11th Grade Lesson Plan Packet

5/11/2020-5/15/2020

GreatHearts Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

May 11-15, 2020
Course: Art
Teacher(s): Ms. Clare Frank

Weekly Plan:

Monday, May 11
☐ Watch the instructional video "Applying Perspective"
☐ Drawing "View through a Window": Establish structured illusion of space space, with eye-level and converging parallel lines * *You may use dry media of your choice, including colored pencils, pastels, or graphite. Use discretion in selecting media. The default is pencil.* *See page 5 for project overview.*

Tuesday, May 12
☐ Drawing "View through a Window": Finish developing general shapes for view outside and fine-tune shapes for interior space

Wednesday, May 13
☐ Watch the instructional video "Livia's Garden Room"
☐ Drawing "View through a Window": Develop the imagery seen outside, with particular attention to shape relationships, specific shape qualities, and drawing style.

Thursday, May 14
☐ Drawing "View through a Window": Develop color, value, mark-making, and linework patterns in the imagery seen outside.

Friday, May 15
☐ attend office hours
☐ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____
Parent Signature

Monday, May 11

1. **Watch the instructional video** for Monday, May 11 in GoogleClassroom.

2. Look over the Project Overview on page 5.

3. (16-18 min.) Continue drawing "View through a Window": **Establish structured illusion of space, with consistent eye-level and converging parallel lines**.

    a. Check the **overall layout** of your drawing, paying special attention to **shape relationships**, the **relationship of interior to exterior**, and to the **overall composition**. Adjust as needed.

    b. Assuming you are showing part of the window structure and possibly part of the wall, window sill, or room, **identify any parallel and perpendicular lines**. Next **ascertain your eye-level**.

    c. **Converging Parallel Lines:** Remember that parallel lines, when seen even at a slight angle, appear to converge. These parallel lines will converge upon a vanishing point on your eye-level or above your head, far outside the picture plane! However, this means that on your page the parallel lines will incline towards each other slightly. Make sure they do so, and that they are inclining toward the vanishing point. This will mean that:

        i. The space between horizontal parallel lines will be wider where the lines are closer to you, and narrower where the lines are farther from you (like the window in Andrew Wyeth's painting *Wind from the Sea*).
        ii. Any horizontal lines below your eye-level will seem to angle upward as the line moves further away from you. In contrast, any horizontal lines above your eye-level will seem to angle downward as the line moves further away from you. However, a horizontal line exactly on your eye-level will remain horizontal, at right angles to the sides of your picture plane and parallel to the top and bottom edges of your picture plane.
        iii. The vertical lines appear to angle in towards each other ever so slightly. In class exercises we usually pretend they are truly vertical on the page, but in fact the vanishing point is directly over your head. For this drawing you may choose to keep the vertical lines vertical, in which case they will be parallel to the right and left sides of the picture plane, and perpendicular to the top edge.

    d. **Show the dimensionality of the window and wall.** While in cartoons or comic strips the windows may be shown as a 2D shape, like a gridded rectangle, in fact any window has dimension. Observe the depth as well as the width and height, by showing all edges seen.

Tuesday, May 12

1. Continue drawing “View through a Window”: **Finish developing general shapes for view outside and fine-tune shapes for interior space**.

a. (15 min.) Check the **overall shapes and shape relationships for the view outside**, and adjust as needed. Pay close attention to the **positive/ negative shape relationships**, seeing even the sky or the lawn as a shape. Eyelevel still applies, both outside and inside!

   i. Remember that things far away appear smaller; the tiny wren singing on my window frame may appear larger than my neighbor’s truck across the street.

   ii. Size relationships should make sense in proportion to each other. Use your physical knowledge to help you. The tabby cat sitting on the curb has a body of about the same thickness as the curb’s height. In contrast, a robin’s body is not as thick as the curb, though it may be as tall as the curb!

   iii. Use shape, value and color to show imagery when you can’t show detail. Perhaps the most important secret in art is how to imply information rather than to show it! Don’t make something larger just because you aren’t sure how otherwise to show the detail - instead, use shape and color/value. To show the bluebird on the telephone wire across the street, I’d place a blue brushmark adjacent to a shorter reddish daub. Just a dab'll do ya!

   iv. Use texture, mark-making and value to differentiate the shape areas outside, beginning to hint at imagery.

b. (5 min.) Develop the **value range and contrast in your interior space** to enhance dimensionality (3D quality). Consider direction of light in relation to the planar surfaces of your interior spaces. Look at Andrew Wyeth’s paintings *Wind from the Sea* and *Frostbitten Apples* for examples. These paintings were in last week’s instructional video.

*** Don’t go cartoony just because you are drawing a scene. It is a thoughtless but easy mistake to jump ahead and define the objects with thick black outlines and stereotyped shapes because that feels “safe”. It isn’t though. It’s just a default mode that limits you and closes doors to growth. Instead really observe and develop the individual shape qualities and textures. Apply the skills we practiced in drawing organic objects and representing form and texture. Pages 7-8 show you helpful ways to simplify form. ***

*** Note: Skills and techniques are tools! Use a variety of techniques, including optical value with mark-making and linework. You don’t have to use color but if you do, use varied saturation levels and be sparing of pure prismatic colors. Instead, focus on muted colors and include chromatic grays.***

Wednesday, May 13

1. **Watch the instructional video** for Wednesday, May 13 in GoogleClassroom.

2. (18 min.) Continue drawing "View through a Window": **Develop the imagery seen outside**, with particular attention to **shape relationships, specific shape qualities, and drawing style**.

   Today you will really focus on the imagery outside, bringing your drawing from general to specific. As you work keep objectives and strategies in mind, including the following:

- **Contrast**
  - **Value**: A wide range of values, with varied levels of value contrast, including high, medium and low contrast.
  - **Color**: If using color, use varied levels of contrast. Include contrasting hue, for which you use complementary color relationships, and contrasting saturation levels. Definitely avoid too much pure prismatic color (your kindergarten crayons didn't just get into a big accident), but a bit of prismatic amongst lower saturation colors does perk things up!
  - **Texture**: Implying a variety of textures increases visual interest. Vary the size, thickness, length, shapes, and density of mark-making.
  - **Line**: Varying weight and length of line increases the illusion of space and texture. Contrasting qualities of line, such as mechanical and organic, help distinguish between types of substances or imagery. Implied line and varied line direction or movement add to visual interest and the illusion of space.
  - **Shape and Size**: Most of your shapes will be irregular and even organic. Straight lined shapes will imply man-made objects, and of course your windows and walls will have straight lines. Size communicates the actual size of the object or how far away it is. A variety of sizes in the shape of a composition is helpful for visual interest.
- **Craftsmanship**
  - **Media application and manipulation**: Take the time to apply your media to obtain beautiful physical effects. Attend to the quality of your marks and lines, and to edges and transitions.
- **Emphasis / Focus**
  - **Placement of Emphasis**: Where your points of emphasis are in an image matters. A triangular arrangement of emphasis points creates more compositional stability and keeps the eye moving. Using the rule of thirds to help you place the points of emphasis within the picture planes can increase compositional harmony and open up the space more. Use emphasis to place importance on certain imagery in your composition - be inspired by what you see!
  - **Creating Emphasis**: Contrast, movement, size, negative space and type of imagery all contribute to the creation of emphasis.

Thursday, May 14

1. (20 min.) Continue drawing "View through a Window": Develop **color, value, mark-making** and **linework patterns** in the imagery seen outside.

   a. **Evaluate before starting**: Take a minute to assess the current state of your work and to determine objectives and priorities. These should be based on the assignment objectives, time remaining (early next week), and your assessment of strengths and areas for growth. Make your strengths work for you!

   b. **Complete the scene outside**. Use the advice from yesterday, together with your observation of the outdoors scene, to reach aesthetic and compositional harmony.

   c. **Attend to craftsmanship**: Apply the media to obtain beautiful effects. Be especially attentive to the edges between shapes and overlapping areas, and to transitions between values, textures or colors.

   d. **Visual Interest**: You are telling a story here - use draftsmanship, craftsmanship and imagery to bring your viewer into your world!

Friday, May 15: Use Friday to attend office hours or to catch up on the week's work.

Have a great weekend!

"View through a Window"

Project Overview

Imagery:
For this project you are drawing a view of the world outside as seen from a window of your home. You may extend the concept of the window to a door, balcony or porch, but the core concept is a window. You will include some part of the indoors, if only part of the window frame and window sill or side wall.

Composition:
You have completed visual research and selected a view and composition. Continue to make decisions that create a strong composition through effective use of the principles and elements of design.

Draftsmanship and Style:
Develop strong draftsmanship, showing the specific line and shape qualities of your subjects, overlapping of shapes, and strong positive/negative shape relationships. Apply principles from linear perspective, such as converging parallel lines, and eye-level. Employ keen observation, but you do not need to have a photographic realism style. Your work can be slightly stylized or simplified, though this should not be in a comic strip or manga style or aesthetic. (With more freedom animation styles can be a temptation or a default, so I bring your attention to it now.) Instead, your style should be based on observation of form and specific shape relationships in the subjects before you. A wide variety of styles are possible within these bounds. For acceptable approaches to simplified form, please see pages 6-7.

Picture Plane:
The picture plane for your drawing should take up a full sketchbook page, though if for compositional reasons you need a border along the lower edge you should establish one, making sure it is straight and perpendicular to the side edges. Your drawing should be at least 8x8, but is more likely to be rectangular. Orient your page appropriately - horizontal for the landscape format, vertical for the portrait format.

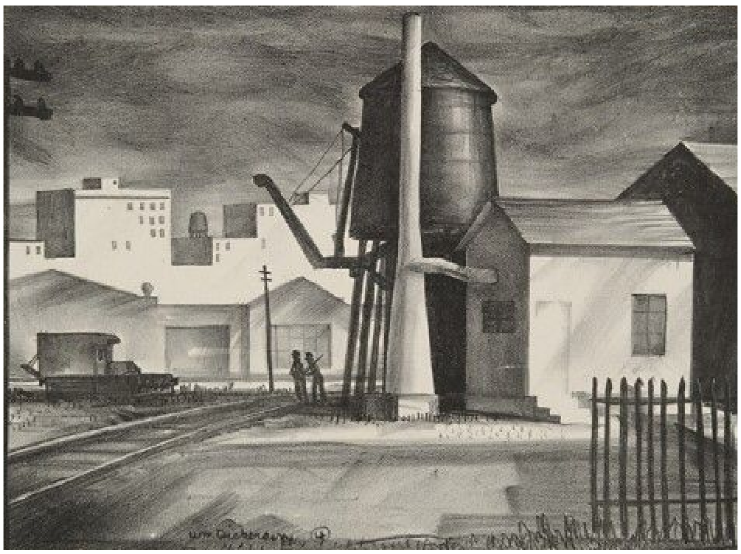
Media:
You will have the opportunity to use dry media of your choice in this project - so colored pencil or pen is also an option (and there are other possibilities depending on what you have at home). Of course, every media requires a certain investment of time and craft, and the pacing of this project is based on pencil. Other media may take longer. As you consider what you would like to use, look back at the examples by the New York artists.

Concept and Expression:
In an excellent artwork, composition, imagery and style should come together to create an individual expression. You might create a particular type of atmosphere or create metaphor. Perhaps you'll include symbolism. Often meaning and expression develop naturally as part of the decision-making process.

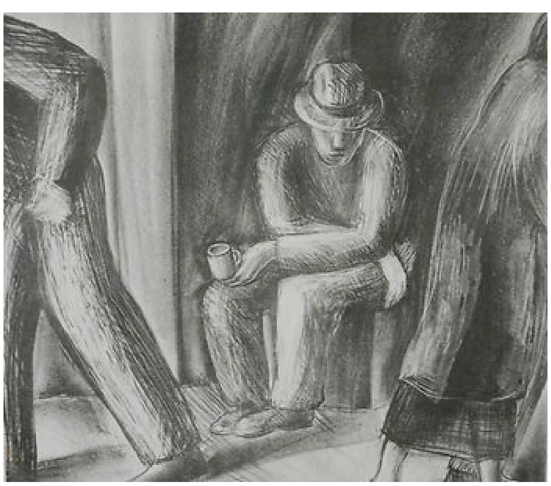
Regarding Simplified Style:

For acceptable approaches to simplified form, I bring your attention to artists from the early 20th century, such as Franz Marc, Georgia O'Keefe, Thomas Hart Benton, or any number of artists from the 1930's.

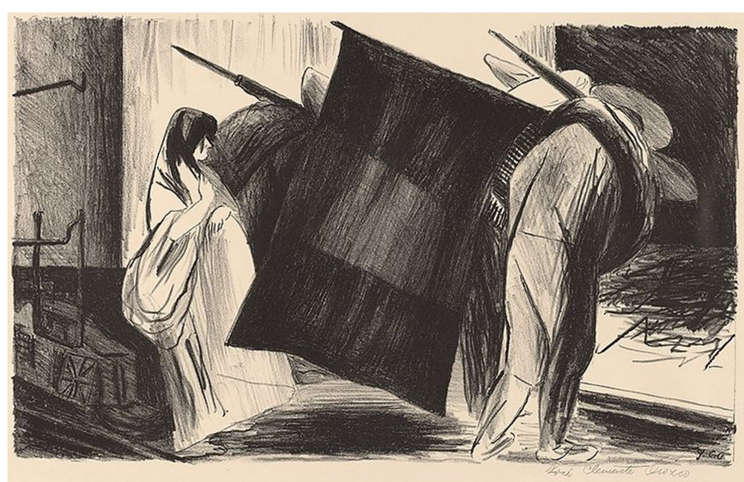
Here are some examples. Remember, I am showing you them as examples of style, not as examples of the project. Look at how **these artists simplified form while capturing the overall energy, lines and shapes in their subjects**. Notice their use of pattern, contrast, movement, and shape overlapping. Notice the way they used block-like forms, but also used **beautiful transitions** with value or mark-making.



