10th Grade Lesson Plan Packet 5/11/2020-5/15/2020

Remote Learning Packet



Please submit scans of written work in Google Classroom at the end of the week.

May 11-15, 2020 Course: 10 Art (HS Art II) Teacher(s): Ms. Clare Frank

Weekly Plan:

Monday, May 11
Watch instructional video about a group of murals from the Works Progress Administration.
Interview an elderly family member; write some notes from the conversation.
Tuesday, May 12
Watch instructional video about narrative art by Faith Ringgold.
Interview an elderly family member; write some notes from the conversation.
Wednesday, May 13
Watch instructional video about Post Office murals.
Interview an elderly family member; write some notes from the conversation.

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 11

- 1. **Watch the instructional video,** found as a Material for Monday, May 11. This video features a selection of murals created during the New Deal Era, through the Works Progress Administration.
- 2. Call an elderly family member, and interview him or her for your "Worst Hard Time" project.*

Tuesday, May 12

- 1. **Watch the instructional video,** found as a Material for Tuesday, May 12. This video is about Faith Ringgold's Story Quilts.
- 2. Interview an elderly family member for your "Worst Hard Time" project.*

Wednesday, May 13

- 1. Watch the instructional video, found as a Material for Wednesday, May 13. This video is about American Post Office Murals.
- 2. Interview an elderly family member for your "Worst Hard Time" project.*

Thursday, May 14

- 1. Follow the link on the Materials tab for Thursday, May 14 to see images from Jacob Lawrence's Migration Series. Notice the style of his artwork. You may look for just a few minutes or explore the website a bit more, depending on your time and inclination.
- 2. Continue with your research interviews as needed.

Friday, May 15 Attend office hours or catch up on the week's work.

* These interviews should be more like conversations. Take notes on your conversation - either during or after, depending on what works best. You may choose to discover stories from more than one elderly relative, or just from one - it is up to you. Consider this daily allotment of time flexible - it is possible you would have two longer conversations rather than four short ones. Also, if you do not have an elderly family member you are able to speak to, you could instead speak with an elderly neighbor or family friend. Please revisit the general overview in the supplemental materials, next page.

"The Worst Hard Time" Storyboard Project: General Overview

Your next creative project involves you designing and drawing a storyboard about a phase of family history. For this project you will conduct interviews with elders (oldest members) of your family - people whose memories reach back - and ask them about the worst hard time they remember in their family history, and what they did to get through those times.

Research:

Whom should you interview? Grandparents or great-grandparents, great-aunts or great-uncles, or elderly friends who are as close to you family as if they are related.

How should you interview them? Call them up, chat a bit about you each are doing, and then let them know you want to know about their lives. Listen! Listen, take notes, ask follow up questions. Once you start listening you'll find people will talk, and some have stories to share that haven't been shared in a long time, if ever. I used to have an elderly neighbor who'd call me up and ask "Baby, you got a minute?" And if I did, he would talk. I got a picture of rural life century deep South I hadn't heard before.

If you need to bring up the topic of hard times, you might on the first call, or maybe on a follow-up call. You can let them know you are working on a project. Find out what were some of the hardest times they lived through and what they did to get through it. What stories do they have?

This week you'll be conducting interviews and taking notes, and then you'll start brainstorming. Following that week you will make a storyboard of at least 6 panels.

Media and Style:

Media will be fairly open on this project - so open a slot in your mind for the ideas to play! There is flexibility on the style, from more realistic to somewhat abstracted, like Jacob Lawrence's work. However, abstract doesn't mean "anything goes". If you look at Lawrence's work with the principles of design in mind you'll realize he has great compositions.

Definitely your storyboard should not look like a comic strip, which is most of what you will come up with if you google "storyboard". Even storyboards of family histories are often unimaginative in style and composition. You, on the other hand, are making art! Use the principles of design and elements of art effectively and beautifully, keeping in mind compositional principles you have worked hard to develop! Play to your skills and stretch both your skills and your imagination.

Website for information on Jacob Lawrence's Migration Series:

https://lawrencemigration.phillipscollection.org/culture/jacob-lawrences-harlem/jacob-lawrence-p anel-23-the-migration-spread-from-the-migration-series-1941

This website is posted as a link on a Materials tab for Thursday, May 14.

Remote Learning Packet



Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: 10 Chemistry **Teacher(s)**: Ms. Oostindie megan.oostindie@greatheartsirving.org

Weekly Plan:

Monday, May 11

Tuesday, May 12 Read 11.1(pp. 333-334) and take notes Balancing nuclear equations worksheet

Wednesday, May 13 Read 11.2 (pp. 334-335) and take notes Watch "History of the Discovery of Radioactivity" video

Thursday, May 14

Read 11.4 (stopping after gamma emission) and take notes

 \Box Diagram the three types of nuclear decay

Friday, May 15

 \Box Catch-up or review the week's work

Statement of Academic Honesty

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Student Signature

Parent Signature

Monday, May 11

Complete the attached worksheet titled "Isotopes." This is a review of material we have already covered in class. If you need to reference your textbook, isotopes can be found in chapter 3, section 3 (pp. 54-55). The key to this worksheet is also attached for you to self-grade your material. *No material will be turned in from this day.

Tuesday, May 12

Read 11.1 (pp. 333-334) in your textbook. Your notes should include bolded vocabulary, sample equations, and the differences between chemical and nuclear reactions.

Complete the balancing nuclear equations worksheet. You will self-grade the first two questions in a different color pen.

*The balancing nuclear equations worksheet will be turned in.

Wednesday, May 13

Read 11.2 (pp. 334-335) in your textbook. Your notes should include bolded vocabulary and a drawing of Figure 11.1. In your notes, outline the three major discoveries of the nature of radioactivity. Include the name of the scientist(s), year of discovery, and a summary of their discovery.

Watch the "History of the Discovery of Radioactivity" video found on Google Classroom to aid you in your note taking.

*No material will be turned in from this day.

Thursday, May 14

Read 11.4 through the section titled "Gamma Emission" (pp. 336-339). Your notes should include bolded vocabulary, worked examples, and a nuclear equation for each of the three types of nuclear decay: alpha, beta, and gamma.

On a separate sheet of paper, diagram the three types of nuclear decay. You may draw simplified versions of Figure 11.3. Label your diagrams.

*Diagrams will be turned in.

Friday, May 15

Use this day to attend office hours, catch up on work from this week, scan your documents, and enjoy the start of your weekend! *You do not need to include notes in your packet submission*, only the documents listed: balancing nuclear equations worksheet and nuclear decay diagrams.

Isotopes

Are all atoms of an element alike?

Why?

The following activity will help you learn the important structural characteristics of an atom. How do we classify atoms? How does the combination of subatomic particles affect the mass and charge of an atom? What are isotopes? This is just a sampling of what we will address. Throughout this activity you will want to keep both Model 1 and a periodic table handy.

Model 1

Isotopes of Hydrogen			
Isotope Symbol		2 1H	³ 1H
Atomic Diagram with Name	Electron cloud Nucleus Hydrogen-1 (protium)	Electron cloud Nucleus Hydrogen-2 (deuterium)	Electron cloud Nucleus Hydrogen-3 (tritium)
Number of Protons 🛛 🔂			
Number of Neutrons O			
	Isotopes of	f Carbon	14
Isotope Symbol	₆ C	¹³ C	¹⁴ ₆ C
Atomic Diagram with Name	Electron cloud	Electron cloud	Electron cloud Vucleus
	Carbon-12	Carbon-13	Carbon-14
Number of Protons 🛛 😝			
Number of Neutrons			
	Isotopes of N	Iagnesium	
Isotope Symbol	²⁴ 12Mg	²⁵ 12Mg	²⁶ 12Mg
Atomic Diagram with Name	Electron cloud Vucleuse Costo of Costo of Costo of Costo of Costo	Electron cloud Vucleus COP COP COP COP COP	Electron cloud
	Magnesium-24	Magnesium-25	Magnesium-26
Number of Protons			
Number of Neutrons O			

- 1. Refer to Model 1. What subatomic particles do the following symbols represent in the Atomic Diagrams?
- Complete the table in Model 1 by counting the protons and neutrons in each atomic diagram. Divide the work evenly among group members.
- 3. Find the three elements shown in Model 1 on your periodic table.

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a. What whole number shown in Model 1 for each element is also found in the periodic table for that element?

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Hydrogen — Carbon — Magnesium —
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- *b.* The whole number in each box of the periodic table is the atomic number of the element. What does the **atomic number** of an element represent?
- *c.* Refer to the isotope symbols in Model 1. Relative to the atomic symbol (H, C, or Mg), where is the atomic number located in the isotope symbol?
- 4. Refer to your periodic table.
 - a. How many protons are in all chlorine (Cl) atoms?
 - b. A student says "I think that some chlorine atoms have 16 protons." Explain why this student is not correct.
- 5. Refer again to Model 1. In the isotope symbol of each atom, there is a superscripted (raised) number. This number is also used in the name of the atom (*i.e.*, carbon-12). It is called the **mass number**.
 - a. How is the mass number determined?
 - b. Why is this number called a "mass" number?

6. Fill in the table for Atom I and Atom II shown below.

	Atom I	Atom II
Number of Protons		
Number of Neutrons		
Mass Number		



- 7. Refer to Model 1.
 - a. Which corner of the isotope symbol contains the mass number?
 - b. How is the mass number of an isotope expressed in the name of an atom?
- 8. Write an isotope symbol (similar to those in Model 1) for each of the atoms in Question 6.
- 9. Write the name of the atom (similar to those in Model 1) for each of the atoms in Question 6.
- 10. Fill in the following table.

Isotope Symbol	40 19 K	18 9 F	
Atomic Number			16
Mass Number			
Number of Protons			
Number of Neutrons			15

TOP

- 11. Consider the examples in Model 1.
 - a. Do all isotopes of an element have the same atomic number? Give at least one example or counter-example from Model 1 that supports your answer.

- *b.* Do all isotopes of an element have the same mass number? Give at least one example or counter-example from Model 1 that supports your answer.
- 12. Considering your answers to Question 11, write a definition of **isotope** using a grammatically correct sentence. Your group must come to consensus on this definition.
- 204 82 78 208 204 205 13. Consult the following list of isotope symbols: ₈₂Pb, ₃₅Br, ₃₅Br, ₈₂Pb, ₇₈Pt, ₈₂Pb.
 - a. Which of the atoms represented by these symbols are isotopes of each other?
 - *b.* Which part(s) of the isotope symbol was the most helpful in answering part *a* of this question?

STOP

Isotopes

Are all atoms of an element alike?

Why?

The following activity will help you learn the important structural characteristics of an atom. How do we classify atoms? How does the combination of subatomic particles affect the mass and charge of an atom? What are isotopes? This is just a sampling of what we will address. Throughout this activity you will want to keep both Model 1 and a periodic table handy.

