Using what we learned from Unit four, we see that the short leg is 1.5 and the long leg is $1.5\sqrt{3}$. And hence our point B can be written in the rectangular plane as $B(1.5, 1.5\sqrt{3})$.

Let us again highlight the results: Remember that $\cos 60° = \frac{1}{2}$? Well, our x-coordinate is that times 3, isn't it? What about $\sin 60°$? What relationship does that have with our result of $1.5\sqrt{3}$?[v]

Isn't it interesting that the rectangular coordinates of a point from the Polar Plane keep coming up as multiples of Trig functions that we've memorized? Let us now generalize the result and make our major point of this section.

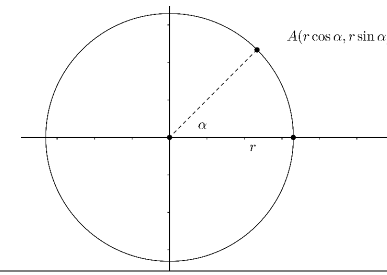
[v] You might ask how we knew about this relationship. Good question! We're not trying to teach a procedure here for you to follow, but only pointing something out, viz. that these familiar numbers keep popping up. Of course, there is a reason we chose 3 in this Example and 2 in the previous… Can you see where we get those two numbers from?

A point on a circle in rectangular coordinates

Any point on any circle is given by the rectangular coordinates

$$(r \cdot \cos\alpha\,, r \cdot \sin\alpha),$$

Where r is the radius of the circle and α is the angle of rotation from the positive x-axis to the point.



We cannot overstate how important this discovery is. So let us restate it another way. We can find the coordinates of any point on a circle using our Trig functions and the information above.

Perhaps more importantly, however, is that this establishes a relationship that we can use to find the values of other angles that we input into Trig functions. In our original definition of Trig functions, we could only use acute angles. This relationship, namely that

$$x = r \cdot \cos\alpha\,, y = r \cdot \sin\alpha,$$

Where x and y have the usual meaning as x- and y-coordinates of a point in the rectangular plane, helps us to find the output of any angle put into a Trig function. With some simple Algebra, we'll make it more clear:

$$\cos\alpha = \frac{x}{r}, \sin\alpha = \frac{y}{r}.$$

Let us now practice this concept.

Example 2a

Evaluate $\sin 120°$.

Did you memorize $\sin 120°$? Because you shouldn't have. There's a better way to deal with these angles than simply memorizing.[vi] Using our Polar Plane from before, we place some point on the $120°$ angle. We did this in Example 1, and, since the previous Example had a radius of 2, let's stick with that.[vii]

Then we can create the right triangle we saw in Figure 68a, which we redraw in Figure 74 with the radius' length added. Note that based on the way we came about our previous definition, that is, that $\cos\alpha = \frac{x}{r}$, we must use the x-axis as the base of our triangle.[viii]

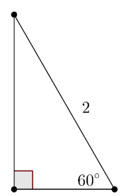


Figure 74

Then, using the relationship previously stated, viz. that

$$\sin\alpha = \frac{y}{r},$$

we can easily get our answer. Although we input $120°$, what we are working with is the $60°$ angle seen in the triangle in Figure 74.

We see that $r = 2$, because that was our chosen radius. But what is y? In this case, it's the vertical leg of the triangle drawn in Figure 74. What is the length of this leg? Hopefully you've not forgotten about $30° - 60° - 90°$ triangles, as we'll find the value of that leg using this technique. We show the lengths of the triangle in Figure 75.

[vi] Although, that being said, don't let us stop you from memorizing if that's what you're really good at.
[vii] Does the radius *need* to be 2? Excellent question! We'll cover that in Example 3.
[viii] By "base" we mean that the right angle must be located on the x-axis. This was not the case in Figure 68b. It is possible to use that Figure, but then we would need to change our definition.

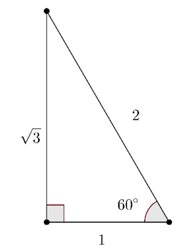


Figure 75

Now we have enough information to write our answer; we get

$$\sin 120° = \frac{\sqrt{3}}{2}.$$ [ix]

Let's do another example.

Example 2b

What is $\cos 225°$?

We follow the same procedure. This time, we need to plot a point on the Polar Plane at $225°$. What radius should we use? How about we stick with 2, for consistency's sake. We show this point, that is, $B(2, 225°)$ in Figure 76.