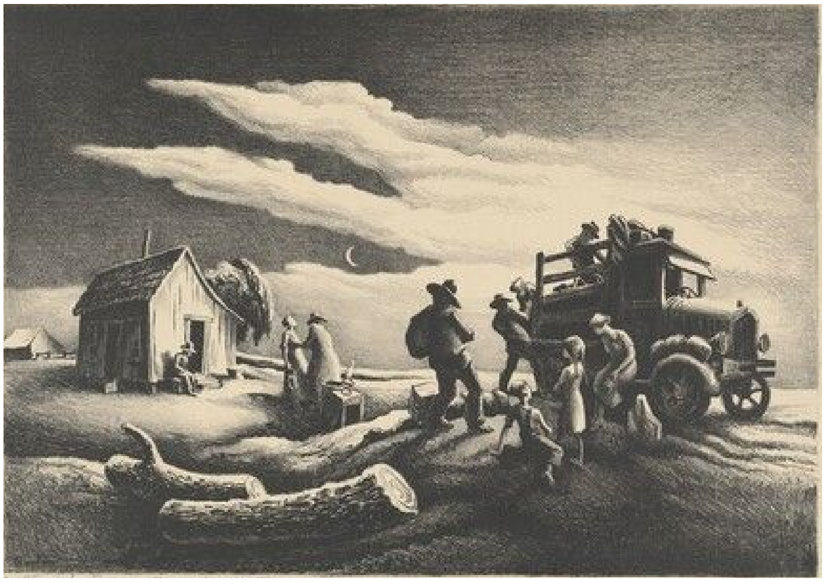
William Judson Dickerson, *Industrial Wichita #4* (1934), lithograph.
Right: Helen Jeanette Noel, depicting Chicago Row Houses in the 1930's

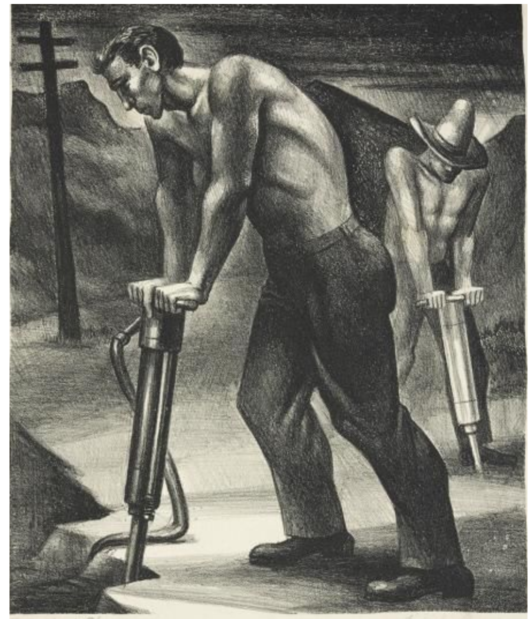


Helen Jeanette Noel, depicting a panhandler on a sidewalk.



Jose Clemente Orozco, *Bandera (Flag)*, lithograph, 1928



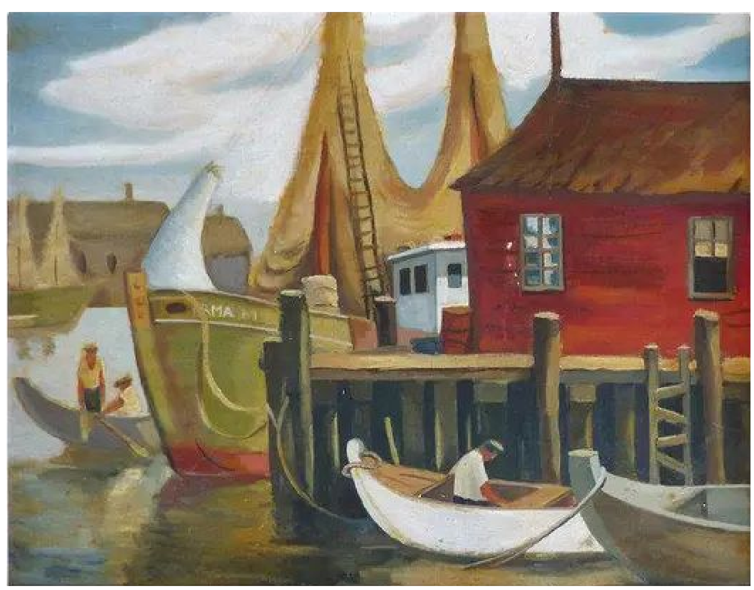


Above: Thomas Hart Benton, *Departure of the Joads*, 1939
Right: Frank Cassara, *Drillers*, 1939

Above: Arthur Runquist, *Lunch*, 1939

Above right: Thelma Johnson Streat, WPA era artist

Right: American School WPA artist, painting of a shipyard



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Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: Calculus I
Teacher(s): Mr. Simmons

Weekly Plan:

Monday, May 11
- [ ] Story time!
- [ ] Check Practice Problems from Week 6
- [ ] Read Example Responses for Week 6
- [ ] Week 7: Practice Problems

Tuesday, May 12
- [ ] Week 7: Conceptual Questions 1-4

Wednesday, May 13
- [ ] Week 7: Conceptual Questions 5-6

Thursday, May 14
- [ ] Week 7: Conceptual Questions 7-9

Friday, May 15
- [ ] Attend office hours
- [ ] Catch up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____
Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____
Parent Signature

Monday, May 11

Happy Monday!

1. If technologically feasible, please send me a story from your life! I love reading them.
2. Check your answer to the practice problems from Week 6.
3. If you find it helpful, read or skim through my example responses for the conceptual questions from Week 6.
4. Complete all the practice problems in this week's review (Week 7).

Tuesday, May 12

1. Answer, in full, complete, grammatical sentences, the first through fourth conceptual questions for Week 7. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone whose only math instruction has been your answers to last week's questions. (If you wish, you may answer these questions together, not in separate answers.)

Wednesday, May 13

1. Answer, in full, complete, grammatical sentences, the fifth and sixth conceptual questions for Week 7. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone whose only math instruction has been your answers to last week's questions. (If you wish, you may answer these questions together, not in separate answers.)

Thursday, May 14

1. Answer, in full, complete, grammatical sentences, the seventh through ninth conceptual questions for Week 7. I will not be surprised if this takes you the whole of today's 40 minutes. As it says in the instructions, you are writing as if you are teaching these concepts to someone whose only math instruction has been your answers to last week's questions. (If you wish, you may answer the first two of these questions together, not in separate answers. Question 9 really ought to be treated separately.)

Week 6 Practice Problems – Answer Key

Calculus I

Mr. Simmons

1. Given $f(x) = 3 - 5x - 2x^2$,

   (a)
   $$\begin{aligned} f(4) &= 3 - 5(4) - 2(4)^2 \\ &= 3 - 20 - 2 \cdot 16 \\ &= -17 - 32 \\ &= -49. \end{aligned}$$

   (b)
   $$\begin{aligned} f(0) &= 3 - 5(0) - 2(0)^2 \\ &= 3 - 0 - 0 \\ &= 3. \end{aligned}$$

   (c)
   $$\begin{aligned} f(-3) &= 3 - 5(-3) - 2(-3)^2 \\ &= 3 + 15 - 2 \cdot 9 \\ &= 18 - 18 \\ &= 0. \end{aligned}$$

   (d)
   $$\begin{aligned} & f(6-t) \\ &= 3 - 5(6-t) - 2(6-t)^2 \\ &= 3 - 30 + 5t - 2\left(36 - 12t + t^2\right) \\ &= -27 + 5t - 72 + 24t - 2t^2 \\ &= -2t^2 + 29t - 99. \end{aligned}$$

   (e)
   $$\begin{aligned} & f(7-4x) \\ &= 3 - 5(7-4x) - 2(7-4x)^2 \\ &= 3 - 35 + 20x - 2\left(29 - 56x + 16x^2\right) \\ &= -32 + 20x - 58 + 112x - 32x^2 \\ &= -32x^2 + 132x - 90. \end{aligned}$$

   (f)
   $$\begin{aligned} f(x+h) &= 3 - 5(x+h) - 2(x+h)^2 \\ &= 3 - 5x - 5h - 2\left(x^2 + 2xh + h^2\right) \\ &= 3 - 5x - 5h - 2x^2 - 4xh - 2h^2. \end{aligned}$$

2. Evaluate $\frac{f(x+h) - f(x)}{h}$ for

   (a) $f(x) = 4x - 9$.
   $$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{[4(x+h) - 9] - (4x - 9)}{h} \\ &= \frac{4x + 4h - 9 - 4x + 9}{h} \\ &= \frac{4h}{h} \\ &= 4 \quad (h \neq 0). \end{aligned}$$

   (b) $f(x) = \frac{2x}{3-x}$.
   $$\begin{aligned} & \frac{f(x+h) - f(x)}{h} \\ &= \frac{\frac{2(x+h)}{3-(x+h)} - \frac{2x}{3-x}}{h} \\ &= \frac{[2(x+h)](3-x) - (2x)[3-(x+h)]}{[3-(x+h)](3-x)} \cdot \frac{1}{h} \\ &= \frac{\left(-2x^2 + 6x - 2xh + 6h\right) - \left(-2x^2 + 6x - 2xh\right)}{[(3-x) - h](3-x)} \cdot \frac{1}{h} \\ &= \frac{6h}{(3-x)^2 - (3-x)h} \cdot \frac{1}{h} \\ &= \frac{6}{(3-x)^2 - (3-x)h} \quad (h \neq 0). \end{aligned}$$

3. Determine the domain of each function:

   (a) $f(x) = 3x^2 - 2x + 1$. The domain of f is the set of all real numbers.

   (b) $f(x) = -x^2 - 4x + 7$. The domain of f is the set of all real numbers.

   (c) $f(x) = 2 + \sqrt{x^2 + 1}$. The domain of f is the set of all real numbers such that $x^2 + 1 \geq 0$, which is all real numbers.

   (d) $f(x) = 5 - |x + 8|$. The domain of f is the set of all real numbers.

4. Find the following limits.

   (a)

$$\begin{aligned}
&\lim_{x\to-5} \frac{x^2 - 25}{x^2 + 2x - 15} \\
&= \lim_{x\to-5} \frac{(x+5)(x-5)}{(x+5)(x-3)} \\
&= \lim_{x\to-5} \frac{x-5}{x-3} \\
&= \frac{(-5)-5}{(-5)-3} \\
&= \frac{-10}{-8} \\
&= \frac{5}{4}.
\end{aligned}$$

   (b)

$$\begin{aligned}
\lim_{x\to1} \frac{x^2 - 1}{x + 1} &= \frac{(1)^2 - 1}{(1) + 1} \\
&= 0.
\end{aligned}$$

   (c)

$$\begin{aligned}
\lim_{x\to2} \frac{x^2 - 4}{x - 2} &= \lim_{x\to2} \frac{(x-2)(x+2)}{x - 2} \\
&= \lim_{x\to2} (x + 2) \\
&= (2) + 2 \\
&= 4.
\end{aligned}$$

   (d)

$$\begin{aligned}
\lim_{x\to3} \frac{x^2 - 4}{x - 2} &= \frac{(3)^2 - 4}{(3) - 2} \\
&= 5.
\end{aligned}$$

   (e)

$$\begin{aligned}
&\lim_{h\to0} \frac{(a+h)^2 - a^2}{h} \\
&= \lim_{h\to0} \frac{a^2 + 2ah + h^2 - a^2}{h} \\
&= \lim_{h\to0} \frac{2ah + h^2}{h} \\
&= \lim_{h\to0} (2a + h) \\
&= 2a + (0) \\
&= 2a.
\end{aligned}$$

5. Let $f(x) = \begin{cases} 7 - 4x & \text{if } x < 1 \\ x^2 + 2 & \text{if } x \geq 1 \end{cases}$. Find the following limits:

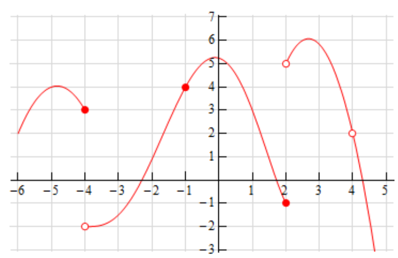
   (a)

$$\begin{aligned}
\lim_{x\to-6} f(x) &= \lim_{x\to-6} (7 - 4x) \\
&= 7 - 4(-6) \\
&= 31.
\end{aligned}$$

   (b)

$$\begin{aligned}
\lim_{x\to1} f(x) &= \lim_{x\to1} \left(x^2 + 2\right) \\
&= (1)^2 + 2 \\
&= 3.
\end{aligned}$$

6. In the function graph below, determine where the function is discontinuous.



   A function is continuous at x if the limit at x equals the value at x. This function is discontinuous at -4 and 2 because the limit does not equal the value, and it is discontinuous at 4 because there it is undefined.

Week 6 Conceptual Questions – Example Responses

Calculus I
Mr. Simmons

There's not just one right answer these questions. Any answers that clearly conveys the concepts such that a non-math person could understand them is a good answer. Below are my example answers to each of the conceptual questions from Week 6.