Model 1

Isotopes of Hydrogen				
Isotope Symbol		2 1H	³ 1H	
Atomic Diagram with Name	Electron cloud Nucleus Hydrogen-1 (protium)	Electron cloud Nucleus Hydrogen-2 (deuterium)	Electron cloud Nucleus Hydrogen-3 (tritium)	
Number of Protons 🛛 🔂	I	l	1	
Number of Neutrons O	0	1	2	
	Isotopes of	f Carbon	14	
Isotope Symbol	₆ C	¹³ ₆ C	¹⁴ ₆ C	
Atomic Diagram with Name	Electron cloud Vicleus	Electron cloud	Electron cloud Wucleus	
	Carbon-12	Carbon-13	Carbon-14	
Number of Protons 🛛 🖨	6	6	6	
Number of Neutrons	<u>ما</u>	7	8	
	Isotopes of N	lagnesium		
Isotope Symbol	²⁴ ₁₂ Mg	²⁵ 12Mg	²⁶ 12Mg	
Atomic Diagram with Name	Electron cloud Mucleuse e constant e constan	Electron cloud	Electron cloud	
	Magnesium-24	Magnesium-25	Magnesium-26	
Number of Protons 🛛 🖨	12	12	12	
Number of Neutrons O	12	13	14	

1. Refer to Model 1. What subatomic particles do the following symbols represent in the Atomic Diagrams?







- 2. Complete the table in Model 1 by counting the protons and neutrons in each atomic diagram. Divide the work evenly among group members.
- 3. Find the three elements shown in Model 1 on your periodic table.
 - a. What whole number shown in Model 1 for each element is also found in the periodic table for that element?

```
Carbon —
                     Magnesium — 12
Hydrogen —
```

b. The whole number in each box of the periodic table is the atomic number of the element. What does the atomic number of an element represent?

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Atomic number represents the number of protons.
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c. Refer to the isotope symbols in Model 1. Relative to the atomic symbol (H, C, or Mg), where is the atomic number located in the isotope symbol?

```
The atoms c number is the subscript. (12Mg)
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- 4. Refer to your periodic table.
 - a. How many protons are in all chlorine (Cl) atoms? There are 17 protons in all chlorine atoms
 - b. A student says "I think that some chlorine atoms have 16 protons." Explain why this student is not correct.

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If an atom had 14 protons, it would not be chlorine.
trylead, that atom would be suffur.
```

- 5. Refer again to Model 1. In the isotope symbol of each atom, there is a superscripted (raised) number. This number is also used in the name of the atom (*i.e.*, carbon-12). It is called the **mass** number.
 - a. How is the mass number determined?

Mass number is the number of protons and neutrons

b. Why is this number called a "mass" number?

It is the mass of the nucleus of the isotope.

6. Fill in the table for Atom I and Atom II shown below.

	Atom I	Atom II
Number of Protons	5	9
Number of Neutrons	4	7
Mass Number	1	16



7. Refer to Model 1.

a. Which corner of the isotope symbol contains the mass number? The top upt corner of the symbol is the mass humber.

b. How is the mass number of an isotope expressed in the name of an atom?

The mass number follows free name of an atom. (Carbon-12)

- 8. Write an isotope symbol (similar to those in Model 1) for each of the atoms in Question 6. aton 2: "F
- atom 1: 5B 9. Write the name of the atom (similar to those in Model 1) for each of the atoms in Question 6.



10. Fill in the following table.

Isotope Symbol	⁴⁰ ₁₉ K	18 9 F	31 16
Atomic Number	19	9	16
Mass Number	40	18	31
Number of Protons	19	9	طا
Number of Neutrons	21	٩	15

- 11. Consider the examples in Model 1.
 - a. Do all isotopes of an element have the same atomic number? Give at least one example or counter-example from Model 1 that supports your answer.

Jes, all the isotopes of hydrogen have one proton

b. Do all isotopes of an element have the same mass number? Give at least one example or counter-example from Model 1 that supports your answer.

No, all the isotopes of hydrogen have different mass numbers.

12. Considering your answers to Question 11, write a definition of isotope using a grammatically correct sentence. Your group must come to consensus on this definition.

Isotopes are different forms of an element that have different mass numbers based on their number us neutrons.

- STOP
 - 78 204 204 82 208 205 13. Consult the following list of isotope symbols: 82Pb, 35Br, 35Br, 82Pb, 78Pt, 82Pb.
 - a. Which of the atoms represented by these symbols are isotopes of each other?

[204 Pb, 208 Pb, 205 Pb] [82 Br, 38 Br] 82 Pb, 82 Pb, 82 Pb] [35 Br, 38 Br]

b. Which part(s) of the isotope symbol was the most helpful in answering part a of this

The atomic number was most helpful in determining. if the isotopes were of the same element.



Balancing Nuclear Equations

Directions: Fill in the missing portion of the following nuclear equations. The first question has been completed for you as an example.





GreatHearts Irving

Balancing Nuclear Equations

Directions: Fill in the missing portion of the following nuclear equations. The first question has been completed for you as an example.

1.
$$212_{84}$$
 Po $\rightarrow 238_{82}$ Pb + 41_{2} He all numbers on
 $82+2=84$

2. 125_{53} I + $^{\circ}_{-1}$ e $\rightarrow 125_{52}$ He around to the same
 $3. \frac{238}{92}$ U $\rightarrow 236_{52}$ He $+ 244e$ symbol: $84Po \rightarrow g_2Pb$

4. $\frac{1}{1}$ encheric $\rightarrow 131_{54}$ Xe $+ ^{\circ}_{-1}$ e

Remote Learning Packet



Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: Economics, 10th Teacher(s): Mr. Loomis

Weekly Plan:

Monday, May 11 Read part 1 of *The Role of Economics*, by E. F. Schumacher.

Tuesday, May 12 ☐ Read part 2 of *The Role of Economics*, by E. F. Schumacher.

Wednesday, May 13

Thursday, May 14

Friday, May 15 Attend office hours Catch-up or review the week's work

Statement of Academic Honesty

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Note:

This and next week's essay are written by E.F. Schumacher, a German-British economist of the early and mid XXth century. In his book *Small is Beautiful*, from which we are reading essays, he wrestles with many of the ideas that we have been discussing this year: anthropology, value, justice, freedom, meaning, etc. Like Hayek, he is dealing with these issues in the context of the XXth century, which was a very turbulent century: totalitarian regimes, the Great Depression, increase in industrialization, globalization, etc. He offers a critical perspective on the ideas that we have been studying, especially in contrast to Marx and Hayek, and our general study of Microeconomics. He was less extreme than Marx, but less fond of the market than Hayek. His rhetoric is sometimes very strong and it may come off as undermining much of what we have been studying. This is alright. This is part of what it means to think *through* a subject (in this case Economics) and to ask, as we did on our first day: *what is Economics*? It is not a very long essay, but it is full of important distinctions. Read carefully and slowly. Pay attention to the reading questions. You do not need to answer all of them, but they are there to help guide you through his argument.

Monday, May 11

Read pp. 42 - 48, to the end of the paragraph ending in "...money is the highest of all values."

Reading questions:

- What is Economics and what does an economist do?
- What does it mean for Economics to "produce" meaning? Does meaning have a place in the study of Economics (supply and demand, opportunity cost, marginal utility, etc.)? (p.44b)
- Do you think it is a fair representation of Economics to say that:
 - the only point of view considered in it is "profit-making," (p.46t)
 - and that the market is the "institutionalization of individualism and irresponsibility?" (p.46b)
- What does the author mean by quantitative vs. qualitative? (p.47b)

Tuesday, May 12

Read pp. 48 - 55, starting with "Economics operates legitimately..."

Reading questions:

- What does it mean to say that a science is derived from something else? How does this relate to the author's definition of "meta-economics?" (p.48b)
- What is the significance of the following statement: "Every science is beneficial within its proper limits, but becomes evil and destructive as soon as it transgresses them? (p.49t)
- What is the Greek word for man and what was the name of the first section in this course? Do you think Schumacher would agree with how we started our program of study? (p.49m)
- Is it possible for Economics to be as precise a science as physics? (p.51b)

- Is the author giving a fair account of the fundamental principle of scarcity in Microeconomics? (p.51m, p.54t)

Wednesday, May 13

Today I want you to work on a two-day reflection. You have a total of 40 minutes over two days to write on the question below, labelled **Question**.

Why does the author want us to think about what he calls "meta-economics?" (p.53b, 48b) To be more specific, he claims that we might say that "Economics does not stand on its own two feet, or that it is a derived science–derived from meta-economics." In distinguishing meta-economics further, he claims that "meta-economics consists of two parts–one dealing with man, and the other dealing with the environment." He then proceeds to speak about the nature aspect of "meta-economics."

Question:

Why does the author want us to focus on a study of nature with respect to the science of Economics? And, why is this distinction essential when it comes to understanding the limitations of the science of Economics?

Your response should be about 2 long paragraphs in length, and no more than 1 page. It should be based from within the text; if you need to supplement it you can use our lessons on Microeconomics. However, you do not have very much time, so I do not expect an essay style response. Nevertheless, I do expect you to demonstrate that you have read carefully, thoughtfully, following along with the reading questions. Your response should also contain specific references to the text.

Note about handing in the reflection:

- You have two and half options:
 - You can write it in the Google Doc provide in Google Classroom, or,
 - You can write it in your own text editor and submit it with the packet, or,
 - You can write it by hand and scan it in this option is not preferable but open if you have no other option.

Thursday, May 14

Continue yesterday's reflection.

Remote Learning Packet

NB: Please keep all work produced this week. Details regarding how to turn in this work will be forthcoming.

May 11-14, 2020 Course: 10 Humane Letters Teacher(s): Mr. Garner ben.garner@greatheartsirving.org

Weekly Plan:

Monday, May 11 Read *Crime and Punishment*, Part Five, chapter 5 Answer chapter 5 reading questions

Tuesday, May 12

Read Crime and Punishment, Part Six, chapters 1-2

Read through seminar discussion questions

Wednesday, May 13

 $\hfill\square$ Review Tuesday's reading assignment and seminar questions

Participate in live seminar

Thursday, May 14 Read *Crime and Punishment*, Part Six, chapters 3-4



Monday, May 11

- Read and annotate Part Five chapter 5 carefully.
- Answer the following reading questions in 3-4 complete sentences each.

Crime and Punishment Part Five, chapter 5

1. What do Raskolnikov and Dunya talk about when she comes to see him one last time at his apartment?

2. Who pays for Katerina Ivanovna's funeral, and why?

Tuesday, May 12

- Read and annotate Part Six chapters 1-2 carefully
- Instead of answering reading questions for this reading, spend extra time annotating and thinking about the reading in preparation for tomorrow's live seminar discussion. The seminar questions for tomorrow's discussion are listed below you do not need to submit written answers to these questions, but I will expect you to come to tomorrow's seminar prepared to discuss these questions.

Seminar Questions:

- 1. What is Raskolnikov's state of mind? To what can we attribute his increased solitude, apathy, and confusion?
- 2. Why does he feel like a "new man" after his conversation with Razumikhin?
- 3. Why has Porfiry Petrovich waited so long to arrest Raskolnikov?
- 4. Porfiry Petrovich seems to have great insight into Raskolnikov's mind. Are his insights correct? How does he seem to know Raskolnikov's mind so well?
- 5. What does Porfiry Petrovich say about the "need for suffering" in some people? Is Raskolnikov one of those people who is seeking out suffering for himself?
 - a. Connected to this discussion of suffering, Porfiry, like Sonya, tells Raskolnikov that he should embrace suffering. Does Porfiry mean the same thing by this as Sonya?
- 6. Porfiry Petrovich suggests that what Raskolnikov needs is "a change of air." (460) Recall that Svidrigailov suggested the same thing to Raskolnikov in the last chapter. What do they mean by this?
- 7. Is Raskolnikov penitent yet for his crime?