[ix] That seems oddly familiar… Wasn't that the same thing as $\sin 60°$…?

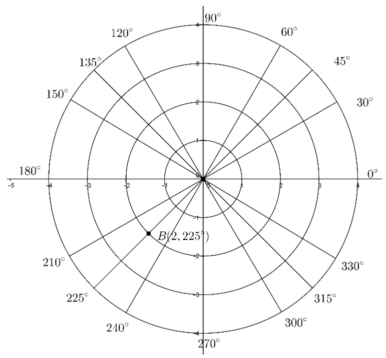


Figure 76

We now draw a right triangle using the x-axis as that base for our right angle. We show this in Figure 77.

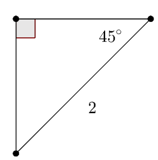


Figure 77

Since we're looking for Cosine, and, based on our findings,

$$\cos\alpha = \frac{x}{r},$$

we need to find the length of the horizontal leg.

Then we just use our knowledge of special right triangles to complete the triangle. We get Figure 78, as shown.

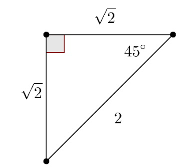


Figure 78

We can now substitute, since we know r (we chose it to be 2) and x, which is the length of the horizontal leg, which is 2. We have

$$\cos 225° = \frac{\sqrt{2}}{2}.$$

But is this correct? One way to check our result is to plot it in the coordinate plane. Our x-coordinate is given by the horizontal distance, which in this case corresponds to the horizontal leg in Figure 78.

A quick glance shows that we must be wrong. The point in Figure 76 was in Quadrant III, and this requires a negative x-value, right? We have an issue.

Or do we? There is an easy way to rectify this error: We simply make our result negative. This seems awfully artificial, and it is. But that's where our Polar Plane comes in handy. We have a picture to see that, clearly, our x-coordinate must be negative. Therefore, our result is

$$\cos 225° = -\frac{\sqrt{2}}{2}.$$

Let's do one more example before introducing you to a special circle.

Example 2c

Evaluate all six Trig functions using $\frac{5\pi}{3}$ as the input.

We are looking for $\sin\frac{5\pi}{3}, \cos\frac{5\pi}{3}$, and so on. We have radians as our input, and this is fine, since we know how to work with them. The first thing we should do is plot the point $C\left(2, \frac{5\pi}{3}\right)$ on the Polar Plane. We do this in Figure 79.

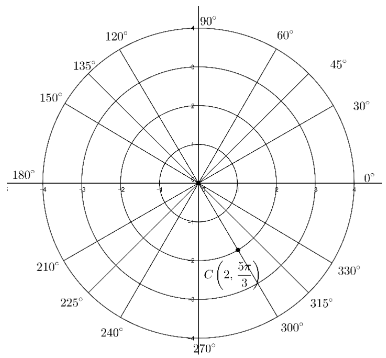


Figure 79

Do you see that $\frac{5\pi}{3}$ is equivalent to 300°?

Before proceeding, do you see how C is in Quadrant IV? Thus $x > 0$ and $y < 0$. You might want to make a note of this for each of the problems like this you do, so that you don't forget.

Now we create a right triangle with the x-axis as our base. We get Figure 80 as shown.

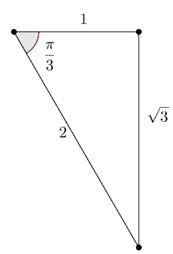


Figure 80

Verify that we've completed this special right triangle correctly.

Now all we need to do is write the appropriate ratios. We see that

$$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

because Sine is opposite to hypotenuse. It must be negative since we are in Quadrant IV. We further see that

$$\cos\frac{5\pi}{3} = \frac{1}{2},$$

and, since we are in Quadrant IV, we must be positive.

What about the other Trig functions? First, recall that $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$. And since $\sin\frac{5\pi}{3}$ is negative while $\cos\frac{5\pi}{3}$ is positive, then Tangent must be negative.[x] Then we just need to find the ratio from the previous Figure, and make it negative. We get

$$\tan\frac{5\pi}{3} = -\sqrt{3}.$$

Finding the reciprocal Trig functions, like Secant, is easy. We just need to find the reciprocal of the previous three results. Note that finding a reciprocal does *not* change its sign. Therefore,

$$\csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}}, \qquad \sec\frac{5\pi}{3} = 2\,, \qquad \cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}.$$

We leave the rationalization to the reader, if they wish.