1. In your own words, what is a function?

2. In your own words, what is a domain?

   Example. Functions are the main object of study in Calculus, but *function* isn't fundamentally a mathematical idea. The length of a person's shadow is a function of their height. When you're driving a car, your location is a function of time. The temperature outside is a function of variables in the weather, like whether clouds are covering up the sun. (*Variable* is also not fundamentally a mathematical idea. It just means something that can change, or vary.) One last example: the amount of money you have is a function of, among other things, the time you've spent earning it or spending it. In general, one thing being a function of another thing just means that the one is dependent on the other. Functions are relations of dependency.

   Where do functions fit into math? In pure math, we deal with abstract variables. Instead of talking about money as a function of time, we just talk about one variable as a function of another variable, and usually the variables are quantifiable, meaning we can assign them a number. And, so that we don't have to keep saying "this variable" or "that variable," we give them names. The names don't necessarily say anything about what the variables represent; they're just names... like proper names. The name "Simmons" doesn't tell you anything about me, really... except maybe that my ancestors were European. In math, if one variable is a function of another, we usually call the first one y and the second one x. (So, I guess the names x and y tell you that the ancestry of these variables is that they were named by a mathematician who likes conventional symbols.)

   One example of a function in pure math is the one represented by the equation

   $$y = 2x.$$

   (An equation is just a statement saying two things are actually just one and the same thing.) This equation says that y is actually just x taken two times. The relation between x and y here is a function. The variable y is dependent on x. It is a function of x. If x is 2, then y

has to be 4. If x is 10, then y has to be 20. We could call this the doubling function, because the dependent variable is double the independent variable.

We could even say that this function doubles things, as if the function is commiting an action. It takes inputs and doubles them to get outputs. The language of inputs and outputs is just another way of talking about functions. It makes them a bit more active, like they're doing something. But we're still just talking about a relation of dependency. The output is dependent on the input.

Finally, sometimes it's helpful to talk about what possible inputs a function can take. If temperature is a function of variables in the weather, what are those variables? (I'm not a meteorologist, so I sincerely don't know.) If money is a function of time, what are the possible inputs for the variable called "time"? Can it have negative numbers for inputs? Can it have fractions of numbers for inputs? We call the collection of all possible inputs to a function its "domain." *Domain* is also not fundamentally a mathematical concept. A king's domain is the land that he has power over. A function's domain is the same thing: it's the land that the function has power over. It's the collection of all things that the function could possible take as an input. You could say that a good king is dependent on his people; a function is dependent on its domain.

In math, we study specifically functions between abstract, variable quantities, but the fundamental idea is still the same as functions in everyday life: a function is a relation of dependency.

3. Exactly how are functions represented by equations?

   Example. If we have the equation $y = 2x$ and we plug in a number for x, there's only one possible number that y could be that could make the equation true. For example, if we plug in 2 for x, y would have to be 4. If we plug in 5 for x, y would have to be 10. For the doubling function to be represented by the equation $y = 2x$ means that it relates inputs that can be plugged in for x to the resulting out values that y would then have to be. So it relates 2 to 4 and 5 to 10, for the exact reasons that plugging 2 in for x yields 4 for y and plugging 5 in for x yields 10 for y, as described above. In general, for a equation to represent a function means that the function relates a given input value to a unique output value if and only if plugging the input value in for x in the equation makes it so that the one and only number that can be plugged in for y in the equation to make the equation true is the output value.

4. Exactly how are functions represented by graphs?

   Example. If a function relates real numbers to real numbers, it can be graphed on a cartesian coordinate plane. Draw a horizontal line and a vertical line and call them axes (singular *axis*). Their point of intersection is called the origin. Now every location on your piece of paper can be assigned a unique pair of numbers as a name based on its location relative to this origin. This unique pair of numbers comes simply from listing that location's horizontal distance from the origin next to its vertical distance from the origin, separated by a comma, all within parentheses. For example, the origin itself has the name $(0, 0)$, since it's zero units distant from itself horizontally and zero units distant from itself vertically. Draw a tick mark on the horizontal line exactly one unit to the right of the origin. (I say "unit" because we're not using this graph to represent any particular physical distance, but rather an abstract distance. If it makes it easier, replace every instance of the word *unit* in this paragraph with the word *inch.*) The point where that tick mark intersects the horizontal axis is called $(1, 0)$ because it is one unit to the right of the origin but not at all vertically distant from it. If you move exactly one unit up from there, you'd get the point $(1, 1)$. To distinguish the point that's one unit to the right of the origin from the one that's one unit to the left, we use negative numbers to mean leftward horizontal distance or downward vertical distance, so that the point one unit to the left of and one unit down from the origin will be called $(-1, -1)$.

That's a cartesian coordinate plane. But what does it have to do with functions? It seems we can represent a pair of numbers visually now, but is a function a pair of numbers?

Actually yes! Kind of. A function is a collection of pairs of numbers. We've described a function before now as taking inputs (maybe infinitely many inputs) and giving outputs, but each time that process happens, we can, as with the points on the plane, assign it a name, like $(0, 0)$. The doubling function would then have the pairs $(0, 0)$, $(1, 2)$, $(2, 4)$, $(3, 6)$, $(-2, -4)$, $(3.142, 6.283)$, and infinitely many more, since you can double any real number. If we fill in each of these points on the cartesian coordinate plane… voila! We've represented the doubling function. (Of course it's impossible to put your pencil down on paper and then pick it up again an infinite number of times, so we cheat by just drawing a line segment with arrows for endpoints… but there are some functions that actually cannot be drawn, because you really would have to put your pencil down and pick it up again an infinite number of times, for example the function whose inputs are real numbers and whose output for a rational input is 1 and whose output for an irrational numbers is 0.)

In general, if we consider a function to be a set of ordered pairs of the form (x, y), where x is an input and y is its unique output, then we can graph any function by filling in on the cartesian coordinate plane all the points whose names (which are pairs of numbers) are in the function.

[You might also mention and describe the vertical and horizontal line tests.]

5. In your own words, what is a limit?

Example. A limit is the tendency of a function. If distance driven is a function of time, then the tendency is for the distance to increase. As time passes, distance increases. Since the output is dependent on the input, limits will always state the tendency of an output as being dependent on the tendency of the input. For $y = 1/x$, as x gets bigger and bigger without bound, y gets closer and closer to 0. As x gets closer and closer to 0 from the positive side (perhaps with values like 1, 1/2, 1/4, 1/8, etc.), y gets bigger without bound. If x approaches 0 again but this time from the negative side of things (graphically the lefthand side of the origin), y gets bigger in the negative (downward) direction without bound. We would say that the limit of y as x approaches infinity is 0, the limit of y as x approaches 0 from the right is positive infinity, and the limit of y as x approaches 0 from the left is negative infinity. If $y = 1/x^2$, since y would approach positive infinity whether x is approaching 0 from the right or from the left, we say simply that y approaches infinity as x approaches 0 (without specifying from which direction). And limits don't have to include infinity: if $y = 2x$, then the limit of y as x approaches 5 is 10. These five statements can be stated symbolically as follows:

$$\lim_{x\to\infty}\frac{1}{x} = 0; \quad \lim_{x\to 0^+}\frac{1}{x} = \infty; \quad \lim_{x\to 0^-}\frac{1}{x} = -\infty; \quad \lim_{x\to 0}\frac{1}{x^2} = \infty; \quad \lim_{x\to 5} 2x = 10.$$

The first of these would be read, "The limit of $1/x$ as x approaches infinity is 0."

We've gotten by with minimal notation until now, but I think it will be helpful to introduce some function notation. If we name a function f, its output can be written as $f(x)$ ("f of x" or "the value of f at x"). So the function equations $y = 2x$, $y = 1/x$, and $y = 1/x^2$ (and many others) have the general equation $y = f(x)$.

So, informally, "$\lim_{x\to c} f(x) = L$" ("the limit of $f(x)$ as x approaches c is L" or "the limit of f near c is L") means that $f(x)$ can be made arbitrarily close to L by making x sufficiently close to, but not equal to, c. When dealing with infinity, simply replace "close to L" or "close to c" with "bigger or smaller without bound." Part of what's so powerful about limits is that phrase "but not equal to." If we had to make x equal to c, then we wouldn't be able to find the limit $\lim_{x\to 0} 1/x$, because $1/0$ is undefined.

To formalize concepts like "aribitrary" and "sufficient," mathematicians use the Greek letter ε ("epsilon") to denote an arbitrarily small quantity, and the Greek letter δ ("delta") to denote a sufficiently small quantity. Then, formally, "$\lim_{x\to c} f(x) = L$" means that given any arbitrarily small but still positive ε, we can find a sufficiently small positive δ such that all the x values within distance δ of c (but not c itself) will have resultant $f(x)$ values within distance ε of L. Still more concisely,

> "$\lim_{x\to c} f(x) = L$" means that for any $\varepsilon > 0$, there exists a $\delta > 0$ such that, for all x, $0 < |x - c| < \delta$ implies $|f(x) - L| < \varepsilon$

($|x|$ being the "absolute value" function that outputs the positive version of the input, helpful for stating distances since they are always positive).

I would love to draw a diagram to make this concept more clear, Mr. Simmons, but we drew a million such diagrams in class and I'm typing on a computer so it would be really hard.

6. In your own words, what is a continuity?

Example. Continuity is when the graph of a function is smooth. All the functions I've mentioned have been continuous, except the one I mentioned in parentheses as being undrawable. If you can graph a function without picking your pencil up, it's continuous, even if you have to use arrow symbols to indicate tendencies toward infinity (as with all the continuous functions we've mentioned).

Another way to phrase it is that a continuous graph doesn't have any jumps or holes. We call those behaviors "discontinuities." Discontinuities happen when a function's behavior changes suddenly; that is, when its tendency, whose entire job is to tell you where you're going, fails at its job. When you're driving a car, the tendency is for distance driven to increase as time increases, and this tendency can be seen as a way of guessing where the car will be in the near future. If the car were to suddenly come to a halt (I'm sure you can imagine some reasons for this happening), the tendency of the time–distance function would have failed to predict what the function actually did. That would be an example of a jump discontinuity. A hole discontinuity might occur if you suddenly vanished from the earth, since then your position would be undefined.

We defined a limit as a function's tendency. So a function is continuous if its limit, which tells you where you're going, is right. I see no need for further delay: formally, a function f is continuous at c if and only if

$$\lim_{x\to c} f(x) = f(c),$$

and a function is a continuous function if it is continuous at every point of its domain.

(A confusing note: the function defined by the equation $f(x) = 1/x$ is continuous, even though it is not continuous at $x = 0$, for the reason that 0 is not in its domain. You might say, or hear it said, that this function is discontinuous at 0, or that it has a discontinuity at 0. I suppose that's fine, since it's certainly not continuous at 0. But the only way to get a function to be not continuous is to have a jump discontinuity, a place in the domain where the function is defined but where the value at that point is not equal to the limit near that point.)

Continuous functions are nice because they are smooth, but they are not always as smooth as we'd like. Consider the absolute value function. The limit near 0 is 0 from both sides, yet the graph is not smooth there: there's a sort of "corner." We'll have to wait for derivatives to be able to identify the smoothest kinds of functions.

Week 7: Derivatives

Mr. Simmons
Calculus I

Practice Problems

Differentiate the following functions.

1. $f(x) = 6x^3 - 9x + 4$
2. $f(x) = 2x^4 - 10x^2 + 13x$
3. $f(x) = 4x^7 - 3x^{-7} + 9x$
4. $f(x) = \sqrt{x} + 8\sqrt[3]{x} - 2\sqrt[4]{x}$
5. $f(x) = \frac{4}{x} - \frac{1}{6x^3} + \frac{8}{x^5}$
6. $f(x) = \left(4x^2 - x\right)\left(x^3 - 8x^2 + 12\right)$
7. $f(x) = \frac{6x^2}{2-x}$
8. $f(x) = \left(6x^2 + 7x\right)^4$
9. $f(x) = \left(4x^2 - 3x + 2\right)^{-2}$
10. $f(x) = \sqrt[3]{1 - 8x}$
11. $f(x) = \frac{\sin(3x)}{1+x^2}$

Conceptual Questions

Answer the following questions in your own words. Try to avoid using symbols to the extent possible. Instead, write in full, complete, grammatical sentences. Answer these questions as if you're teaching these concepts to someone whose only math training has been to read and understand your answers to the conceptual questions from last week. That might mean giving examples, counterexamples, or analogies, for example. If you use any notation, it means explaining that notation, unless you explained it last week. (This is the most important part of the review.)

1. What is a slope?
2. What is a rate of change?
3. What is an average rate of change?
4. What is an instantaneous rate of change?
5. What is a derivative?
6. What is differentiation? (This question in particular you might be tempted to answer with only one sentence. That would be like answering "What is labor?" with "work; especially hard, physical work," without giving any examples like yardwork, carpentry, coal mining, factory work, etc. Be a teacher. Give examples. Name some known derivatives. I won't accept "finding a derivative" as a complete answer for a topic we spent multiple weeks on.)
7. What is an implicitly defined function?
8. What is implicit differentiation?
9. What is a differential?