Wednesday, May 13

- Review yesterday's reading and annotations, as well as the list of seminar questions.
- Participate in the live seminar! The link for the Zoom meeting can be found on the Google Classroom page.

Thursday, May 14

- Read and annotate Part Six chapters 3-4 carefully, paying special attention to the following points:
 - In these chapters the character of Svidrigailov comes to the forefront again. Continue to consider his earlier claim, that he and Raskolnikov were alike at heart; do they seem more alike now as they converse? Why is Raskolnikov so repulsed by the possibility of their alikeness?
 - Svidrigailov talks about "depravity" in his conversation with Raskolnikov. Consider whether Svidrigailov's depravity is different than Raskolnikov's, and if one form of depravity is worse than the other.
 - Svidrigailov begins to appear in this chapter as a "Napoleon" of sorts that is to say, it seems as though he, much more than Raskolnikov, has abandoned the conventional constraints of morality and is quite comfortable "stepping over" whenever he feels like it. Is this the sort of man Raskolnikov thought he might become?

Remote Learning Packet



Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020 Course: 10 Latin IV Teacher(s): Ms. Mueller mariel.mueller@greatheartsirving.org Supplemental Links: <u>Aeneid I.102-123 Online Grammar Reference</u> <u>Aeneid Online Vocabulary Reference</u>

Weekly Plan:

Monday, May 11
Check answer keys to last week's *Aeneid* I. 113-123 worksheet and translation
Prepare for Tuesday's and Wendesday's *Aeneid* I.34-123 Assessment
Log into Google Classroom and watch an instructional video on lines I.102-123
Tuesday, May 12
Log into Google Classroom and complete "*Aeneid* I. 34-123 Assessment: Part I"
Wednesday, May 13
Log into Google Classroom and complete "*Aeneid* I. 34-123 Assessment: Part I"
Thursday, May 14
Read the attached translation of *Aeneid* I. 124-156
Read *Aeneid* lines I.57-58, and 170-179 in Latin (pp. 26-28) and the translation of lines 159-169
Complete "*Aeneid* I. 157-158 and 170-179 Questions" worksheet

No new assignments, attend office hours and/or get caught up on previous work

Statement of Academic Honesty

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Parent Signature

Monday, May 11

1. Review the answer keys for last week's "*Aeneid* I. 113-123 Grammar and Reading Questions" and the *Aeneid* I. 113-123 translation for the *Aeneid* I.34-123 Assessment tomorrow and Wednesday. If you still have access to those assignments from last week, I encourage you to compare your answers to those on the answer keys.

The assessment tomorrow and Wednesday will be open book and open note, but as you review today, you want to make sure you are familiar enough with lines I.34-123 to:

- a. answer reading comprehension and grammar questions in a multiple choice or true/ false format
- b. identify the use of literary and rhetorical devices
- c. scan lines of dactylic hexameter
- d. write a thoughtful essay supported by the Latin text in response to a specific prompt
- 2. Log into Google Classroom and watch an instructional video guiding you through lines I.102-123 of the *Aeneid*.

Tuesday, May 12

- 1. Take some time to get out the materials you will want to reference for the open book/open note *"Aeneid* I.34-123 Assessment: Part I." This part of the assessment will test reading comprehension, grammar analysis, recognition of literary devices, and scansion. Please note that while you can reference any of your notes or materials, you may not ask for or receive help from anyone during the assessment.
- 2. Log into Google Classroom and complete the "Aeneid I. 34-123 Assessment: Part I."

Wednesday, May 13

- 1. Take some time to get out the materials you will want to reference for the open book/open note "Aeneid I.34-123 Assessment: Part II." For this part of the assessment, you will write a short essay responding to a prompt over a specific passage from lines 34-123. Your essay should be well developed and make frequent references to the Latin passage demonstrating your ability to understand the Latin as well as offer thoughtful analysis on the passage as a piece of literature.
- 2. Log into Google Classroom and complete the "Aeneid I. 34-123 Assessment: Part II."

Thursday, May 14

- 1. Read the attached translation of Book I. 124-156
- 2. Read lines I. 157-158 and 170-179 in Latin and a translation of lines I. 159-169. Complete the *"Aeneid* I. 157-158 and 170-179 Questions" worksheet.

Friday, May 15

No new assignments! Use this day to attend office hours and/or get caught up on previous work from the week!

KEY	
	Aeneid I.113-123 Reading and Grammar Questions

I. Comprehension Questions: Answer the following questions about lines 113-123.

- What happens to the ship that is carrying Orontes (lines 113-115)?
 A huge wave strikes against the ship.
- 2. What happens to the helmsman of Orontes' ship (lines 115-117)?

The helmsman is cast out of the ship and is turned onto his head.

- 3. Name three things that appear among the waves in lines 118-119.
 - 1. Scattered men (swimming) 2. the men's weapons 3. Trojan wealth
- 4. Vergil mentions four more ships that were damaged in the storm. With what comrade was each associated (lines 120-123)?

Ilioneus, brave Achates, Abas, and old Aletes.

II. Grammar Questions: Indicate True or False by marking a "T" or an "F" in the space provided.

- 1. F ipsius (line 114) refers to Oronten (line 113). ipsius refers to Aeneas
- 2. T Line 118 can be translated "There appear scattered men floating in the huge whirlpool."
- 3. F The object of vicit in line 122 is hiems (line 122). hiems is the subject of vicit
- 4. F In line 122, *laterum* is accusative. laterum *is genitive plural*

III. Scansion: After watching the instructional video on Google Classroom, scan the following lines of dactylic hexameter.

Iam vali | d(am) Īlio | neī nā | vem, iam | fortis A | chātae, et quā | vectus A | bās, et | quā gran | daevus A | lētēs, vīcit hi | ems; la | xīs late | rum com | pāgibus | omnēs accipi | unt ini | mīc(um) imb | rem rī | mīsque fa | tīscunt.

Aeneid Book I.113-123 Translation

(Lines 113-117) One (ship), which was carrying the Lycians and faithful Orontes, before the eyes of (Aeneas) himself, a huge wave from its summit strikes against the stern: the helmsman is cast out and leaning forward is turned onto his head, but a wave twists that (ship) three times in the same place driving (it) around and a swift whirlpool swallows (it) in the sea. (Lines 118-119) There appear through the waves scattered men floating in the vast whirlpool, the weapons of men and planks and Trojan wealth. (Lines 120-123) Now the mighty ship of Ilioneus, now (the ship) of brave Achates, and (the ship) in which Abas is carried, and (the ship) in which old Aletes (is carried) the storm conquered; all (the ships) receive the hostile flood water with the loose seams of their sides and they gape with fissures.

Translation of Aeneid Book I.124-156 (by A. S. Kline)

Bkl:124-156 Neptune Intervenes

Neptune, meanwhile, greatly troubled, saw that the sea was churned with vast murmur, and the storm was loose and the still waters welled from their deepest levels: he raised his calm face from the waves, gazing over the deep. He sees Aeneas's fleet scattered all over the ocean, the Trojans crushed by the breakers, and the plummeting sky. And Juno's anger, and her stratagems, do not escape her brother. He calls the East and West winds to him, and then says: 'Does confidence in your birth fill you so? Winds, do you dare, without my intent, to mix earth with sky, and cause such trouble, now? You whom I – ! But it's better to calm the running waves: you'll answer to me later for this misfortune, with a different punishment. Hurry, fly now, and say this to your king: control of the ocean, and the fierce trident, were given to me, by lot, and not to him. He owns the wild rocks, home to you, and yours, East Wind: let Aeolus officiate in his palace, and be king in the closed prison of the winds.' So he speaks, and swifter than his speech, he calms the swollen sea, scatters the gathered cloud, and brings back the sun. Cymothoë and Triton, working together, thrust the ships from the sharp reef: Neptune himself raises them with his trident, parts the vast quicksand, tempers the flood, and glides on weightless wheels, over the tops of the waves. As often, when rebellion breaks out in a great nation, and the common rabble rage with passion, and soon stones

and fiery torches fly (frenzy supplying weapons), if they then see a man of great virtue, and weighty service, they are silent, and stand there listening attentively: he sways their passions with his words and soothes their hearts: so all the uproar of the ocean died, as soon as their father, gazing over the water, carried through the clear sky, wheeled his horses, and gave them their head, flying behind in his chariot.

Translation of Aeneid Book I.159-169 (by A. S. Kline)

Bkl:159-169 Shelter on the Libyan Coast

There is a place there in a deep inlet: an island forms a harbour with the barrier of its bulk, on which every wave from the deep breaks, and divides into diminishing ripples. On this side and that, vast cliffs and twin crags loom in the sky, under whose summits the whole sea is calm, far and wide: then, above that, is a scene of glittering woods, and a dark grove overhangs the water, with leafy shade: under the headland opposite is a cave, curtained with rock, inside it, fresh water, and seats of natural stone, the home of Nymphs. No hawsers moor the weary ships here, no anchor, with its hooked flukes, fastens them.

Aeneid I.157-158 and 170-179 Questions

I. Comprehension Questions: Answer the following questions about lines 157-158 and 170-179.

- 1. Why do Aeneas and his followers end up on the shores of Libya (lines 157-158)?
- 2. What does the phrase magno telluris amore (line 171) tell us about the shipwrecked Trojans?
- 3. What is Achates doing in lines 174-176?
- 4. Why do you think Vergil goes into such detail describing the preparation of the food in lines 174-179?

II. Answer the following multiple choice questions on lines 157-158 and 170-179.

- 1. The best translation of lines 157-158 (Defessi . . . oras) is
 - a. Aeneas's tired followers strive toward the shores in their course, which is very near, and they are turned toward Libya's coast
 - b. Aeneas's tired followers, who are nearest to the shore in their course, aim toward it, and they are turned toward Libya's coast.
 - c. The weary followers of Aeneas strive to seek with their course the shores which are nearest, and they are turned toward the coast of Libya.
 - d. The weary followers of Aeneas seek in their haste the nearest shores, which they strive toward, and they are turned toward the coast of Libya.

- 2. In line 70, *omni* modifies
 - *a. huc* (line 70)
 - *b. septem* (line 170)
 - *c. numero* (line 171)
 - d. amore (line 171)
- 3. A figure of speech that occurs in line 177 is
 - a. personification
 - b. anaphora
 - c. litotes
 - d. metonymy
- 4. The metrical pattern of the first four feet of line 179 is
 - a. dactyl-spondee-dactyl-spondee
 - b. spondee-dactyl-spondee-dactyl
 - c. spondee-dactyl-spondee-spondee
 - d. dactyl-dactyl-spondee-dactyl

Remote Learning Packet



Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: 10 Precalculus **Teacher(s)**: Mr. Simmons

Weekly Plan:

Monday, May 11 Story time! Check out the "Trig Cheat Sheet." Check answers from previous assignments.

Tuesday, May 12 Complete problems 1-8 (A-G) from "Polar Plane."

Wednesday, May 13

Thursday, May 14

Friday, May 15
Attend office hours
Catch up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 11

Happy Monday! I hope everyone's doing well.

1. Story time! If technologically feasible, please email me and let me know how you're doing. I love hearing from each and every one of you.