Up to this point, we've always used a radius of 2. Just because. But certainly there has to be a better choice, right? Indeed, there is, and we call it the **unit circle**.

The unit circle
A circle centered at the origin whose radius is 1.

This simple definition has some profound implications. For example, recall that, using a circle of radius r, we have the relationship

$$\cos\alpha = \frac{x}{r}.$$

But if we have a unit circle, where $r = 1$, then we have the simpler relationship

$$\cos\alpha = x.$$

This also holds with $\sin\alpha$, of course:

$$\sin\alpha = y.$$

[x] The quotient of a negative number and a positive must be a negative number, right?

Not only is this easier to write, but it also helps us to find the Sine (or Cosine, or…) of any angle pretty easily. We show this intuition in Figure 81.

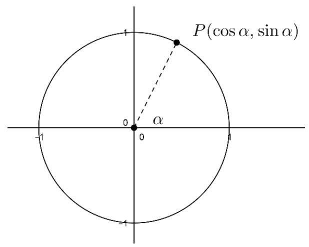


Figure 81

Any point on the circle has an x-coordinate of $\cos\alpha$ and a y-coordiante of $\sin\alpha$.

Example 3a

Evaluate $\sin 135°$.

The unit circle is a picture to place into your head so you can evaluate Trig functions like this very quickly, efficiently, and accurately. At first, you'll need to draw it out and it might take some time. But eventually it becomes second-nature, and it is indispensable in Calculus. So while someone adept at math might not need to draw it out, we will do so in each of these examples.

We draw a 135° angle on the unit circle in Figure 82.

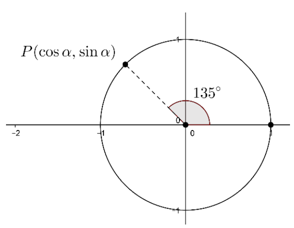


Figure 82

Since P has x- and y-coordinates of $\cos\alpha$ and $\sin\alpha$, respectively, all we need to do to evaluate $\sin 135°$ is to find the y-coordinate of P. But this amounts to the same thing we in the previous set of Examples, using the Polar Plane. The only difference is that our

radius will always be 1.[xi] Thus we need to make a right triangle with the x-axis as our base as shown in Figure 83.

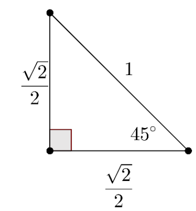


Figure 83

If you have difficulties seeing what type of special right triangle you get, use your Polar Plane. It will be more evident since each angle is marked. Also note we rationalized the denominators on each leg.

Now we just use substitute. We get that

$$\sin 135° = \frac{\sqrt{2}}{2}.$$

And yes, our result should be positive. Another way to see that our result must be positive is because, if we start at the origin, we had to travel up to get to P, correct? And isn't up a positive direction?

Example 3b

Evaluate $\cos\frac{11\pi}{6}$.

We first draw an angle of $\frac{11\pi}{6}$ on our unit circle in Figure 84.

[xi] As opposed to whatever we want. Why choose 1, again?

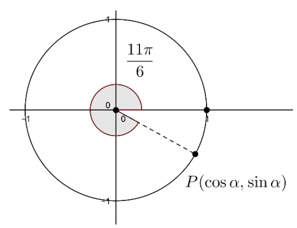


Figure 84

Again, use your Polar Plane to help you find $\frac{11\pi}{6}$. However, you'll want to have a very firm grasp of radians so don't completely rely on you Polar Plane.

Again, we now create a right triangle using the x-axis as our base. We get the special right triangle shown in Figure 85.

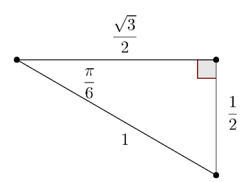


Figure 85

Then we just substitute, knowing what ratio we get with the Cosine function. Hence

$$\cos\frac{11\pi}{6} = \frac{1}{2}.$$

This, also, should be positive, since we went to the right to get to P.

Be patient and resilient as you learn the unit circle. Mastery will come, but only with practice and perseverance. Once you master the unit circle, Trigonometry becomes your plaything.

§3 **Exercises**

1.) Plot the following points on the Polar Plane, then create a right triangle where the x-axis serves as the base.
   (A) $A(2,135°)$
   (B) $B(3,300°)$
   (D) $D\left(3,\frac{\pi}{6}\right)$