GreatHearts
Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: 11th Drama
Teacher(s): Mrs. Jimenez (margaret.cousino@greatheartsirving.org)

Weekly Plan:

Monday, May 11
- [ ] Prepare private performance
- [ ] Practice lines for 20 minutes if not in rehearsal

Tuesday, May 12
- [ ] Act 4 Zoom Rehearsal at 11am *OR*
- [ ] Practice lines for 20 minutes

Wednesday, May 13
- [ ] Act 5 Zoom Rehearsal at 11am *OR*
- [ ] Practice lines for 20 minutes

Thursday, May 14
- [ ] Prepare for next week's Zoom performance! (see checklist)
- [ ] Practice lines for 20 minutes if not in rehearsal

Friday, May 15
- [ ] Attend office hours
- [ ] Catch-up or review the week's work
- [ ] Scan and submit work by Sunday, May 17

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 11

1. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to.

2. REMEMBER: Private performance recordings are due by Wednesday, May 27.

3. SAVE THE DATE: We will be recording the play via Zoom on Friday, May 22 from 2-5pm

Tuesday, May 12

1. Actors in Act 4 need to attend today's Zoom rehearsal. Zoom link can be found on Google Classroom.

2. Characters in Act 4: Student director, Clown, Sebastian, Maria, Sir Toby, Malvolio, Sir Andrew, Olivia

3. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to. Everyone still has to submit a line memorizing sheet, however.

Wednesday, May 13

1. Actors in Act 5 need to attend today's Zoom rehearsal. Zoom link can be found on Google Classroom.

2. Characters in Act 5: Student director, Fabian, First officer, Second officer, Malvolio, Clown, Orsino, Viola, Antonio, Olivia, Priest, Sir Andrew, Sir Toby, Sebastian

3. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to. Everyone still has to submit a line memorizing sheet, however.

Thursday, May 14

1. If you're not attending either rehearsal this week, practice your lines for 20 minutes. Focus on mastering the lines for your private performance. If you are attending rehearsals, no need to.

2. Be prepared for next week's Zoom performance on Friday, May 22, 2-5pm. Go through the following checklist:

   - [ ] I have found all my costume pieces and they are ready for filming
   - [ ] I have found/made all my props and they are ready for filming
   - [ ] I will do my hair (all actors) and make-up (girls only; boys don't worry)
   - [ ] I have a quiet place to film with a white/beige/light-colored wall I can use for the background
   - [ ] I have a device (preferably a laptop or computer) I can use for filming
   - [ ] I will have a stable internet connection so I can use video/audio
   - [ ] My lines and character development are ready for the stage! (Or camera, in our case)

Friday, May 15

1. Finish any work from this week that you need to.
2. If you haven't turned in your parental permission slip for recording, ***please do so ASAP.***
3. Attend office hours at 11am if you have questions.

Drama Weekly Line Memorization

NB: If you are attending Zoom rehearsals this week and so are exempt from the 20 mins a day, please write "Zoom rehearsal" by the day(s) you attended and still submit this through Google Classroom so I have a record. Thanks!

Name: **Week: 5/11-5/17**

Day:	Minutes practiced:
Monday	
Tuesday	
Wednesday	
Thursday	
Friday	
Saturday	
Sunday	

Minimum time: 20 mins/day, 4 days/week

I verify that this is a true and accurate account of the time I have spent memorizing my lines this past week.

Signature: **Date:**

Recording Permission Slip

GreatHearts Irving

Dear Great Hearts Parent/Guardian -

Instead of putting on a live production for the Spring 2020 Junior Drama class, we will be recording a virtual performance over Zoom and students will be recording some of their own lines for a private performance at home. These may be used on social media or shared with the Great Hearts Irving community.

Please indicate your consent below:

☐ I give my consent for my child to participate in these recordings of the Great Hearts Irving Junior Play: William Shakespeare's *Twelfth Night* and for their part to be shared on social media.

☐ I give my consent for my child to participate in these recordings of the Great Hearts Irving Junior Play: William Shakespeare's *Twelfth Night* but do NOT want their part to be shared on social media.

PRINT Student full name: ____.

PRINT Parent/Guardian name: ____.

SIGN Parent/Guardian name: ____.

Today's date:_________________________.

GreatHearts Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: 11 Greek
Teacher(s): Miss Salinas annie.salinas@greatheartsirving.org

Weekly Plan:

Monday, May 11
☐ Optional video on Ch 9 vocab
☐ Make Ch 9α vocab flashcards

Tuesday, May 12
☐ Optional intro video to this week's story
☐ Translate lines 1-26 of 9α

Wednesday, May 13
☐ Translate lines 27-54 of 9α

Thursday, May 14
☐ Translate lines 55-72 of 9α

Friday, May 15
☐ attend office hours
☐ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____
Student Signature

____
Parent Signature

χαιρετε φιλοι! Believe it or not, this is our second-to-last full week of schoolwork. We've spent a solid few months in Chapter 8, and this week we'll be bravely forging into Chapter 9 for some new vocab and a new passage.

Monday, May 11

Optional: watch today's video, in which I'll read aloud and discuss our vocab for this chapter. (I promise my videos are shorter this week!)

Make vocab flashcards or a vocab list for Chapter 9α. If you make flashcards, you can submit them in a picture

Tuesday, May 12

Optional: watch today's video introducing our 9α passage. I'll be explaining our new grammar for this chapter, which we won't be focusing on this week, but will be useful for translation.

After many weeks away, we're back to Dikaiopolis and fam! I've glossed the story (hopefully) thoroughly enough that it'll be a challenge, but not *too* much of a challenge.

Translate lines 1-26 of 9α, provided for you as part of this packet

Wednesday, May 13

Translate lines 27-54 of our story, in which the family reaches the top of the Acropolis, explores the Pantheon, and prays to Athena.

Thursday, May 14

Translate lines 55-72 of our story, in which the family finds their grandfather again and decides what to do next.

Friday, May 15

Have questions about the week's work? Want to go over something to make sure you got it? Just simply desperate to converse with someone you're not related to? Come to Greek Office Hours at 10:30am - link available in the stream of our Google Classroom! See you there.

Monday

Chapter 9α vocab

Feel free to use this space to write your vocab list for Chapter 9α; alternatively, feel free to do it on a sheet of notebook paper or on flashcards.

Greek (don't forget your breathing marks!)	English	derivatives/mnemonics

Tuesday-Thursday

Chapter 9α passage

Feel free to use this space to write your translation for lines 1-72 of Chapter 9α; alternatively, feel free to do so on a sheet of notebook paper.

55

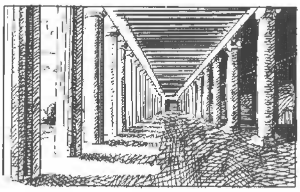
60

65

70

Ὁρῶσι τὴν εἰκόνα τῆς Ἀθηνᾶς, ἐνοπλίου οὔσης καὶ Νίκην τῇ δεξιᾷ φερούσης.
They see the image of Athena, being armed and carrying Nike in her right hand.

ἡ Νίκη
(τῆς Νίκης)
Nike, the winged goddess of victory



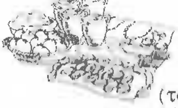
ἡ στοά
(τῆς στοᾶς)
the stoa (colonnade, pillared walkway)

σπεύδων (*m.*)
σπεύδουσα (*f.*)
σπεῦδον (*n.*)
(*gen.*: σπεύδοντος, σπευδούσης, σπεύδοντος)

βοῶν (*m.*, < βοά-ων)
βοῶσα (*f.*, < βοά-ουσα)
βοῶν (*n.*, < βοά-ον)
(*gen.*: βοῶντος, βοώσης, βοῶντος)

ποιῶν (*m.*, < ποιέ-ων)
ποιοῦσα (*f.*, < ποιέ-ουσα)
ποιοῦν (*n.*, < ποιέ-ον)
(*gen.*: ποιοῦντος, ποιούσης, ποιοῦντος)

ὁ ἀλλᾱντοπώλης
(τοῦ ἀλλᾱντοπώλου)
the sausage-seller



τὰ ὤνια
(τῶν ὠνίων)
things for sale

Η ΠΑΝΗΓΥΡΙΣ (α)

the public gathering (for a religious festival)

πορευομαι = I go; -μενος = -ing

Οὕτως οὖν πορευόμενοι ἀφικνοῦνται εἰς τὴν ἀγοράν. Ἐκεῖ δὲ τοσοῦτός ἐστιν ὁ ὅμῑλος ὥστε μόλις προχωροῦσι πρὸς τὴν Ἀκρόπολιν. Τέλος δὲ τῷ Δικαιοπόλιδι ἑπόμενοι εἰς στοάν τινα ἀφικνοῦνται, καὶ καθιζόμενοι θεῶνται τοὺς ἀνθρώπους σπεύδοντας καὶ βοῶντας καὶ θόρυβον ποιοῦντας.

thus; so great; crowd; that; hardly
ἕπομαι = I lead; -μενος = -ing
a certain stoa (see picture)
καθιζομαι = I sit down; -μενος = -ing
they see
σπευδω = I hurry; -ων, -οντος = -ing
noise

Ἤδη δὲ μάλα πεινῶσιν οἱ παῖδες. Ὁ δὲ Φίλιππος ἀλλᾱντοπώλην ὁρᾷ διὰ τοῦ ὁμίλου ὠθιζόμενον καὶ τὰ ὤνια βοῶντα. Τὸν οὖν πατέρα καλεῖ καί, «πάππα φίλε,»

they are hungry
a sausage-seller
crowd; shoving his way; wares, things for sale

ὁ ἀλλᾱντοπώλης διὰ τοῦ ὁμίλου ὠθίζεται
the sausage-seller shoves his way through the crowd

ἐνόπλιος, ἐνόπλιον *armato*
ἡ πανήγυρις *l'adunanza pubblica (per una festa religiosa)*
πεινάω *ho fame*
ὠθίζομαι *mi fo largo a spintoni*

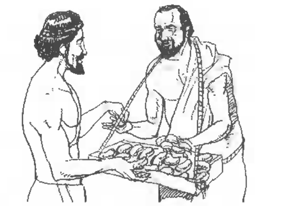
φησίν, «ἰδού, ἀλλᾱντοπώλης προσχωρεῖ.
a sausage-seller
Ἆρ᾿οὐκ ἐθέλεις σῖτον ὠνεῖσθαι; Μάλα
to buy
γὰρ πεινῶμεν.» Ὁ οὖν Δικαιόπολις τὸν
we are hungry
ἀλλᾱντοπώλην καλεῖ καὶ σῖτον ὠνεῖται.
buys
Οὕτως οὖν ἐν τῇ στοᾷ καθίζονται
ἀλλᾶντας ἐσθίοντες καὶ οἶνον πίνοντες.
ἐσθιω = I eat; -ων, -οντος = -ing wine πινω = I drink; -ων, -οντος = -ing

Μετὰ δὲ τὸ δεῖπνον ὁ Δικαιόπολις,
After
«ἄγετε,» φησίν, «ἆρ᾿οὐ βούλεσθε ἐπὶ τὴν
Come on
Ἀκρόπολιν ἀναβαίνειν καὶ τὰ ἱερὰ
to go up the temples
θεᾶσθαι;» Ὁ μὲν πάππος μάλα κάμνει
to see
καὶ οὐκ ἐθέλει ἀναβαίνειν, οἱ δὲ ἄλλοι
the others
λείπουσιν αὐτὸν ἐν τῇ στοᾷ καθιζόμενον
leave
καὶ διὰ τοῦ ὁμίλου ὠθιζόμενοι ἐπὶ τὴν
crowd shoving their way
Ἀκρόπολιν ἀναβαίνουσιν.

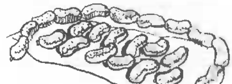
Ἐπεὶ δὲ εἰς ἄκρᾱν τὴν Ἀκρόπολιν
the top
ἀφικνοῦνται καὶ τὰ προπύλαια
the Proylaea (see picture)
gen
διαπερῶσιν, τὸ τῆς Παρθένου ἱερὸν
they cross over temple
gen
ὁρῶσιν ἐναντίον καὶ τὴν τῆς Ἀθηνᾶς
opposite of Athena
εἰκόνα, μεγίστην οὖσαν, ἐνόπλιον καὶ
image/icon very large being armed
dat of means φερω = I carry; -ουσα, -σης = -ing
δόρυ δεξιᾷ φέρουσαν. Πολὺν οὖν χρόνον
a spear carrying
ἡσυχάζουσιν οἱ παῖδες τὴν θεὸν θεώμενοι,
they stay quiet

πίνω *bevo*
ἄγε, (plur.) **ἄγετε** *suvvia!, ovvia!, avanti!*
ἐναντίος, **ἐναντίᾱ**, **ἐναντίον** *(che si trova) dirimpetto*

τὸ δόρυ (τοῦ δόρατος)
spear

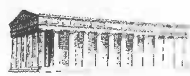


Dikaiopolis buys the sausages
ὁ Δ. ὠνεῖται τοὺς ἀλλᾶντας



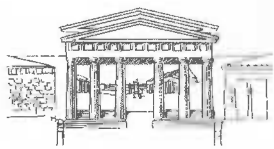
ἐσθίων (*m.*)
ἐσθίουσα (*f.*)
ἐσθίον (*n.*)
(*gen.*: ἐσθίοντος, ἐσθιούσης, ἐσθίοντος)

οἱ ἀλλᾶντες (ὁ ἀλλᾶς, τοῦ ἀλλᾶντος)
the sausages



πίνων (*m.*)
πίνουσα (*f.*)
πῖνον (*n.*)
(*gen.*: πίνοντος, πῑνούσης, πίνοντος)

τὸ ἱερόν (τοῦ ἱεροῦ)
the temple



τὰ προπύλαια (τῶν προπυλαίων)
the Propylaea (massive gateway to the Athenian acropolis)