I heard from a few students (thanks so much for filling out the survey!), and one comment about the readings was that the information from each section wasn't all available in one place for reference while working problems. I hope the "Trig Cheat Sheet" included in this packet will help. It has all the formulas you're learning, and then some. (Don't feel like you need to go memorize the ones that the text I gave you doesn't cover.) I am also including answer keys for all previous problem sets (sorry I didn't have those earlier). From now on I'll post an answer key the day after each problem set. So:

- 2. At least glance at the cheat sheet. Better yet, print it out. Have it at the ready. Maybe frame it. Put it on your wall.
- 3. Check your answers to all previous problems using the answer keys included herein.

Tuesday, May 12

1. Complete Problems 1-8 from "Introduction to the Polar Plane" (pp. 144-46). For some of the problems, there are multiple exercises, labeled with capital letters. For each of those types of problems, please complete only A through G. (That means you're still doing every exercise labeled in small roman numerals.)

Wednesday, May 13

1. Check your answers from yesterday. (I'll post the answer key.)

Thursday, May 14

1. Read "The Unit Circle." Remember to read slowly! Try to work each of the example problems yourself before looking at the right answer.

Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition



For this definition θ is any angle. (x, y)х x $\sin\theta = \frac{y}{1} = y$ $\csc\theta = \frac{1}{y}$

Unit circle definition

$\cos\theta = \frac{x}{1} = x$ $\sec\theta = \frac{1}{x}$ $\tan \theta = \frac{y}{x} \qquad \cot \theta = \frac{x}{y}$

Domain

adjacent

The domain is all the values of θ that can be plugged into the function.

$\sin heta$,	θ can be any angle
$\cos\theta$,	θ can be any angle
$\tan \theta$,	$\theta \neq \left(n + \frac{1}{2}\right)\pi, n = 0, \pm 1, \pm 2, \dots$
$\csc \theta$,	$\theta \neq n \pi$, $n = 0, \pm 1, \pm 2, \dots$
$\sec\theta$,	$\theta \neq \left(n + \frac{1}{2}\right)\pi, n = 0, \pm 1, \pm 2, \dots$
$\cot \theta$,	$\theta \neq n \pi$, $n = 0, \pm 1, \pm 2, \dots$

Range

The range is all possible values to get out of the function.

 $\csc \theta \ge 1$ and $\csc \theta \le -1$ $-1 \le \sin \theta \le 1$ $\sec\theta \ge 1$ and $\sec\theta \le -1$ $-1 \le \cos \theta \le 1$ $-\infty < \tan\theta < \infty$ $-\infty < \cot \theta < \infty$

Period

Facts and Properties

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$
$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$
$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Fangent and Cotangent Identities		
$\tan \theta = \frac{\sin \theta}{2}$	$\cot \theta = \frac{\cos \theta}{\cos \theta}$	
$\cos\theta$	$\sin\theta$	
Reciprocal Ident	ities	
$\csc\theta = \frac{1}{\sin\theta}$	$\sin\theta = \frac{1}{\csc\theta}$	
$\sec\theta = \frac{1}{\cos\theta}$	$\cos\theta = \frac{1}{\sec\theta}$	
$\cot\theta = \frac{1}{\tan\theta}$	$\tan\theta = \frac{1}{\cot\theta}$	
Pythagorean Identities		

sin² θ + cos² θ = 1 tan² θ + 1 = sec² θ 1 + cot² θ = csc² θ

Even/Odd Formulas

$\sin\left(-\theta\right) = -\sin\theta$	$\csc(-\theta) = -\csc\theta$
$\cos(-\theta) = \cos\theta$	$\sec(-\theta) = \sec\theta$
$\tan\left(-\theta\right) = -\tan\theta$	$\cot(-\theta) = -\cot\theta$

Periodic Formulas

If <i>n</i> is an integer.	
$\sin\left(\theta+2\pi n\right)=\sin\theta$	$\csc(\theta+2\pi n)=\csc\theta$
$\cos(\theta + 2\pi n) = \cos\theta$	$\sec(\theta + 2\pi n) = \sec\theta$
$\tan\left(\theta+\pi n\right)=\tan\theta$	$\cot(\theta + \pi n) = \cot\theta$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= 2\cos^2\theta - 1$$
$$= 1 - 2\sin^2\theta$$
$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

 $\frac{\pi}{180} = \frac{t}{x} \implies t = \frac{\pi x}{180} \text{ and } x = \frac{180t}{\pi}$

 $\sin\frac{\theta}{2} = \pm \sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$ $\cos\frac{\theta}{2} = \pm \sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$ $\tan\frac{\theta}{2} = \pm \sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$ Sum and Difference Formulas $\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$ $\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1\mp \tan\alpha \tan\beta}$

Half Angle Formulas (alternate form)

Product to Sum Formulas 1 = 1

$$\sin \alpha \sin \beta = \frac{1}{2} \left[\cos(\alpha - \beta) - \cos(\alpha + \beta) \right]$$
$$\cos \alpha \cos \beta = \frac{1}{2} \left[\cos(\alpha - \beta) + \cos(\alpha + \beta) \right]$$
$$\sin \alpha \cos \beta = \frac{1}{2} \left[\sin(\alpha + \beta) + \sin(\alpha - \beta) \right]$$
$$\cos \alpha \sin \beta = \frac{1}{2} \left[\sin(\alpha + \beta) - \sin(\alpha - \beta) \right]$$
Sum to Product Formulas
$$\sin \alpha + \sin \beta = 2 \sin\left(\frac{\alpha + \beta}{2}\right) \cos\left(\frac{\alpha - \beta}{2}\right)$$
$$\sin \alpha - \sin \beta = 2 \cos\left(\frac{\alpha + \beta}{2}\right) \sin\left(\frac{\alpha - \beta}{2}\right)$$

$$\cos \alpha + \cos \beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$
$$\cos \alpha - \cos \beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$





For any ordered pair on the unit circle (x, y): $\cos \theta = x$ and $\sin \theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

115	I uncuo	unctions	
	Inverse	Prop	

$y = \sin^{-1} x$	is equivalent	to $x = \sin y$
$y = \cos^{-1} x$	is equivalent	to $x = \cos y$
$y = \tan^{-1} x$	is equivalent	to $x = \tan y$

Domain

 $y = \sin^{-1} x$ $-1 \le x \le 1$ $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

 $y = \cos^{-1} x \qquad -1 \le x \le 1 \qquad 0 \le y \le \pi$

 $y = \tan^{-1} x$ $-\infty < x < \infty$ $-\frac{\pi}{2} < y < \frac{\pi}{2}$

Range

Inverse Properties $\cos(\cos^{-1}(x)) = x$ $\cos^{-1}(\cos(\theta)) = \theta$ $\sin(\sin^{-1}(x)) = x$ $\sin^{-1}(\sin(\theta)) = \theta$ $\tan(\tan^{-1}(x)) = x$ $\tan^{-1}(\tan(\theta)) = \theta$

Domain and Range

Function

Alternate Notation $\sin^{-1} x = \arcsin x$ $\cos^{-1} x = \arccos x$ $\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



b

Law of Sines $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c}$ Law of Cosines $a^{2} = b^{2} + c^{2} - 2bc \cos c$

$$a^{2} = b^{2} + c^{2} - 2bc \cos \alpha$$
$$b^{2} = a^{2} + c^{2} - 2ac \cos \beta$$
$$c^{2} = a^{2} + b^{2} - 2ab \cos \gamma$$

Mollweide's Formula $\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha - \beta)}{\sin\frac{1}{2}\gamma}$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$
$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$
$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

4.1.1–5 odd – Answer Key

Precalculus Mr. Simmons

1. (a)

$$12^{2} + 15^{2} = c^{2}$$

$$144 + 225 = c^{2}$$

$$369 = c^{2}$$

$$c = \sqrt{369}$$

$$= 3\sqrt{41}$$

$$\approx 19.21$$

(b)
$$c = \sqrt{6.54} \approx 2.56$$

(c) $c = \sqrt{640} = 8\sqrt{10} \approx 25.30$
(d) $b = \sqrt{204.75} \approx 14.31$
(e) $a = \sqrt{27632} = 4\sqrt{1727} \approx 166.23$
(f) $b = \sqrt{19.25} \approx 4.39$
(g) $a = 3\sqrt{17} \approx 12.37$

- (h) $c = \sqrt{101} \approx 10.05$
- 3. No, not all triangles are possible. You cannot have a right triangle with legs of length 10 and 20 and a hypotenuse of 15, because $10^2 + 20^2 \neq 15^2$. Even outside of right triangles, not every triangle is possible. Try drawing a triangle with side lengths 5, 10, and 20.
- 5. (a) c = 5
 - (b) a = 5
 - (c) a = 7, b = 24, c = 25; a = 8, b = 15, c = 17; a = 9, b = 40, c = 41; a = 11, b = 60, c = 61; a = 12, b = 35, c = 37; a = 13, b = 84, c = 85 (and there are others).

4.2.1-5 – Answer Key

Precalculus

Mr. Simmons

- 1. (a) $\sin \alpha = 3/5$, $\cos \alpha = 4/5$, $\tan \alpha = 3/4$; $\sin \beta = 4/5$, $\cos \beta = 3/5$, $\tan \beta = 4/3$. (b) $\sin \alpha = 1/\sqrt{82} \approx 0.11$, $\cos \alpha = 9/\sqrt{82} \approx 0.99$, $\tan \alpha = 1/9$;
 - $\sin \beta = 9/\sqrt{82} \approx 0.99,$ $\cos \beta = 1/\sqrt{82} \approx 0.11,$ $\tan \beta = 9.$
 - (c) $\sin \alpha = 2/\sqrt{5} \approx 0.89$, $\cos \alpha = 1/\sqrt{5} \approx 0.45$, $\tan \alpha = 2$; $\sin \beta = 1/\sqrt{5} \approx 0.45$, $\cos \beta = 2/\sqrt{5} \approx 0.89$, $\tan \beta = 1/2$.
 - (d) $\sin \alpha = 2/4.5 \approx 0.44$, $\cos \alpha = 4/4.5 \approx 0.89$, $\tan \alpha = 1/2$; $\sin \beta = 4/4.5 \approx 0.89$, $\cos \beta = 2/4.5 \approx 0.44$, $\tan \beta = 2$.
- 2. (a) The "really nice angles" are 30° , 45° , and 60° .
 - (b) [Drawing of any two right triangle with acute angles 30° and 60° or both 45° . Such triangles could have a variety of side lengths, two examples being side lenths 1-1- $\sqrt{2}$ and 3-4-5.]

	α	$\sin lpha$	$\cos \alpha$	an lpha
(c)	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
(0)	45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

- 3. (a) It will decrease.
 - (b) Zero.
 - (c) Zero.
 - (d) $\cos \alpha = 7/7.04 \approx 0.99$
 - (e) The length of the hypotenuse will approach the length of the adjacent side.
 - (f) One.
- 4. (a) An example of such a triangle is one with side lengths 1-10- $\sqrt{101}$. $(\sqrt{101} \approx 10.05.)$ Here, α will be 84°.
 - (b) $\sin 90^\circ = 1; \cos 90^\circ = 0.$
- 5. Adding to the earlier table and approximating when appropriate, we have the following table.

α	$\sin lpha$	$\cos lpha$	$\tan lpha$
0°	0	1	0
5°	~ 0.09	~ 0.996	~ 0.09
10°	~ 0.17	~ 0.98	~ 0.18
15°	~ 0.25	~ 0.97	~ 0.27
20°	~ 0.34	~ 0.94	~ 0.36
25°	~ 0.42	~ 0.91	~ 0.47
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{\sqrt{3}}{3} \approx 0.58$
35°	~ 0.57	~ 0.82	~ 0.70
40°	~ 0.64	~ 0.77	~ 0.84
45°	$\frac{\sqrt{2}}{2} \approx 0.71$	$\frac{\sqrt{2}}{2} \approx 0.71$	1
50°	~ 0.77	~ 0.64	~ 1.19
55°	~ 0.82	~ 0.57	~ 1.43
60°	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{1}{2} = 0.5$	$\sqrt{3} \approx 1.73$
65°	~ 0.91	~ 0.42	~ 2.14
70°	~ 0.94	~ 0.34	~ 2.75
75°	~ 0.97	~ 0.25	~ 3.73
80°	~ 0.98	~ 0.17	~ 5.67
85°	~ 0.996	~ 0.09	~ 11.43
90°	1	0	undefined

4.3.1-14,16 - Answer Key

Precalculus

Mr. Simmons

1. (a)
$$\sin \alpha = 12/13$$
,
 $\cos \alpha = 5/13$,
 $\tan \alpha = 12/5$,
 $\csc \alpha = 13/12$,
 $\sec \alpha = 13/5$,
 $\cot \alpha = 5/12$.
(b) $\sin \alpha = 4/5$,
 $\cos \alpha = 3/5$,
 $\tan \alpha = 4/3$,
 $\csc \alpha = 5/4$,
 $\sec \alpha = 5/3$,
 $\cot \alpha = 3/4$.
(c) $\sin \alpha = 24/25$,
 $\cos \alpha = 7/25$,
 $\tan \alpha = 24/7$,
 $\csc \alpha = 25/24$,
 $\sec \alpha = 25/7$,
 $\cot \alpha = 7/24$.
(d) $\sin \alpha = 1/4$,
 $\cos \alpha = \sqrt{17}/4$,
 $\tan \alpha = 1/\sqrt{17}$,
 $\csc \alpha = 4$,
 $\sec \alpha = 4/\sqrt{17}$,
 $\cot \alpha = \sqrt{17}/4$,
 $\tan \alpha = \sqrt{69}/13$,
 $\cos \alpha = 10/13$,
 $\tan \alpha = \sqrt{69}/10$,
 $\csc \alpha = 13/\sqrt{69}$,
 $\sec \alpha = 13/\sqrt{69}$,
 $\sec \alpha = 5/\sqrt{61}$,
 $\cos \alpha = \sqrt{61}/5$,
 $\sec \alpha = \sqrt{61}/5$,
 $\cot \alpha = 6/5$.

- (g) $\sin \alpha = 2/7$, $\cos \alpha = 3\sqrt{5}/7$, $\tan \alpha = 2/(3\sqrt{5})$, $\csc \alpha = 7/2$, $\sec \alpha = 7/(3\sqrt{5})$, $\cot \alpha = 3\sqrt{5}/2$. (h) $\sin \alpha = \sqrt{51}/10$, $\cos \alpha = 7/10$, $\tan \alpha = \sqrt{51}/7$, $\csc \alpha = 10/\sqrt{51}$, $\sec \alpha = 10/7$, $\cot \alpha = 7/\sqrt{51}$.
- 2. The triangle in Example 1c has opposite leg length a and hypotenuse length 1. If we call the adjacent leg length b, we have $a^2 + b^2 = 1^2$, so $b = \sqrt{1 - a^2}$. So the six trig ratios are as follows: $\sin \alpha = a$, $\cos \alpha = \sqrt{1 - a^2}$.

$$\cos \alpha = \sqrt{1 - a^2},$$

$$\tan \alpha = a/\sqrt{1 - a^2},$$

$$\csc \alpha = 1/a,$$

$$\sec \alpha = 1/\sqrt{1 - a^2},$$

$$\cot \alpha = \sqrt{1 - a^2}/a.$$

- 3. (a) If $\tan \alpha = d$, then we can consider the triangle with leg lengths d and 1, with hypotenuse $\sqrt{d^2 + 1}$. Then have the following trig ratios: $\sin \alpha = d/\sqrt{d^2 + 1}$, $\cos \alpha = 1/\sqrt{d^2 + 1}$, $\tan \alpha = d$, $\csc \alpha = \sqrt{d^2 + 1}/d$, $\sec \alpha = \sqrt{d^2 + 1}$, $\cot \alpha = 1/d$.
- 4. (a) $\sin \alpha = a/c$, $\cos \alpha = b/c$,

- $\tan \alpha = a/b,$ $\csc \alpha = c/a$ $\sec \alpha = c/b$ $\cot \alpha = b/a.$
- 5. (a) $\sin 30^{\circ} = \cos 60^{\circ}$
 - (b) $\cos 10^\circ = \sin 80^\circ$
 - (c) $\cot 7^\circ = \tan 83^\circ$
 - (d) $\sec 64^\circ = \csc 26^\circ$
 - (e) $\cos 31^\circ = \sin 59^\circ$
 - (f) $\tan 14^\circ = \cot 76^\circ$
 - (g) $\csc 47^\circ = \sec 43^\circ$
 - (h) $\sin 25^\circ = \cos 65^\circ$
 - (i) $\tan(\beta + \gamma) = \cot(90^\circ \beta \gamma)$
 - (j) $\sin \beta = \cos (90^\circ \beta)$
- 6. $\sin (90^{\circ} \theta) = \cos \theta,$ $\cos (90^{\circ} \theta) = \sin \theta,$ $\csc (90^{\circ} \theta) = \sec \theta,$ $\sec (90^{\circ} \theta) = \csc \theta,$ $\tan (90^{\circ} \theta) = \cot \theta,$ $\cot (90^{\circ} \theta) = \tan \theta.$
- 7. $\csc \theta = 1/\sin \theta$, $\sin \theta = 1/\csc \theta$, $\sec \theta = 1/\cos \theta$, $\cos \theta = 1/\sec \theta$, $\cot \theta = 1/\tan \theta$, $\tan \theta = 1/\cot \theta$.
- 8. (a) $\sin^2 30^\circ = (1/2)^2 = 1/4$
 - (b) $\cos^2 30^\circ = (\sqrt{3}/2)^2 = 3/4$
 - (c) $\tan^2 30^\circ = \left(\sqrt{3}/3\right)^2 = 1/3$
 - (d) $\cos^2 45^\circ = \left(\sqrt{2}/2\right)^2 = 1/2$
 - (e) $\tan^2 45^\circ = 1^2 = 1$
 - (f) $\sin^2 60^\circ = \left(\sqrt{3}/2\right)^2 = 3/4$
 - (g) $\cos^2 60^\circ = (1/2)^2 = 1/4$
 - (h) $\tan^2 60^\circ = \sqrt{3}^2 = 3$
 - (i) $\sin 30^{\circ 2} = \sin 900^{\circ} =$ undefined (until we have the unit-circle definition of sine)

(j) $\cos^2 \alpha = (\cos \alpha)^2$

	α	$\sin^2 lpha$	$\cos^2 \alpha$	$\tan^2 \alpha$
	0°	0	1	0
	5°	~ 0.01	~ 0.99	~ 0.01
	10°	~ 0.03	~ 0.93	~ 0.03
	15°	~ 0.07	~ 0.88	~ 0.07
	20°	~ 0.12	~ 0.82	~ 0.13
	25°	~ 0.18	~ 0.67	~ 0.22
	30°	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	$\frac{1}{3} \approx 0.33$
	35°	~ 0.33	~ 0.59	~ 0.49
9	40°	~ 0.41	~ 0.59	~ 0.70
5.	45°	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$	1
	50°	~ 0.59	~ 0.41	~ 1.42
	55°	~ 0.67	~ 0.33	~ 2.04
	60°	$\frac{3}{4} = 0.75$	$\frac{1}{4} = 0.25$	3
	65°	~ 0.82	~ 0.18	~ 4.60
	70°	~ 0.88	~ 0.12	~ 7.55
	75°	~ 0.93	~ 0.07	~ 13.93
	80°	~ 0.97	~ 0.03	~ 32.16
	85°	~ 0.99	~ 0.01	~ 130.65
	90°	1	0	undefined

- 10. (a) 1; 1; no maximum
 - (b) 0; 0; 0
 - (c) The sin and sin² functions have the same shape, but sin² has only positive values and is shorter. It also has hills/valleys twice as frequently. (We say that its "period" is half the length.)
- 11. $\sin^2 \theta + \cos^2 \theta = 1$
- 12. These expressions are all of the form $\sin^2 \theta + \cos^2 \theta$, so they all equal one.
- 13. Since $\sin^2 \theta + \cos^2 \theta = 1$, we have

$$\sin^2\theta = 1 - \cos^2\theta$$

and

$$\cos^2\theta = 1 - \sin^2\theta$$

14. (a)

$$\tan \alpha \cdot \csc \alpha = \frac{\sin \alpha}{\cos \alpha} \cdot \frac{1}{\sin \alpha}$$
$$= \frac{1}{\cos \alpha}$$
$$= \sec \alpha.$$

(b)

- $(\sin \alpha + \cos \alpha)^2 = \sin^2 \alpha + 2 \sin \alpha \cos \alpha + \cos^2 \alpha$ $= 2 \sin \alpha \cos \alpha + 1.$
- 16. (a) True. Although this is not the cofunction identity (that would be $\sin \alpha = \cos (90^\circ - \alpha)$), the statement here is

still true because

$$\sin \alpha = \cos (90^\circ - \alpha)$$
$$= \cos (- [90^\circ - \alpha])$$
$$= \cos (\alpha - 90^\circ).$$

This fact is, however, dependent on the unit-circle definition of cosine, so, strictly speaking, since cosine is undefined for negative angle measures in right-triangle trigonometry, "false" would also be an acceptable answer, as long as you understood the reason.

- (b) False. Counterexample: $\alpha = \beta = 30^{\circ}$.
- (c) False. Counterexample: $\alpha = 5^{\circ}$
- (d) False. Squares are never negative.

5.1.1-14 – Answer Key

Precalculus

Mr. Simmons

- 1. (a) 2
 - (b) $1/3 \approx 0.33$
 - (c) $20/3 \approx 6.67$
 - (d) $64/3 \approx 21.33$
- 2. (a) 10
 - (b) 9
 - (c) 5/4 = 1.25
 - (d) 5
- 3. (a) 5
 - (b) 1
 - (c) $10/3 \approx 3.33$
 - (d) 12
- 4. See 1, 2, and 3 sketched on pp. 129–30. Come to office hours if you want to see the rest sketched.
- 5. (a) $10\pi/9$
 - (b) $5\pi/9$
 - (c) $5\pi/18$
 - (d) $270/\pi^{\circ}$
 - (e) 15°
 - (f) 4π
 - (g) 36°
 - (h) 180°
- 6. (a) $\pi/6$, $\pi/4$, $\pi/3$, $\pi/4$, $\pi/2$, $3\pi/2$, 2π .
 - (b) $\pi/2$
 - (c) 120°, 135°, 150°, 210°, 225°, 240°, 300°, 315°, 330°.