ὤν (*m.*)
οὖσα (*f.*)
ὄν (*n.*)
(*gen.*: ὄντος, οὔσης, ὄντος)
< εἰμι

φέρων (*m.*)
φέρουσα (*f.*)
φέρον (*n.*)
(*gen.*: φέροντος, φερούσης, φέροντος)

ἡ τῆς Ἀθηνᾶς εἰκών (τῆς εἰκόνος)
the image of Athena

τέλος δὲ ὁ Δικαιόπολις, «ἄγετε,» φησίν,
«ἆρ' οὐ βούλεσθε τὸ ἱερὸν θεᾶσθαι;» Καὶ
the temple
ἡγεῖται αὐτοῖς πόρρω.
leads onwards

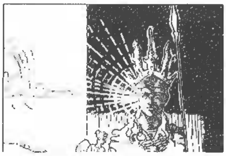
Μέγιστόν ἐστι τὸ ἱερὸν καὶ κάλλιστον.
Very large very beautiful
Πολὺν χρόνον τὰ ἀγάλματα θεῶνται, ἃ
the statues which
τὸ πᾶν ἱερὸν κοσμεῖ. Ἀνεῳγμέναι εἰσὶν
make the whole temple beautiful opening
αἱ πύλαι· ἀναβαίνουσιν οὖν οἱ παῖδες καὶ
walk up
εἰσέρχονται. Πάντα τὰ εἴσω σκοτεινά
walk in the things inside dark
ἐστιν, ἀλλ' ἐναντίαν μόλις ὁρῶσι τὴν τῆς
opposite hardly
Ἀθηνᾶς εἰκόνα, τὸ κάλλιστον ἔργον τοῦ
statue/image most beautiful work
gen dat of means
Φειδίου. Ἡ θεὸς λάμπεται χρυσῷ, τῇ μὲν
of Pheidias (sculptor) gleams with gold
δεξιᾷ Νίκην φέρουσα τῇ δὲ ἀριστερᾷ τὴν
Nike carrying
ἀσπίδα. Ἅμα τ' οὖν φοβοῦνται οἱ παῖδες
shield Together
θεώμενοι καὶ χαίρουσιν. Ὁ δὲ Φίλιππος
dat. ind. obj.
προχωρεῖ καὶ τὰς χεῖρας ἀνέχων τῇ θεῷ
his hands lifting
εὔχεται· «Ὦ Ἀθηνᾶ Παρθένε, παῖ Διός,
prays child of Zeus
+gen
πολιοῦχε, ἵλεως ἴσθι καὶ ἄκουέ μου
patron goddess gracious/favorable
εὐχομένου· σῷζε τὴν πόλιν καὶ σῷζε ἡμᾶς
praying save
ἐκ πάντων κινδύνων.» Ἐνταῦθα δὴ πρὸς
from dangers At that very moment
τὴν Μέλιτταν ἐπανέρχεται καὶ ἡγεῖται
he comes back
αὐτῇ ἐκ τοῦ ἱεροῦ.

πόρρω ↔ ὄπισθεν

τὸ ἄγαλμα (τοῦ ἀγάλματος) = ἡ εἰκών

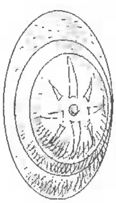
κοσμέω (< κόσμος) : καλὸν ποιέω

σκοτεινός, -ή, -όν < σκότος



ἡ θεὸς λάμπεται

ὁ Φειδίας (τοῦ Φειδίου)



ἡ ἀσπίς (τῆς ἀσπίδος)

ἀνέχω = αἴρω

πολιοῦχος, -ον : ἡ Ἀθηνᾶ πολιοῦχός ἐστιν = ἡ Ἀ. τὴν πόλιν φυλάττει

ἀνέχων (*m.*) ἀνέχουσα (*f.*) ἀνέχον (*n.*) (*gen.*: ἀνέχοντος, ἀνεχούσης, ἀνέχοντος)

ἅ *che*
ἀνεῳγμέναι *aperte*
τὰ εἴσω *l'interno, le parti interne*

κάλλιστος, καλλίστη, κάλλιστον *bellissimo, il più bello*

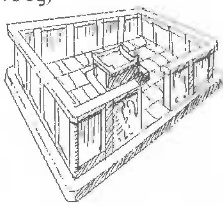
Πολύν τινα χρόνον τοὺς τεκόντας
their parents
ζητοῦσιν, τέλος δὲ εὑρίσκουσιν αὐτοὺς
they search for they find
+gen
ὄπισθεν τοῦ ἱεροῦ καθορῶντας τὸ τοῦ
behind looking down
Διονύσου τέμενος. Ὁ δὲ Δικαιόπολις,
sacred ground
«ἰδού, ὦ παῖδες,» φησίν, «ἤδη
συλλέγονται οἱ ἄνθρωποι εἰς τὸ τέμενος.
are reuniting sacred ground
Καιρός ἐστι καταβαίνειν καὶ ζητεῖν τὸν
The right time to search for
πάππον.»

Καταβαίνουσιν οὖν καὶ σπεύδουσι
πρὸς τὴν στοάν· ἐκεῖ δὲ εὑρίσκουσι τὸν
when they find
πάππον ὀργίλως ἔχοντα. «Ὦ τέκνον,»
having anger (he's angry) children
φησίν, «τί ποιεῖς; Διὰ τί με λείπεις
Why did you leave
τοσοῦτον χρόνον; Διὰ τί τὴν πομπὴν οὐ
so much Why the procession
θεώμεθα;» Ὁ δὲ Δικαιόπολις, «θάρρει,
cheer up
πάππα,» φησίν· «νῦν γὰρ πρὸς τὸ τοῦ
Διονύσου τέμενος πορευόμεθα· δι᾽ὀλίγου
sacred ground
γὰρ γίγνεται ἡ πομπή. Ἄγε δή.» Οὕτω
is happening the procession
λέγει καὶ ἡγεῖται αὐτοῖς πρὸς τὸ τέμενος.
leads

οἱ τεκόντες, τῶν τεκόντων (< τίκτω) : ὁ πατὴρ καὶ ἡ μήτηρ

καθ-οράω

τὸ τέμενος (τοῦ τεμένους)



ὁ πάππος ὀργίλως ἔχει

ὀργίλως ἔχω = ἀγανακτέω

ἡ πομπή (τῆς πομπῆς)

συλλέγομαι *mi raccolgo, mi riunisco*

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: 11 Humane Letters

Teacher(s): Mr. Brandolini david.brandolini@greatheartsirving.org

Mr. Mercer andrew.mercer@greatheartsirving.org

Weekly Plan:

Monday, May 11

☐ Read and annotate *Job* 1-14

Tuesday, May 12

☐ Read and annotate *Job* 15-28

Wednesday, May 13

☐ Read and annotate *Job* 29-42

☐ Prepare for seminar

Thursday, May 14

☐ Attend Zoom seminar on *Job* readings

☐ Make at least one edit or addition to your Project Essay

Friday, May 15

☐ Attend office hours (optional)

☐ Catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 11

This week we are making our final transition for the year through an encounter with the Old Testament book of *Job*. This ancient work, which likely predates most of the Greek literature we read this year, gives us a glimpse into early human attempts to grapple with some of life's most difficult questions. How do we make sense of what seems like unjust suffering, and what role does the divine play in the drama of human suffering? *Job* has been the source of many reflections on these and other related questions for many centuries and continues to serve as a key text for philosophers, theologians, and artists, whose ranks you may join this week as you wrestle with the reading. It holds a special place in the religious Scriptures of most Christian and Jewish traditions.

Because the dialogue in the book of *Job* is dense and requires careful attention, and in order to make the most of our seminar time this week, we will spend Monday, Tuesday, and Wednesday of this week becoming as familiar as we can with the text. Your only assignment these first three days is to read the text as closely as possible and annotate as you go. Your annotations should include questions you may wish to raise during the seminar as well as observations aimed at understanding the positions of the various characters (namely Job, Job's friends, and God). We look forward to our discussion of this rich work with you, as we reach the culmination of our readings for the year.

Begin today by reading and annotating *Job* chapters 1-14.

Tuesday, May 12

Read and annotate *Job* 15-28.

Wednesday, May 13

Read and annotate *Job* 29-42. Prepare to participate in tomorrow's seminar.

Thursday, May 14

Attend and participate in our Zoom seminar on *Job*.

Take some time today and/or this weekend to review your project essay draft. Make at least one concrete addition or edit to your work so far.

Friday, May 15

Use today to catch up on work (including your project essay). Please attend optional office hours if there is anything you need to discuss with your teacher.

GreatHearts Irving

Physics Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: 11 Physics
Teacher: Miss Weisse natalie.weisse@greatheartsirving.org
Resource: *Miss Weisse's Own Physics Textbook* — new pages found at the end of this packet

Weekly Plan:

Monday, May 11
☐ Read *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
☐ Complete Worksheet 3 #1-5
☐ Email Miss Weisse with Questions and to Ask for Solutions

Tuesday, May 12
☐ Read *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
☐ Complete Worksheet 3 #6-8
☐ Email Miss Weisse with Questions and to Ask for Solutions

Wednesday, May 13
☐ Review *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
☐ Complete Worksheet 3 #9-10
☐ Complete Worksheet 4 #1-2
☐ Email Miss Weisse with Questions and to Ask for Solutions

Thursday, May 14
☐ Review *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
☐ Complete Worksheet 4 #3-4
☐ Email Miss Weisse with Questions and to Ask for Solutions

Friday, May 15
☐ Attend Office Hours at 9:30 AM! W
☐ Turn in your assignments on Google Classroom by the end of the day Sunday May 17.

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

Student Signature

Parent Signature

Monday, May 11

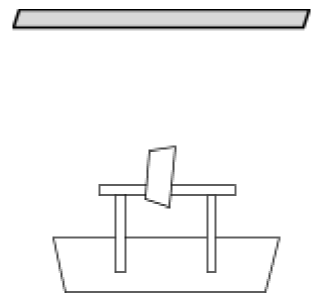
- ➔ Read *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
- ➔ Complete Worksheet 3 #1-5 showing all your work.
- ➔ Email Miss Weisse with Questions and to Ask for Solutions

Energy Storage and Transfer Model Worksheet 3 #1-5:

Quantitative Energy Calculations & Energy Conservation

Be careful with units and unit conversions!

1. How much kinetic energy does a 2000 kg SUV traveling 70 mph have? (1 mile = 1600 meters)



2. Consider your 3 kg physics binder resting on the table in your bedroom. Determine the gravitational energy of the earth-book system if the zero reference level is chosen to be:
    a. the table
    b. the floor, 0.68 meters below the book
    c. the ceiling, 2.5 meters above the book

3. A bungee cord stretches 25 meters and has a spring constant of 140 N/m. How much energy is stored in the bungee?

4. How fast does a 50 gram arrow need to travel to have 40 joules of kinetic energy?

5. How much energy is stored when a railroad car spring is compressed 10.0 cm? (The spring requires about 10,000 N to be compressed 3.0 cm.)

Tuesday, May 12

- ➔ Read *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
- ➔ Complete Worksheet 3 #6-8
- ➔ Email Miss Weisse with Questions and to Ask for Solutions

Energy Storage and Transfer Model Worksheet 3 #6-8: Quantitative Energy Calculations & Energy Conservation

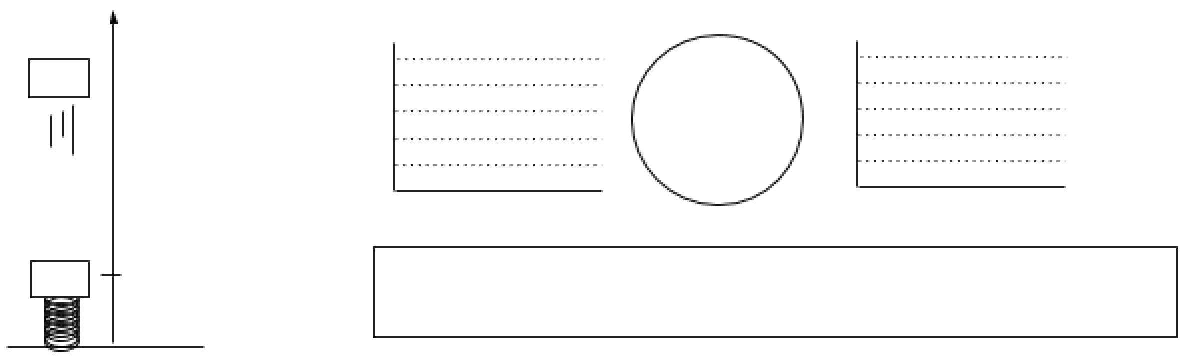
Directions: For each problem,

→ identify what is part of your system inside the circle

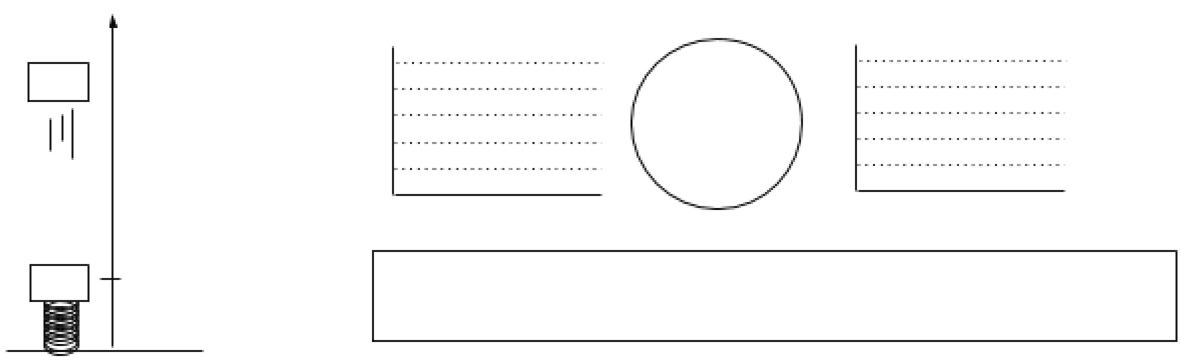
→ create bar graphs for the initial (A) and final (B) conditions of the object (don't forget to label your axes!)