- (d) $2\pi/3$, $3\pi/4$, $5\pi/6$, $7\pi/6$, $5\pi/4$, $4\pi/3$, $5\pi/3$, $7\pi/4$, $11\pi/6$.
- 7. (a) 1/2
 - (b) The more pieces you divide a whole into, by necessity the smaller the pieces will be (if they are of equal size—if they are unequally sized, still the average size will be smaller).
 - (c) $\pi/3$
 - (d) 2
 - (e) π
 - (f) $15\pi/4$
- 8. A half rotation would be π , which, written as $6\pi/6$, is obviously greater than $5\pi/6$.
- 9. Quadrant IV. One way to tell quickly is that $11\pi/6$ is very nearly 2π (which fact is made obvious by writing 2π as $12\pi/6$).
- 10. $5\pi/3 = 300^{\circ} < 315^{\circ} = 7\pi/4$
- 11.

$$\frac{5\pi}{3} = \frac{20\pi}{12} < \frac{21\pi}{12} = \frac{7\pi}{4}$$

- 12. (a) 24π in
 - (b) $12\pi \, \text{in}$
 - (c) 120π in
 - (d) 3 (radians)
- 13. (a) 10 in
 - (b) 10π (radians)
- 14. (a) $60/(13\pi)$ ft ≈ 1.47 ft = 17.64 in (b) 80/3 ft ≈ 26.67 ft = 320.04 in



then we will have found the x and y distance, and thus, the x- and y-coordinate

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§2 Exercises

- Plot the following points. You may use the same Polar Plane if you wish. 1.)
 - H(3.5, 135°)
 - (G) $(G, G, 0.5, 60^{\circ})$ (H) $H(3.5, 135^{\circ})$ (I) $I(20, 180^{\circ})$ (I) $I(20, 180^{\circ})$ (I) $I(1, 260^{\circ})$ (I) $L(\frac{7}{2}, 350^{\circ})$ (A) $A(2, 30^{\circ})$ (B) $B(1, 45^{\circ})$ (C) $C(3, 300^{\circ})$ (D) $D(2, 150^{\circ})$ (E) $E(4, 315^{\circ})$ (F) $F(2, 210^{\circ})$ C(3,300°)
 - $D(2, 150^{\circ})$
 - $E(4, 315^{\circ})$
- $F(2, 210^{\circ})$
- in this unit, except instead of using 30°, 45°, ... use the corresponding radian Before you start plotting points using radians, create a Polar Plane just like we did measures. Make sure each angle is labeled. í)
 - Now that you have a Polar Plane with radians, plot the following points. з:)

 - (H) $H\left(2,\frac{3\pi}{4}\right)$ (I) $I\left(2,\frac{\pi}{6}\right)$ (A) $A \left(2, \frac{\pi}{2}\right)$ (B) $B \left(3, \frac{\pi}{3}\right)$ (C) $C \left(4, \frac{2\pi}{3}\right)$ (D) $D \left(1, \frac{5\pi}{4}\right)$
- $J\left(300, \frac{3\pi}{2}\right)$
 - 5
- (K) $K\left(5, \frac{5\pi}{3}\right)$
 - (L
 - (E) E(4,1)
- (F) $F\left(3, \frac{11\pi}{6}\right)$

 $M\left(3,\frac{7\pi}{6}\right)$

Ξ

 $L\left(2,\frac{7\pi}{4}\right)$

- (G) *G*(2, π)
- Now let's work with negative angles.

4

- Now create a Polar Plane just like you did in 2.), except this time label each angle as -30° , -45° ,... as we started to do in the reading. Make sure each (A) What does a negative angle represent, or tell you to do? 8
- Create another Polar Plane, except this time use negative radian measures. angle is labeled. Q
 - Again, make sure each angle is labeled.
 - Plot the following points. .. .
- G(5, -25°) H(2, -2) $I(3, -100^{\circ})$ \widehat{E} \widehat{E} \widehat{E} \widehat{E} \widehat{E} \widehat{E} \widehat{E} \widehat{E} (A) $A(2, -45^{\circ})$ (B) $B\left(3, -\frac{2\pi}{3}\right)$ (C) $C(1, -180^{\circ})$
- $J(4, -2\pi)$ $K(3, -135^{\circ})$ $L\left(2, -\frac{3\pi}{4}\right)$ $E(3, -300^{\circ})$ $D\left(2,-\frac{7\pi}{4}\right)$ Ô E
- trigonometry, and was one of the main reasons we chose to work with the Polar Now let's work with coterminal angles. This is incredibly important for our work in Plane before working with the Unit Circle. Use the Polar Planes you created from previous exercises to help you. (F) $F\left(4, -\frac{5\pi}{6}\right)$ 6.)

of our point? Converting a Polar point to a rectangular one, then, amounts to finding lengths of a triangle.

But we know how to do this!

What, then, are the rectangular coordinates of the given Polar point? Find the length of each leg of the triangle shown above. 3 8

Using the same method above (and drawing a picture), convert the following Polar points into rectangular points. (; 8

- $A(1,60^{\circ})$
- $B(1,45^\circ)$ $C(1,90^\circ)$ () () () ()
 - $D\left(2,\frac{\pi}{6}\right)$ $E\left(1,\frac{\pi}{3}\right)$ Ô
 - (E
- Did any of your previous results seem familiar to what you've already learned? How so? £
- Can you generalize the process of converting Polar coordinates into rectangular coordinates? It seems like there's some treasure hidden in this Exercise. J

The unit circle 8

We now embark on perhaps the most important section in Trigonometry. It is imperative that you learn and master the techniques in this section, as it will make much of Trigonometry (and, subsequently, Calculus) much easier. In this section we provide a tool to help visualize and efficiently evaluate most of the Trig functions you'll run into. Of course, as has been the case in many of these Trig sections, you must be adept at the previous lessons as well, including quickly evaluating sin 30°, for example.¹

Jp to this point, we've worked with only a few Trig functions, such as cos 30°. And, as we mentioned, there are many Trig functions that we simply can't evaluate with any sort of efficiency. In this section we'll learn how to evaluate a handful more – an infinite amount, actually! – quickly and efficiently.

can find the length of the sides for very quickly. Then we just have to write the Recall that the main reason we're able to evaluate a Trig expression like tan 45° is because we can create a special right triangle (in this case, a $45^\circ - 45^\circ - 90^\circ$ triangle) which we

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corresponding ratio." Without these special right triangles, we would have to resort to approximation methods, which aren't very interesting to study.

But let us bring back our Polar Plane from the previous section in Figure 66. Then let us see if we can't create any other special right triangles.



Can you create any special right triangles using this picture? Try making a point on one of the angles.

It turns out that not only can we make a few, but we can make many of them! We consider one example below

Example 1

Given the point $A(2, 120^\circ)$, create some special right triangle.

We first plot the given point. We show this in Figure 67.

" We must also bring up the fact that the size of the triangle we choose does not matter.

ⁱ Although, frankly, at this point, this warning should not be necessary.



Two possible right triangle present themselves, and we show them both in Figure 68a and b.



To draw these right triangles, we just dropped a straight line down from the point the xaxis (in Figure a) and then a straight line to the right from the point to the y-axis. Right triangles are great, and this allows us to use what we've learned in previous sections. But it would be even better if they were special right triangles, right?

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But that's exactly what each of them are! And it is made even more evident by drawing them on our Polar Plane like we did. In Figure 68a, for example, you can see that we have a $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangle, which we draw separately in Figure 69.



This is significant, and we'll demonstrate why as we proceed through this section. Hold on to this thought for a moment because we need to establish something else before making our greater point.

Figure 69

One of the Exercises from the previous section saw you converting Polar coordinates into rectangular coordinates. This was done by forming a right triangle (not unlike our previous Example) and then using your memorized Trig functions to find the missing lengths of the right triangle (which corresponded to the x- and y-values of the point^[11]). Let us reconsider that with some more formality and see if we can't uncover some truth.

<u>Example 2a</u>

Convert the Polar coordinate (2, 45°) to rectangular coordinates.

Although you did this problem in the previous section, let us formalize this process. We first plot the given point, and then create a right triangle, as shown in Figure 70.

ii Huh? Did you miss something? There's something very interesting going on here; can you feel it?



Now we can redraw that triangle by itself, adding in what we know, viz. the radius. We show this in Figure 71.



Figure 71

Recall that when we graph something on the coordinate plane, we go right some distance, and then up some distance. In the case of our picture, do you recognize that the origin is the point to the left of the right angle? Then if we travel right to the right angle, and then up, wouldn't we have just went through the process of plotting a point on the rectangular plane?^{1V} Thus, if we find the length of each leg, we'll have our *x*- and *y*-coordinates, right?

So that's what we'll do. And this is quite easy, since this is an isosceles right triangle, we just divide the hypotenuse by $\sqrt{2}$, and find that each leg has a length of

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<u>2</u> <u>
</u>
2 Let us rationalize this number so we can perhaps recognize it after we're done with it:

$$\frac{2}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}.$$

So each leg measures $\sqrt{2}$ in length, and this tells us that the point $A(2, 45^{\circ})$ can also be written as $A(\sqrt{2}, \sqrt{2})$.

This is neat, but more importantly that number $\sqrt{2}$ reminds us of a number that popped up quite a few times in the previous unit: $\frac{\sqrt{2}}{2}$. In fact, our result was twice that of $\frac{\sqrt{2}}{2}$. Why is this interesting? Because what is sin 45°? And what is $\cos 45°$? And is this a coincidence??

<u>Example 2b</u>

Convert the Polar coordinate $B(3, 30^{\circ})$ into rectangular coordinates.

Let's do one more test before trying to generalize and formalize our results. Following our previous procedure, then, we end up with Figure 72.



From here, we once again draw a right triangle, and, once again, in Figure 73 we see a special right triangle, don't we?

¹⁶ That is confusing in words. Try doing what I wrote to help you see that you're just following the same procedure you've used perhaps thousands of times to plot a point. Or ask your teacher to demonstrate.



Figure 73

Again, what we're looking for are the legs of this right triangle, as that will tell us the xand y-coordinates of the point we're looking for. Using what we learned from Unit four, we see that the short leg is 1.5 and the long leg is $1.5\sqrt{3}$. And hence our point B can be written in the rectangular plane as $B(1.5,1.5\sqrt{3})$. Let us again highlight the results: Remember that $\cos 60^\circ = \frac{1}{2}$? Well, our *x*-coordinate is that times 3, isn't it? What about $\sin 60^\circ$? What relationship does that have with our result of $1.5\sqrt{3}$?

Isn't it interesting that the rectangular coordinates of a point from the Polar Plane keep coming up as multiples of Trig functions that we've memorized? Let us now generalize the result and make our major point of this section.



A point on a circle in rectangular coordinates

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We cannot overstate how important this discovery is. So let us restate it another way. We can find the coordinates of any point on a circle using our Trig functions and the information above.

Perhaps more importantly, however, is that this establishes a relationship that we can use to find the values of other angles that we input into Trig functions. In our original definition of Trig functions, we could only use acute angles. This relationship, namely that

$$x = r \cdot \cos \alpha$$
, $y = r \cdot \sin \alpha$,

Where x and y have the usual meaning as x- and y-coordinates of a point in the rectangular plane, helps us to find the output of any angle put into a Trig function. With some simple Algebra, we'll make it more clear:

$$\cos \alpha = \frac{x}{r}, \sin \alpha = \frac{y}{r}.$$

Let us now practice this concept.

<u>Example 2a</u>

Evaluate sin 120°.

^{*} You might ask how we knew about this relationship. Good question! We're not trying to teach a procedure here for you to follow, but only pointing something out, viz. that these familiar numbers keep popping up. Of course, there is a reason we chose 3 in this Example and 2 in the previous... Can you see where we get those two numbers from?

point on the 120° angle. We did this in Example 1, and, since the previous Example had a these angles than simply memorizing.^{vi} Using our Polar Plane from before, we place some Did you memorize sin 120°? Because you shouldn't have. There's a better way to deal with radius of 2, let's stick with that.^{vii} Then we can create the right triangle we saw in Figure 68a, which we redraw in Figure 74 with the radius' length added. Note that based on the way we came about our previous definition, that is, that $\cos \alpha = \frac{x}{r}$, we must use the x-axis as the base of our triangle.^{viii}



Figure 74

Then, using the relationship previously stated, viz. that
$$\sin \alpha = \frac{\gamma}{r},$$

we can easily get our answer. Although we input 120° , what we are working with is the 60° angle seen in the triangle in Figure 74.

you've not forgotten about $30^\circ-60^\circ-90^\circ$ triangles, as we'll find the value of that leg We see that r = 2, because that was our chosen radius. But what is y? In this case, it's the vertical leg of the triangle drawn in Figure 74. What is the length of this leg? Hopefully using this technique. We show the lengths of the triangle in Figure 75.





Now we have enough information to write our answer; we get

$$\sin 120^\circ = \frac{\sqrt{3}}{3}.$$

Let's do another example.

Example 2b

What is cos 225°?

We follow the same procedure. This time, we need to plot a point on the Polar Plane at 225°. What radius should we use? How about we stick with 2, for consistency's sake. We show this point, that is, B(2, 225°) in Figure 76.

 $^{\rm ix}$ That seems oddly familiar... Wasn't that the same thing as sin 60°...?

[&]quot; Although, that being said, don't let us stop you from memorizing if that's what you're really good at " Does the radius *need* to be 2? Excellent question! We'll cover that in Example 3. " By "base" we mean that the right angle must be located on the *x*-axis. This was not the case in Figure 68b. It is possible to use that Figure, but then we would need to change our definition.





We now draw a right triangle using the x-axis as that base for our right angle. We show this in Figure 77.



Figure 77

Since we're looking for Cosine, and, based on our findings,

$$\cos \alpha = \frac{x}{r}$$

we need to find the length of the horizontal leg.

Then we just use our knowledge of special right triangles to complete the triangle. We get Figure 78, as shown.

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We can now substitute, since we know r (we chose it to be 2) and x, which is the length of the horizontal leg, which is 2. We have

Figure 78

$$\cos 225^\circ = \frac{\sqrt{2}}{2}.$$

But is this correct? One way to check our result is to plot it in the coordinate plane. Our *x*-coordinate is given by the horizontal distance, which in this case corresponds to the horizontal leg in Figure 78.

A quick glance shows that we must be wrong. The point in Figure 76 was in Quadrant III, and this requires a negative x-value, right? We have an issue.

Or do we? There is an easy way to rectify this error: We simply make our result negative. This seems awfully artificial, and it is. But that's where our Polar Plane comes in handy. We have a picture to see that, clearly, our *x*-coordinate must be negative. Therefore, our result is

$$\cos 225^\circ = -\frac{\sqrt{2}}{2}.$$

Let's do one more example before introducing you to a special circle.

Example 2c

Evaluate all six Trig functions using $\frac{5\pi}{3}$ as the input.

We are looking for $\sin \frac{5\pi}{3}$, $\cos \frac{5\pi}{3}$, and so on. We have radians as our input, and this is fine, since we know how to work with them. The first thing we should do is plot the point $C\left(2, \frac{5\pi}{3}\right)$ on the Polar Plane. We do this in Figure 79.

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Jnit



Do you see that $\frac{5\pi}{3}$ is equivalent to 300° ?

want to make a note of this for each of the problems like this you do, so that you don't Before proceeding, do you see how C is in Quadrant IV? Thus x > 0 and y < 0. You might forget.

Now we create a right triangle with the x-axis as our base. We get Figure 80 as shown.



Now all we need to do is write the appropriate ratios. We see that



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because Sine is opposite to hypotenuse. It must be negative since we are in Quadrant IV. We further see that

$$\cos\frac{5\pi}{3} = \frac{1}{2}$$

and, since we are in Quadrant IV, we must be positive.

What about the other Trig functions? First, recall that $\tan \alpha = \frac{\sin \alpha}{\cos \alpha}$. And since $\sin \frac{5\pi}{3}$ is negative while $\cos \frac{5\pi}{3}$ is positive, then Tangent must be negative.^x Then we just need to find the ratio from the previous Figure, and make it negative. We get

$$\tan\frac{5\pi}{3} = -\sqrt{3}.$$

Finding the reciprocal Trig functions, like Secant, is easy. We just need to find the reciprocal of the previous three results. Note that finding a reciprocal does not change its sign. Therefore,

$$\csc \frac{5\pi}{3} = -\frac{2}{\sqrt{3}}, \quad \sec \frac{5\pi}{3} = 2, \quad \cot \frac{5\pi}{3} = -\frac{1}{\sqrt{3}}.$$

We leave the rationalization to the reader, if they wish.

Up to this point, we've always used a radius of 2. Just because. But certainly there has to be a better choice, right? Indeed, there is, and we call it the unit circle.

The unit circle

A circle centered at the origin whose radius is 1. This simple definition has some profound implications. For example, recall that, using a circle of radius r, we have the relationship

 $\cos \alpha = \frac{x}{r}$.

But if we have a unit circle, where r = 1, then we have the simpler relationship

 $\cos \alpha = x$.

This also holds with sin α , of course:

 $\sin \alpha = y$.

 $^{ imes}$ The quotient of a negative number and a positive must be a negative number, right?

Not only is this easier to write, but it also helps us to find the Sine (or Cosine, or...) of any angle pretty easily. We show this intuition in Figure 81.



Any point on the circle has an x-coordinate of $\cos \alpha$ and a y-coordiante of $\sin \alpha$.

<u>Example 3a</u>

Evaluate sin 135°.

The unit circle is a picture to place into your head so you can evaluate Trig functions like this very quickly, efficiently, and accurately. At first, you'll need to draw it out and it might take some time. But eventually it becomes second-nature, and it is indispensable in Calculus. So while someone adept at math might not need to draw it out, we will do so in each of these examples.

We draw a 135° angle on the unit circle in Figure 82.



Since *P* has *x*- and *y*-coordinates of $\cos \alpha$ and $\sin \alpha$, respectively, all we need to do to evaluate $\sin 135^{\circ}$ is to find the *y*-coordinate of *P*. But this amounts to the same thing we in the previous set of Examples, using the Polar Plane. The only difference is that our

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radius will always be 1.^{xi} Thus we need to make a right triangle with the x-axis as our base as shown in Figure 83.



If you have difficulties seeing what type of special right triangle you get, use your Polar Plane. It will be more evident since each angle is marked. Also note we rationalized the denominators on each leg.

Now we just use substitute. We get that

$$\sin 135^\circ = \frac{\sqrt{2}}{2}.$$

And yes, our result should be positive. Another way to see that our result must be positive is because, if we start at the origin, we had to travel up to get to P, correct? And isn't up a positive direction?

<u>Example 3b</u>

Evaluate $\cos \frac{11\pi}{6}$.

We first draw an angle of $\frac{11\pi}{6}$ on our unit circle in Figure 84.

^{xi} As opposed to whatever we want. Why choose 1, again?



Again, use your Polar Plane to heb you find ¹¹⁴ However, you'll want to have a very firm grasp of radians so don't completely rely on you Polar Plane.

Figure 84

Again, we now create a right triangle using the x-axis as our base. We get the special right triangle shown in Figure 85.



Figure 85

Then we just substitute, knowing what ratio we get with the Cosine function. Hence

$$\cos\frac{11\pi}{6} = \frac{1}{2}.$$

This, also, should be positive, since we went to the right to get to P.

Be patient and resilient as you learn the unit circle. Mastery will come, but only with practice and perseverance. Once you master the unit circle, Trigonometry becomes your plaything.

§3 Exercises

- 1.) Plot the following points on the Polar Plane, then create a right triangle where the *x*-axis serves as the base. (A) $A(2,135^{\circ})$ (B) $B(3,300^{\circ})$



Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: Spanish III

Teacher(s): Ms. Barrera <u>anna.barrera@greatheartsirving.org</u> Supplemental links: <u>www.spanishdict.com</u> <u>www.lingt.com/barreratumble</u> Weekly Plan:

Monday, May 11

Capítulo 4B - Part I Read about the life and customs of Spain.

Capítulo 4B - Reading comprehension of vocabulary classification and matching.

Tuesday, May 12

Capítulo 4B - Part II Read about the life and customs of Spain.

Capítulo 4B - Reading comprehension by completing the statements and definition of festivals.

Wednesday, May 13

☐ Capítulo 4B- Listening Activity: Spanish storytime from the book titled Vida o muerte en el Cusco. ☐ Capítulo 4B- Speaking Activity: Listen and answer questions in Spanish.

Thursday, May 14
Capítulo 4B- Painting by Carmen Lomas Garza, *Tamalada*, 1990, color lithograph, Smithsonian American Art Museum.
Capítulo 4B- Read statements about the painting and decide whether true or false and correct if false.

Friday, May 15

attend office hours

□ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently. I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Monday, May 11

Capítulo 4B - Part I - Read about the life and customs of Spain. Comprehension of the reading using matching, vocabulary classification and matching.

I. **Reading Handout:** *La vida y las costumbres de España.* Reading comprehension. **Exercise A**. Indicate for each of the 15 words if they belong under alimentation, beverages, garments, religious festivals or social customs. **Exercise B**. Match the 10 vocabulary words with the words on the right side of the column.

Tuesday, May 12

Capítulo 4B - Part II - Read about the life and customs of Spain. Comprehension of the reading by completing the statements and definition of festivals.

I. **Reading Handout:** *La vida y las costumbres de España.* Reading comprehension. **Exercise C**. Complete the following 10 sentences with the appropriate vocabulary. **Exercise D.** Explain each of these 10 religious holidays. For example; **romería** Cada pueblo tiene su santo patrón, en el dia del santo se va a la tumba del santo a hacer picnics religiosas. Esos picnics se llaman romerías.

Wednesday, May 13

Capítulo 4B- Listening Activity: Storytime in Spanish from the book titled Vida o muerte en el Cusco. Speaking Activity: Listen and answer questions in Spanish.

I. Storytime: Listen to chapters 2 and 3 of *Vida o muerte en el Cusco*. Video link in google classroom.II. Speaking Assignment: Answer questions in Spanish about chapter 2 and 3. Questions are in lingt.

Thursday, May 14

Capítulo 4B- Painting by Carmen Lomas Garza, *Tamalada*, 1990, color lithograph, Smithsonian American Art Museum. Read statements about the painting and decide whether true or false and correct if false.

I.**Textbook**, **p.222** - **Activity 13** - *La Tamalada*, Painting by Carmen Lomas Garza. Read the following statements and decide if they are true or false. If they are false please correct and write the entire sentence. These sentences are in the preterit and imperfect tense.

- 1. Entre en la cocina con mi papa.
- 2. Había tres personas alrededor de la mesa.
- 3. Todos ayudaban a hacer galletas.
- 4. Todos mis parientes tenían el pelo negro.
- 5. Las personas tenían diferentes trabajos.
- 6. Las paredes de la cocina eran amarillas.