→ write an equation for the conservation of energy

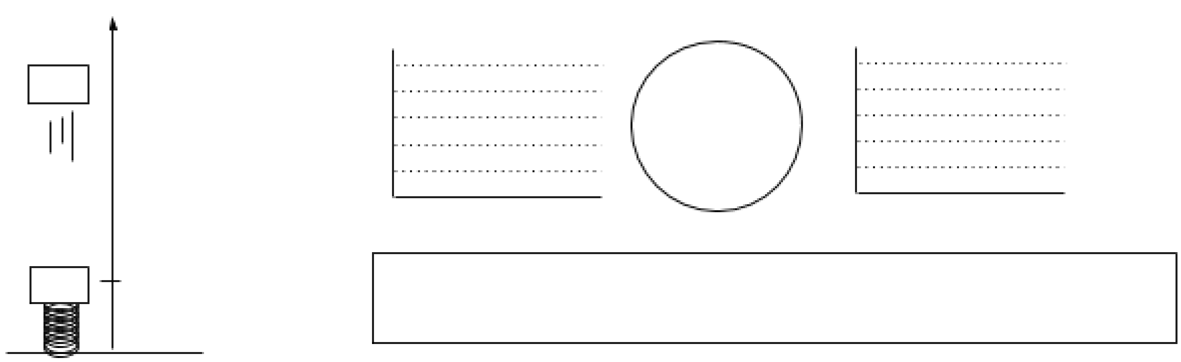
6. A load of bricks rests on a tightly coiled spring and is then launched into the air. Assume a system that includes the spring, the bricks and the earth. Do this problem without friction.



7. Repeat problem 7 with friction.



8. Repeat problem 7 for a system that does not include the spring.



Wednesday, May 13

- ➔ Review *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
- ➔ Complete Worksheet 3 #9-10
- ➔ Complete Worksheet 4 #1-2.
- ➔ Email Miss Weisse with Questions and to Ask for Solutions

Energy Storage and Transfer Model Worksheet 3 #6-8:
Quantitative Energy Calculations & Energy Conservation

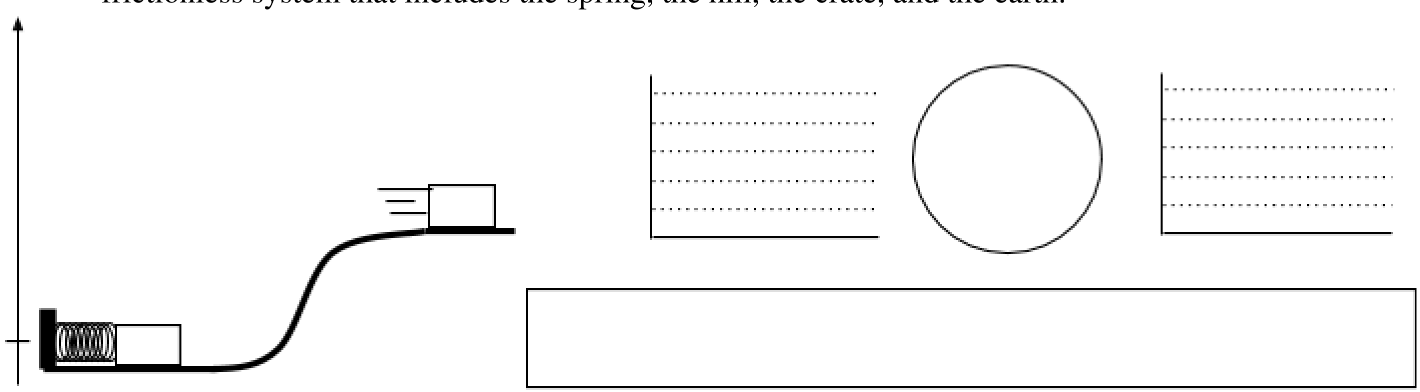
Directions: For each problem,

→ identify what is part of your system inside the circle

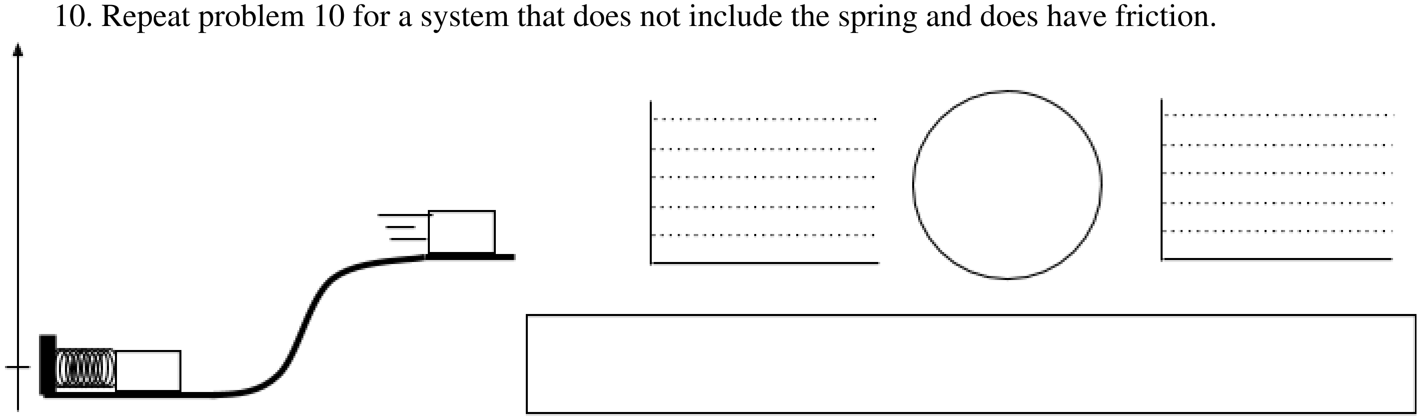
→ create bar graphs for the initial (A) and final (B) conditions of the object (don't forget to label your axes!)

→ write an equation for the conservation of energy

9. A crate is propelled up a hill by a tightly coiled spring. Analyze this situation for a frictionless system that includes the spring, the hill, the crate, and the earth.

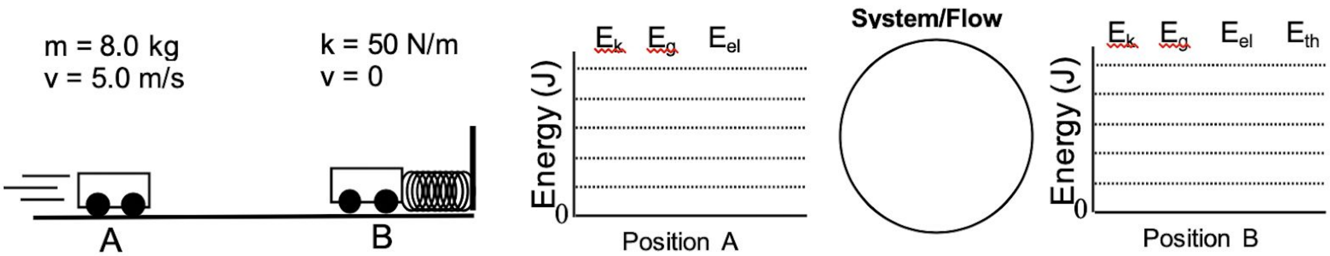


10. Repeat problem 10 for a system that does not include the spring and does have friction.



Energy Storage and Transfer Model Worksheet 4 #1-2:
Quantitative Energy Conservation

1. A cart moving at 5.0 m/s collides with a spring. At the instant the cart is motionless, what is the largest amount that the spring could be compressed? Assume no friction.

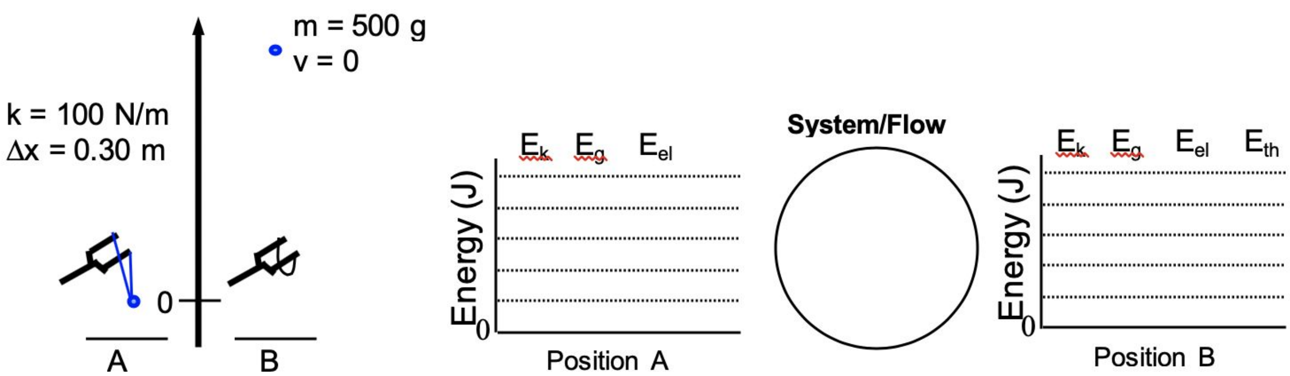


a. Define the system with the energy flow diagram, then complete the energy bar graphs qualitatively.

b. Quantitative Energy Conservation Equation:

c. Determine the maximum compression of the spring.

2. A rock is shot straight up into the air with a slingshot that had been stretched 0.30 m. Assume no air resistance.



a. Qualitatively complete the energy flow diagram and the energy bar graphs.

b. Quantitative Energy Conservation Equation:

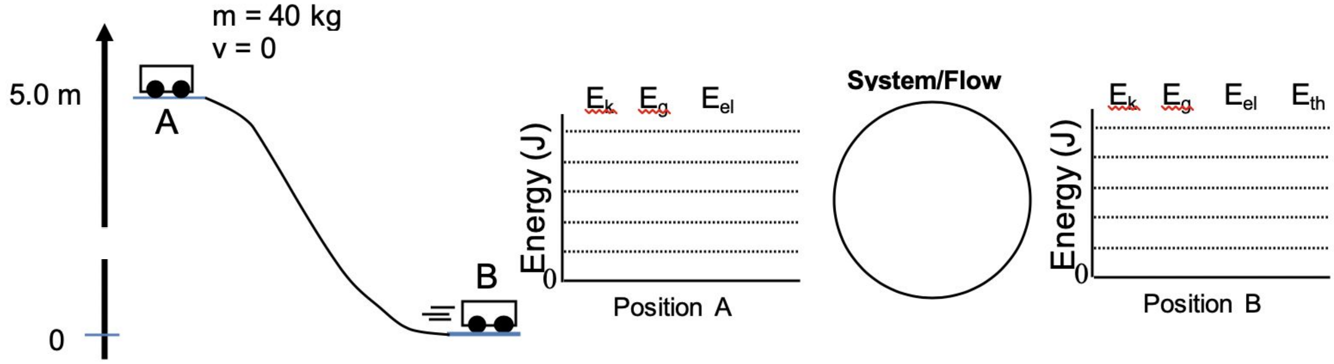
c. Determine the greatest height the rock could reach.

Thursday, May 14

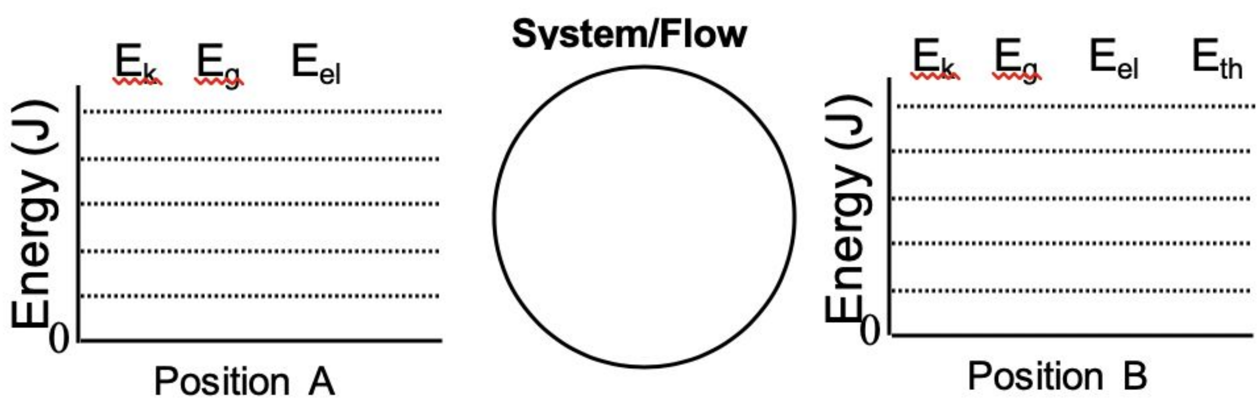
- ➔ Review *Unit 8 Part 6* of *Miss Weisse's Own Physics Textbook*
- ➔ Complete Worksheet 4 #3-4
- ➔ Email Miss Weisse with Questions and to Ask for Solutions

Energy Storage and Transfer Model Worksheet 4 #3-4:
Quantitative Energy Conservation

3. Determine final velocity of the rollercoaster, assuming a 10% loss to friction.



4. The moon could be an ideal spaceport for exploring the solar system. A moon launching system could consist of a magnetic rail gun that shoots items into moon orbit. How much energy would be needed from the rail gun to get a 10,000 kg capsule into an orbit 100 km above the moon surface? The moon's gravitational field strength is 1.6 N/kg and the orbital velocity for this altitude is 1700 m/s. Hint: Put the rail gun outside of the system.



Unit 8 - Energy
Part 7
Conservation of Energy Bar Graphs, Work, and System Flow Diagrams

Conservation of Energy

A few times, now, Conservation of Energy has come up – in the momentum unit, in making energy pie charts, and in justifying our $E_i = E_f$ assumption in the Kinetic Energy versus velocity lab.

As a formal definition

Energy cannot be created or destroyed in a closed system, but can be altered from one form to another.

It is important to point out the idea of a SYSTEM again. We get to define our system however we want to (or however the problem tells us to). The energy in the system remains constant unless it is an open system which means either

1) something outside the system does WORK on the system

·OR·

2) the system does WORK on something outside the system

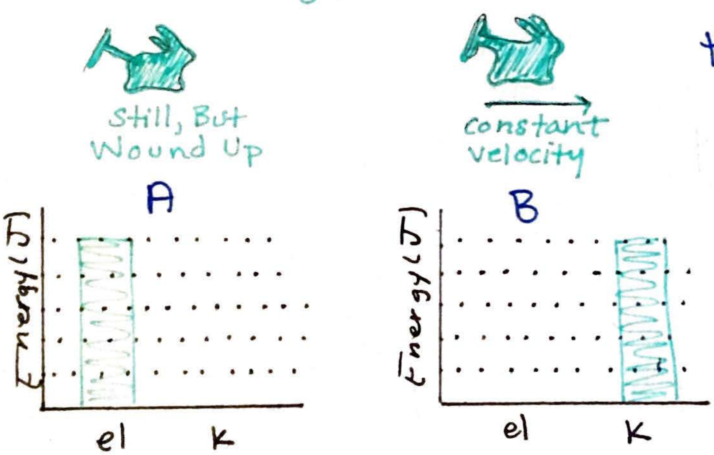
In both of these cases, if we made our system more inclusive, if we made it bigger, energy would be conserved. BUT, by this argument we might find ourselves in the position of saying the universe is our system so nothing is left out and energy <u>has</u> <u>to</u> <u>be</u> conserved... but the math would be a little trickier with some <u>very</u> <u>large</u> <u>sums</u>. Moral of the story—

WE LIKE SMALL SYSTEMS.

Energy Bar Graphs

Thus far we have used pie charts to track energy in a system. Now we are going to use bar graphs because bar graphs can be more easily quantified. Let us <u>jump</u> (hop?) into an example with our old friend the wind-up bunny toy.

The wind-up bunny toy is fully wound up and being held at rest at point A. The toy is let go (on a frictionless surfaces) and at point B we see it is moving at a constant speed. Draws bar graphs tes quantitatively show the conservation of energy.

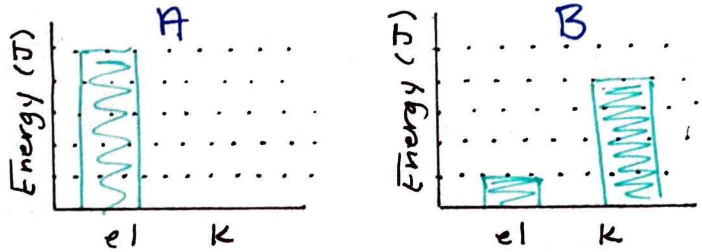


It is important that we define our system. Our system includes the bunny and the frictionless floor.

We can also how write an equation:

$$(E_{el})_A = (E_K)_B$$

Let us change the scenario so that the bunny is still increasing speed at point B. If there is Δspeed, there is acceleration, and if there is acceleration, there is a net Force. It makes sense to assume that net force comes from the elastic force.



notice the sum of all bars at B equal the bars at A

$$(E_{el})_A = (E_{el} + E_K)_B$$

Work & System Flow Diagrams

Earlier in this section it was mentioned that energy is conserved in a system UNLESS WORK is done to or done by the system.

What is WORK?

In non-physics terms, you are doing work right now as you complete your physics assignment. We can think about the work you are doing in two ways. First, you are focusing your energy, using your energy, to think about these concepts. Second, you are accomplishing something; when you finish the assignment something will have changed from before you started the assignment.

The same is true in physics terms. WORK is energy transferred by force.

As an equation

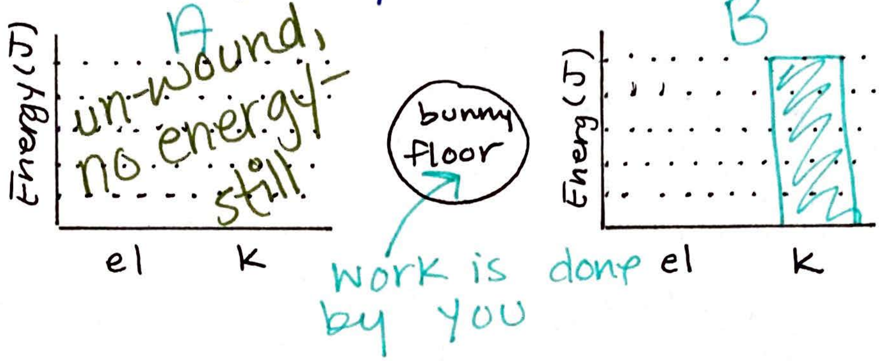
$$\boxed{\begin{gathered} W = \Delta E \\ W = F \cdot d \end{gathered}}$$

System Flow Diagrams show work being done to or done by a system by showing energy coming into or out of a system.

We draw our system flow diagrams as circles (kind of like our pie charts) and we put in in between two bar graphs showing the conservation of energy. Inside the circle we list everything that is included in the system. If energy is coming into the system between points A and B, we show this with an arrow pointing into the circle (we also want to mark where that energy is coming from/ what is doing the work).

With the System Flow Diagram, the examples we were just thinking about look like this:

Let us change the problem so work is done. The wind-up bunny toy is sitting, unwound on a frictionless surface. You then wind up the bunny and at point B we again see the bunny traveling at a constant speed. But YOU are NOT part of the system.

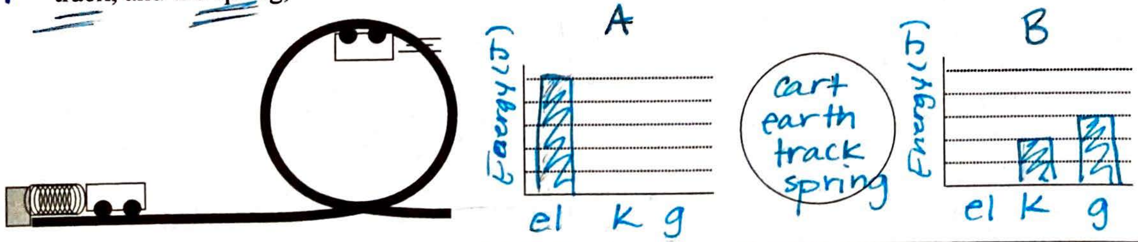


$$W = (E_K)_B$$

Even though the bars of the bar graphs don't add up at points A and B, we are accounting for the energy by showing work was done to the system, therefore adding energy to the system.

Now let us do a bunch of examples.

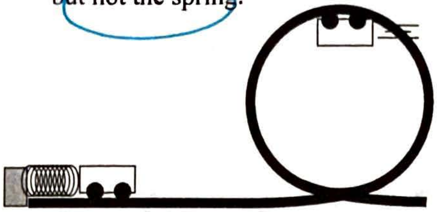
Ex 1a) In the situation shown below, a spring launches a roller coaster cart from rest on a frictionless track into a vertical loop. Assume the system consists of the cart, the earth, the track, and the spring,

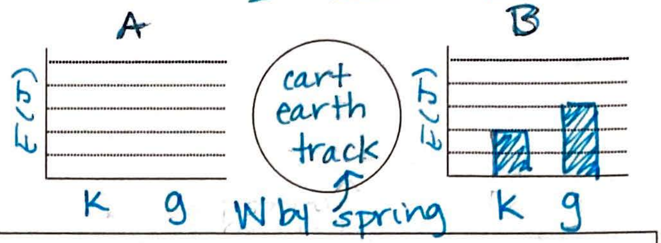


$$\boxed{(E_{el})_A = (E_K + E_g)_B}$$

Ex 1b)

Repeat problem 1a for a frictionless system that includes the cart, the earth, and the track, but not the spring.

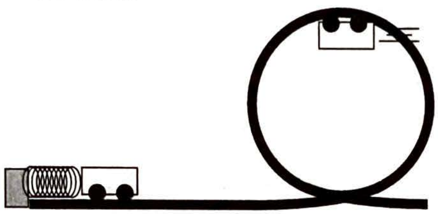


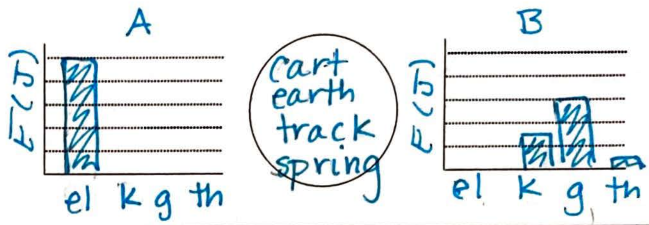


$$W_{spring} = (E_K + E_g)_B$$

Ex 1c)

Use the same system as problem 1a, but assume that there is friction between the cart and the track.

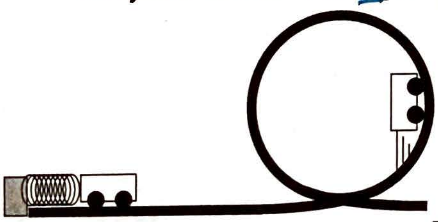


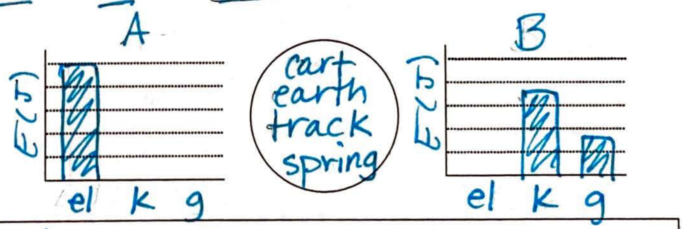


$$(E_{el})_A = (E_K + E_g + E_{th})_B$$

Ex 1d)

This situation is the same as problem 1a except that the final position of the cart is lower on the track. Make sure your bars are scaled consistently between problem 1a and 1d. Assume the system consists of the cart, the earth, the track, and the spring.

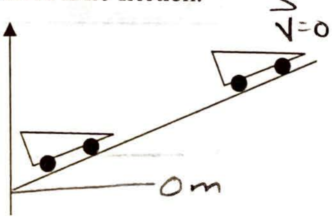


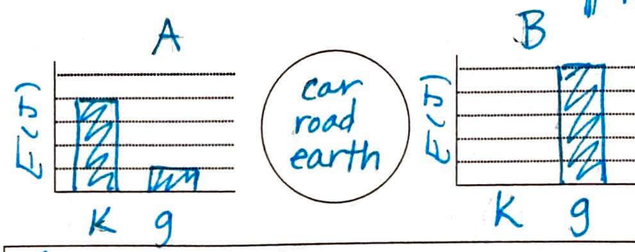


$$(E_{el})_A = (E_K + E_g)_B$$

Ex 2a)

A moving car rolls up a hill until it stops. Do this problem for a system that consists of the car, the road, and the earth. Assume that the engine is turned off, the car is in neutral, and there is no friction. ‖ no force forward

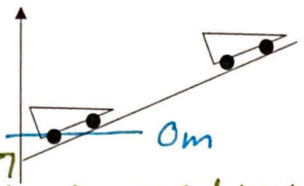


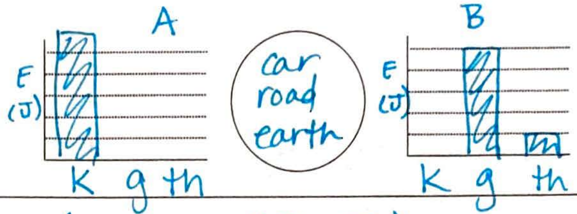


$$(E_K + E_g)_A = (E_g)_B$$

Repeat problem 2a for the same system with friction.

Ex 2b)



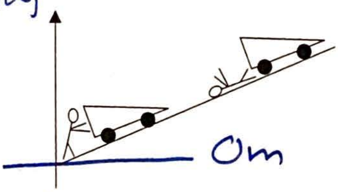


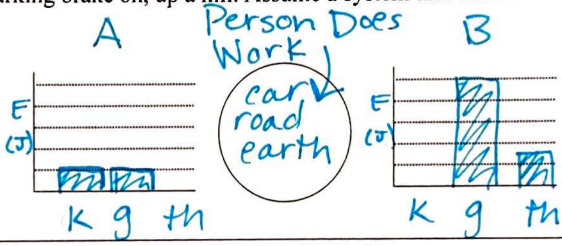
$(E_K)_A = (E_g + E_{th})_B$

In this example compared to example (2a), if the car is going to make it to the same height <u>WITH</u> friction, it must have started with more energy <u>OR</u> work was done on the system.