FRANCIA

Chapter 37 La vida y las costumbres de España

Hoy día la vida diaria de España es muy semejante a la vida diaria de los Estados Unidos. Sin embargo, cada país tiene algo de particular. En España existen muchas costumbres y tradiciones interesantes.

LA CASA Y LA FAMILIA

Las casas de las ciudades grandes se parecen a las de las otras ciudades del mundo. Hay edificios de apartamentos y casas particulares. Muchos españoles viven en su condominio o piso que forma parte de un edificio alto.

En los pueblos pequeños, las ciudades antiguas y en las partes antiguas de las ciudades grandes las casas están en calles estrechas. Generalmente, estas casas son de un solo piso, con balcones, ventanas con rejas y pintorescos patios interiores. En muchas casas, las paredes están cubiertas de azulejos.

LOS NOMBRES Y LOS APELLIDOS

Los nombres españoles son diferentes a los de los anglosajones, y esta costumbre de los nombres se extiende a todos los países hispanos. Además del nombre de pila, los españoles e hispanos llevan dos apellidos (el apellido del padre seguido del apellido de la madre): por ejemplo, Rafael *Hernández Silva*. Este nombre se archiva bajo «H» porque se considera más importante el apellido del padre.

Cuando una mujer se casa, ella retiene el apellido de su padre y añade el «de» seguido por el apellido de su esposo. Por ejemplo, si Rafael se casa con Marisa Trujillo Rodríguez, su nombre sería Marisa Trujillo de Hernández. Ella no usa ninguno de los apellidos maternos.

Por lo general, el nombre de pila de la mayoría de los españoles (y de los hispanoamericanos) es el nombre de un santo. Además de su propio cumpleaños, celebran el día del santo.

TIPOS PINTORESCOS

La tuna es un grupo de músicos ambulantes que ha sido parte de la vida estudiantil universitaria desde el siglo XVI. Los tunos usan un traje tradicional de color negro y una capa. Tocan música romántica y alegre. Sus instrumentos incluyen el laúd, la bandurria, el requinto, la guitarra y la pandereta.

Los gitanos viven en el sur de España, principalmente cerca de las ciudades de Sevilla y Granada. Conservan muchas de sus tradiciones dentro de las que la música y el baile son muy importantes. Hablan un idioma llamado romani.

ESPAÑA

COSTUMBRES

PORTUGAL

La vida social es muy importante para los españoles. Son muy sociales y amigables. Les gusta pasar mucho tiempo charlando con los amigos en los cafés y en los bares. Cenan mucho más tarde que en nuestro país, y por consiguiente, se acuestan más tarde.

En muchas ciudades hay un **ateneo**, un club intelectual donde se reúnen grupos literarios y científicos.

La tertulia es una reunión informal con el propósito de charlar y divertirse. Muchas veces no termina hasta después de la medianoche.

La siesta es la tradición de acostarse por la tarde durante las horas de mayor calor. Las tiendas y las oficinas se cierran, y los trabajadores regresan a sus casas para comer, descansar o dormir la siesta. Después de la siesta, las tiendas y oficinas vuelven a abrir y quedan abiertas hasta muy tarde. La tradición de la siesta ha ido desapareciendo en las ciudades grandes.

La lotería está dirigida por el gobierno y es muy popular. Se usan las ganancias de la lotería para el beneficio de las personas pobres y los niños huérfanos. El sorteo tiene lugar tres veces al mes y hay muchos premios. El premio mayor se llama «el premio gordo».

«Pelar la pava» es una tradición antigua que se usaba cuando el novio cortejaba a la novia. El novio le hablaba a la novia por medio de la reja. El estaba de pie afuera y la mujer estaba sentada dentro de la casa al otro lado de la reja. Hoy día muchas de las costumbres de cortejar a las mujeres están desapareciendo, especialmente en las ciudades grandes.

COMIDAS Y BEBIDAS

La tradición de comer tres comidas al día existe tanto en España como en los Estados Unidos. Principalmente, la diferencia está en el horario. En España existe la costumbre de comer más tarde. Se toma el desayuno a eso de las ocho de la mañana y generalmente consiste en café con leche o chocolate y pan con mantequilla y mermelada o bollos. Se toma la comida a eso de las dos de la tarde y no se toma la cena hasta después de las ocho o nueve de la noche. Estas pletas. Pueden co con arroz o verdu de la tarde los esp ligero como un s bocados que llama itivo antes de la co España tiene una y deliciosa. Entro puchero que es cional de España diario, especialm llama también ol El plato más

con pollo. En ingredientes al a Las bebidas

el café, el té, l chata es una b dras, agua y azúv verano como ru una bebida pop Lo preparan mu con **panecillos**

LA ROPA

La ropa de los Europa. Sin en La **mantilla** e jes que la mu **mantilla** se u que se llama **p** ricamente bon **boina** es una «beret» francé sandalia hech bajadores en

FIESTAS R

Puesto que l religión cató portantes en 1. La **Navi**

la fiesta pios de **Nacimi** tan el **Nochel misa d** Grupos tando v acostum las pers

de la noche. Estas dos comidas son comidas completas. Pueden consistir en sopa, ensalada, carne con arroz o verduras y postre. Alrededor de las seis de la tarde los españoles toman la merienda, algo ligero como un sándwich. Se acostumbra comer bocados que llaman «tapas» cuando toman el aper-

España tiene una gastronomía variada, interesante y deliciosa. Entre los platos tradicionales está el puchero que es considerado quizás el plato nacional de España. Es un guisado que se sirve casi a diario, especialmente entre los campesinos. Se llama también olla o cocido.

El plato más conocido de España es el arroz con pollo. En Valencia añaden mariscos y otros ingredientes al arroz con pollo y lo llaman paella.

Las bebidas que toman los españoles incluyen el café, el té, la leche y los refrescos. La horchata es una bebida tradicional hecha de almendras, agua y azúcar. La horchata se toma fría en el verano como refresco. El chocolate caliente es una bebida popular del desayuno y de la merienda. Lo preparan muy espeso y lo toman muy caliente con panecillos, bizcochos o churros.

LA ROPA

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La ropa de los españoles se parece a la del resto de Europa. Sin embargo, existen prendas tradicionales. La mantilla es un pañuelo grande de seda y encajes que la mujer lleva en la cabeza. Debajo de la mantilla se usa un peine alto, ricamente adornado, que se llama peineta. El mantón es un chal grande, ricamente bordado. Sirve de adorno o de abrigo. La boina es una gorra de lana redonda parecida al «beret» francés. Las alpargatas son una especie de sandalia hecha de lona. Son comunes entre los trabajadores en muchas partes de España.

FIESTAS RELIGIOSAS

Puesto que la mayoría de los españoles practican la religión católica, las fiestas religiosas son muy importantes en el país.

1. La Navidad se celebra el 25 de diciembre y es la fiesta más importante del año. Desde principios de diciembre en cada casa se preparan los Nacimientos (grupos de figuras que represende Jesucristo). La nacimiento el Nochebuena (Christmas Eve) la gente asiste a la tan misa del gallo (midnight mass) en las iglesias. Grupos de personas caminan por las calles cantando villancicos (Christmas carols). También se acostumbra dar regalos, llamados aguinaldos, a las personas que han servido a la familia durante el año (el cartero, los criados y otros). Los niños reciben sus regalos el 6 de enero, que se llama el Día de los Reyes Magos. Los Reyes Magos corresponden a nuestro Santa Claus.

2. El Carnaval es un período de tres días de diversión antes del Miércoles de Ceniza, que comienza la Cuaresma (Lent). La Cuaresma son los cuarenta días que siguen. Termina el domingo de Resurrección, la Pascua Florida. La Semana Santa, que precede a la Pascua Florida, se celebra con mucha solemnidad y devoción, sobre todo en Sevilla. El Viernes Santo hay procesiones religiosas en muchos pueblos y ciudades.

Cada pueblo tiene su santo patrón, cuyo día se celebra con una fiesta. La noche anterior se celebra una verbena (evening festival). El día del santo hay romerías (religious picnics) a la tumba del santo.

3. El Día de los Difuntos o Muertos (All Souls' Day) se celebra el 2 de noviembre, en memoria de los muertos.

FIESTAS NACIONALES

- 1. El dos de mayo es la fiesta nacional de España. Conmemora un suceso patriótico, el comienzo de la resistencia contra los franceses en 1808.
- 2. El Día de la Raza, que se celebra el 12 de octubre, corresponde a nuestro «Columbus Day». Se celebra esta fiesta en todo el mundo hispano.

DEPORTES Y DIVERSIONES

1. La corrida de toros es un espectáculo muy típico de España. Generalmente hay corridas los domingos por la tarde y los días de fiesta. Tiene lugar en la plaza de toros. La corrida comienza con un desfile de todos los participantes por la arena, mientras se escucha la música de pasadobles.

La corrida tiene tres partes o lo que llaman suertes. Los picadores entran, montados a caballo en la primera suerte. Llevan picas largas. Los banderilleros entran a pie en la segunda suerte. Ellos llevan banderillas. En la tercera suerte el matador (el torero final), armado de una espada de acero muy fina, y llevando una pequeña muleta roja, exhibe su arte y su valor. Ejecuta varios pases con la muleta, hasta que el momento ideal llega para matar al toro.

2. El jai-alai es un juego de pelota de origen vasco que se juega con una pelota dura en un

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gran frontón de tres paredes. Es semejante al «handball» pero el jugador usa una cesta larga y estrecha, que está atada a la muñeca, para tirar y coger la pelota.

3. El fútbol es muy popular no sólo en España, sino también en el resto de Europa y en Latinoamérica. Se puede decir que es el deporte nacional de la mayoría de esos países.

EXERCISE A. Identifique cada una de las siguientes palabras como alimento, bebida, prenda de vestir, fiesta religiosa, o costumbre social.

1. boina		
2. verbena		
3. paella	``````````````````````````````````````	
4. tertulia		
5. cocido		
6. romería		
7. horchata		
8. el Día de los Difuntos		č.
9. alpargatas		
10. arroz con pollo		-
11. mantilla		-
12. Carnaval		-
13. merienda		-
14. peineta		
15. chocolate		

EXERCISE B. A la izquierda de cada expresión de la lista A, escriba la letra de la palabra o expresión de la lista B que tenga relación con ella.

Α	В
1. apellido 2. pelar la pava 3. lotería 4. Nochebuena 5. banderillero 6. jai-alai 7. día del santo patrón 8. puchero 9. tuna	 B a. premio gordo b. frontón c. sandalias d. músicos ambulantes e. reja f. nombre g. alimento h. Navidad i. corrida de toros j. verbena
10. alpargatas	

EXERCISE C. Complete las frases siguientes. 1. Antonio Moreno y Villa está casado con Luisa Gómez y Vega, y tienen un hijo, Juan. El nombre completo del hijo es Juan _____. 2. Un alimento popular en Valencia, hecho de arroz, pollo y mariscos se llama ______. 3. La fiesta nacional de España se celebra _____. 4. Los niños españoles reciben regalos el 6 de enero, el Día de los ______. 5. El torero que va montado a caballo se llama el _____. 6. El 12 de octubre se celebra _____. 7. En vez de celebrar su cumpleaños, los niños españoles celebran su _____ 8. _____ es el nombre que se da a un club científico y literario. 9. La horchata es una bebida fría que se hace de _____. 10. El jai-alai es un deporte que se juega en un _____. Explique cada uno de los siguientes. EXERCISE D. 1. Día de los Muertos _____ 2. matador

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