A person pushes a car, with the parking brake on, up a hill. Assume a system that includes the car, the road, and the earth, but does not include the person.

Ex 3a)





$W_{person} + (E_K + E_g)_A = (E_g + E_{th})_B$

Ex 3b) Let's add numbers to problem 3a. You are pushing a 1500kg car up a hill with a force of 10,000N. The car is initially at a height of 1m and moving at 0.5 m/s, and by the time you collapse in exhaustion having pushed the car 3.5m, its final height is 3m and, thanks to that parking break, is at rest. Find the amount of thermal energy in the system at B.

$$W_{person} + (E_K + E_g)_A = (E_g + E_{th})_B$$

$$F \cdot d + \tfrac{1}{2}mv^2 + mgh = mgh + E_{th}$$

$$(10{,}000N)(3.5m) + \tfrac{1}{2}(1500kg)(0.5 m/s)^2 + (1500)(10)(1m) = (1500)(10)(3m) + E_{th}$$

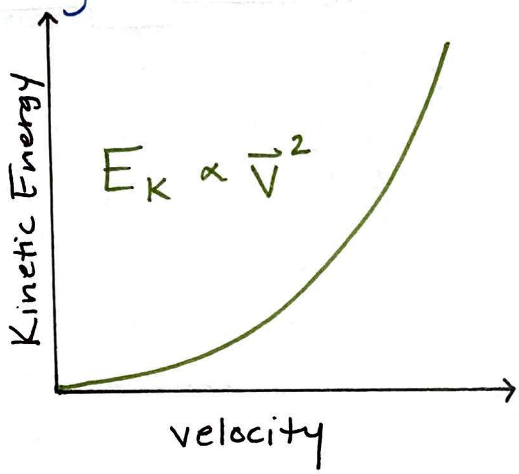
$$3500 + 250 + 15{,}000 = 45{,}000 + E_{th}$$

$$-45{,}000 \qquad 45{,}000$$

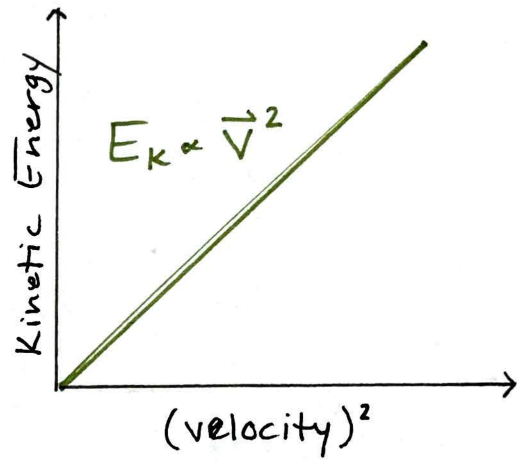
$$\boxed{E_{th} = \cdot 5{,}187 \text{ J}}$$

Unit 8 - Energy
Part 6
Kinetic Energy Post-Lab Discussion

In the kinetic energy versus velocity lab we determined $E_K \propto \vec{V}^2$ and, hopefully, $E_K \propto$ mass. The first relation came from the shape of our graph. The second relation came from the analysis of the slope of the linearized graph. If done correctly, the data would form graphs like below –



$\Rightarrow$



The y-intercept being at (or near with the 5% rule) makes sense. If there is no motion, there is no E_K creating that lack of motion.

The slope is more interesting. Let's first consider the units.

$$\text{slope} = \frac{\Delta E_K}{\vec{V}^2}\left(\frac{J}{\left(\frac{m}{s}\right)^2}\right)$$

we know a $(J) = N \cdot m$

$= \frac{kg \cdot m}{s^2} \cdot m$

$= \left(\frac{kg \cdot m^2}{s^2}\right)$

$$\Rightarrow \left(\frac{\left(\frac{kg \cdot m^2}{s^2}\right)}{\left(\frac{m}{s}\right)^2}\right)$$

$$\Rightarrow \frac{kg \cdot \cancel{m^2}}{\cancel{s^2}} \cdot \frac{\cancel{s^2}}{\cancel{m^2}}$$

$$\boxed{\Rightarrow kg}$$

THE UNIT OF SLOPE IS <u>KILOGRAMS</u>!

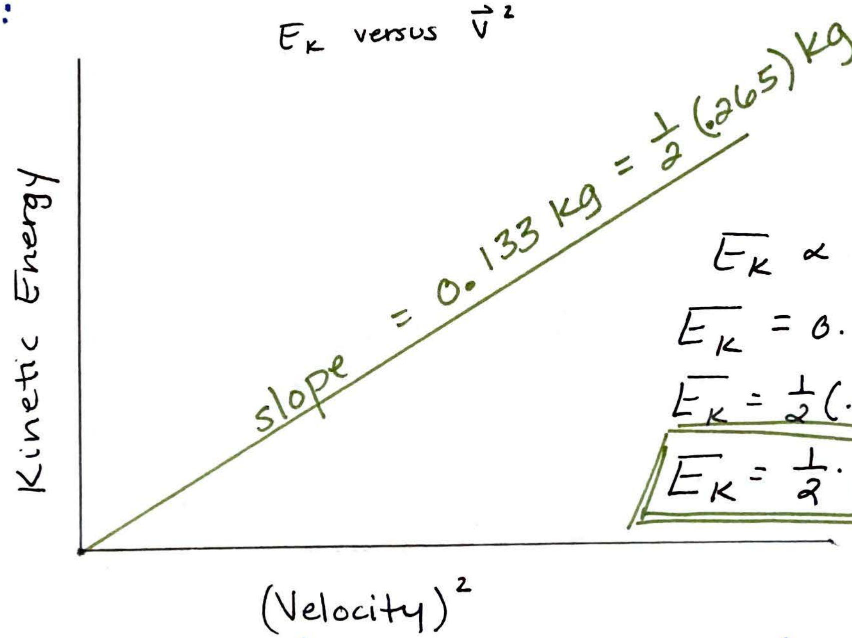
Since the unit of slope is kilograms it only makes sense that the slope is mass! If the slope is mass, is it the mass of our buggy?

* I have a confession to make.
I was quickly making measurements and being very sloppy - I was eyeballing heights, measuring to the back wheel at the top of the ramp and measuring to the front wheel at the bottom of our ramp, and, well,

OUR DATA IS BAD!

From our data, it does seem to be pretty close to the mass of our buggy... but it is supposed to be $\frac{1}{2}$(mass of the buggy).

So let's just say our linearized graph looks like this:



$E_K \propto \vec{v}^2$

$E_K = 0.133 \cdot \vec{v}^2$

$E_K = \frac{1}{2}(.265) \cdot \vec{v}^2$

$\boxed{E_K = \frac{1}{2} \cdot m \cdot \vec{v}^2}$

Kinetic Energy = $\frac{1}{2}$(mass)(velocity^2).

It should not actually be very surprising that mass is part of the kinetic energy equation, we already knew it was part of the gravitational potential energy equation.

Now we know two equations to calculate different types of energies.

$$\boxed{\begin{aligned} E_g &= mgh \\ E_K &= \tfrac{1}{2}mv^2 \end{aligned}}$$

Side Note

Without getting too mathy – when you multiply two vector quantities the output is a scalar quantity (a number WITHOUT direction). For both of these energy calculations we have two vectors multiplied together:

$$\vec{g} \cdot \vec{h} \quad \text{and} \quad \vec{v} \cdot \vec{v}.$$

Therefore, E_g and E_K are both scalars (they do not have direction) even though they are calculated with vectors.

GreatHearts
Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: 11 Precalculus
Teacher(s): Mr. Simmons

Weekly Plan:

Monday, May 11
- ☐ Story time!
- ☐ Check out the "Trig Cheat Sheet."
- ☐ Check answers from previous assignments.

Tuesday, May 12
- ☐ Complete problems 1-8 (A-G) from "Polar Plane."

Wednesday, May 13
- ☐ Check answers. (An answer key will be posted.)

Thursday, May 14
- ☐ Read "The Unit Circle."

Friday, May 15
- ☐ Attend office hours
- ☐ Catch up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

____

Student Signature

I affirm that, to the best of my knowledge, my child completed this work independently

____

Parent Signature

Monday, May 11

Happy Monday! I hope everyone's doing well.

1. Story time! If technologically feasible, please email me and let me know how you're doing. I love hearing from each and every one of you.

I heard from a few students (thanks so much for filling out the survey!), and one comment about the readings was that the information from each section wasn't all available in one place for reference while working problems. I hope the "Trig Cheat Sheet" included in this packet will help. It has all the formulas you're learning, and then some. (Don't feel like you need to go memorize the ones that the text I gave you doesn't cover.) I am also including answer keys for all previous problem sets (sorry I didn't have those earlier). From now on I'll post an answer key the day after each problem set. So:

2. At least glance at the cheat sheet. Better yet, print it out. Have it at the ready. Maybe frame it. Put it on your wall.
3. Check your answers to all previous problems using the answer keys included herein.

Tuesday, May 12

1. Complete Problems 1-8 from "Introduction to the Polar Plane" (pp. 144-46). For some of the problems, there are multiple exercises, labeled with capital letters. For each of those types of problems, please complete only A through G. (That means you're still doing every exercise labeled in small roman numerals.)

Wednesday, May 13

1. Check your answers from yesterday. (I'll post the answer key.)

Thursday, May 14

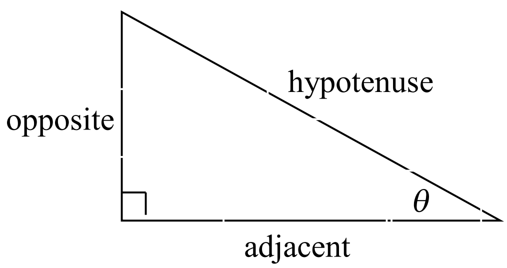
1. Read "The Unit Circle." Remember to read slowly! Try to work each of the example problems yourself before looking at the right answer.

Trig Cheat Sheet

Definition of the Trig Functions

Right triangle definition
For this definition we assume that

$0 < \theta < \frac{\pi}{2}$ or $0° < \theta < 90°$.

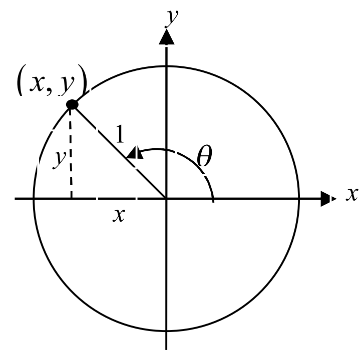


$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

Unit circle definition
For this definition θ is any angle.



$$\sin\theta = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

Facts and Properties

Domain
The domain is all the values of θ that can be plugged into the function.

$\sin\theta$, θ can be any angle
$\cos\theta$, θ can be any angle
$\tan\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\csc\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\sec\theta$, $\theta \neq \left(n + \frac{1}{2}\right)\pi$, $n = 0, \pm 1, \pm 2, \ldots$
$\cot\theta$, $\theta \neq n\pi$, $n = 0, \pm 1, \pm 2, \ldots$

Range
The range is all possible values to get out of the function.

$-1 \le \sin\theta \le 1 \qquad \csc\theta \ge 1$ and $\csc\theta \le -1$
$-1 \le \cos\theta \le 1 \qquad \sec\theta \ge 1$ and $\sec\theta \le -1$
$-\infty < \tan\theta < \infty \qquad -\infty < \cot\theta < \infty$

Period
The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods.

$$\sin(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \rightarrow T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \rightarrow T = \frac{\pi}{\omega}$$

Formulas and Identities

Tangent and Cotangent Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even/Odd Formulas

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

Periodic Formulas

If n is an integer.

$$\sin(\theta + 2\pi n) = \sin\theta \qquad \csc(\theta + 2\pi n) = \csc\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta \qquad \sec(\theta + 2\pi n) = \sec\theta$$

$$\tan(\theta + \pi n) = \tan\theta \qquad \cot(\theta + \pi n) = \cot\theta$$

Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Degrees to Radians Formulas

If x is an angle in degrees and t is an angle in radians then

$$\frac{\pi}{180} = \frac{t}{x} \quad \Rightarrow \quad t = \frac{\pi x}{180} \quad \text{and} \quad x = \frac{180t}{\pi}$$

Half Angle Formulas (alternate form)

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}} \qquad \sin^2\theta = \frac{1}{2}\left(1 - \cos(2\theta)\right)$$

$$\cos\frac{\theta}{2} = \pm\sqrt{\frac{1+\cos\theta}{2}} \qquad \cos^2\theta = \frac{1}{2}\left(1 + \cos(2\theta)\right)$$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \qquad \tan^2\theta = \frac{1-\cos(2\theta)}{1+\cos(2\theta)}$$

Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Product to Sum Formulas

$$\sin\alpha\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$$

$$\cos\alpha\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta) + \cos(\alpha+\beta)\right]$$

$$\sin\alpha\cos\beta = \frac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$$

$$\cos\alpha\sin\beta = \frac{1}{2}\left[\sin(\alpha+\beta) - \sin(\alpha-\beta)\right]$$

Sum to Product Formulas

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

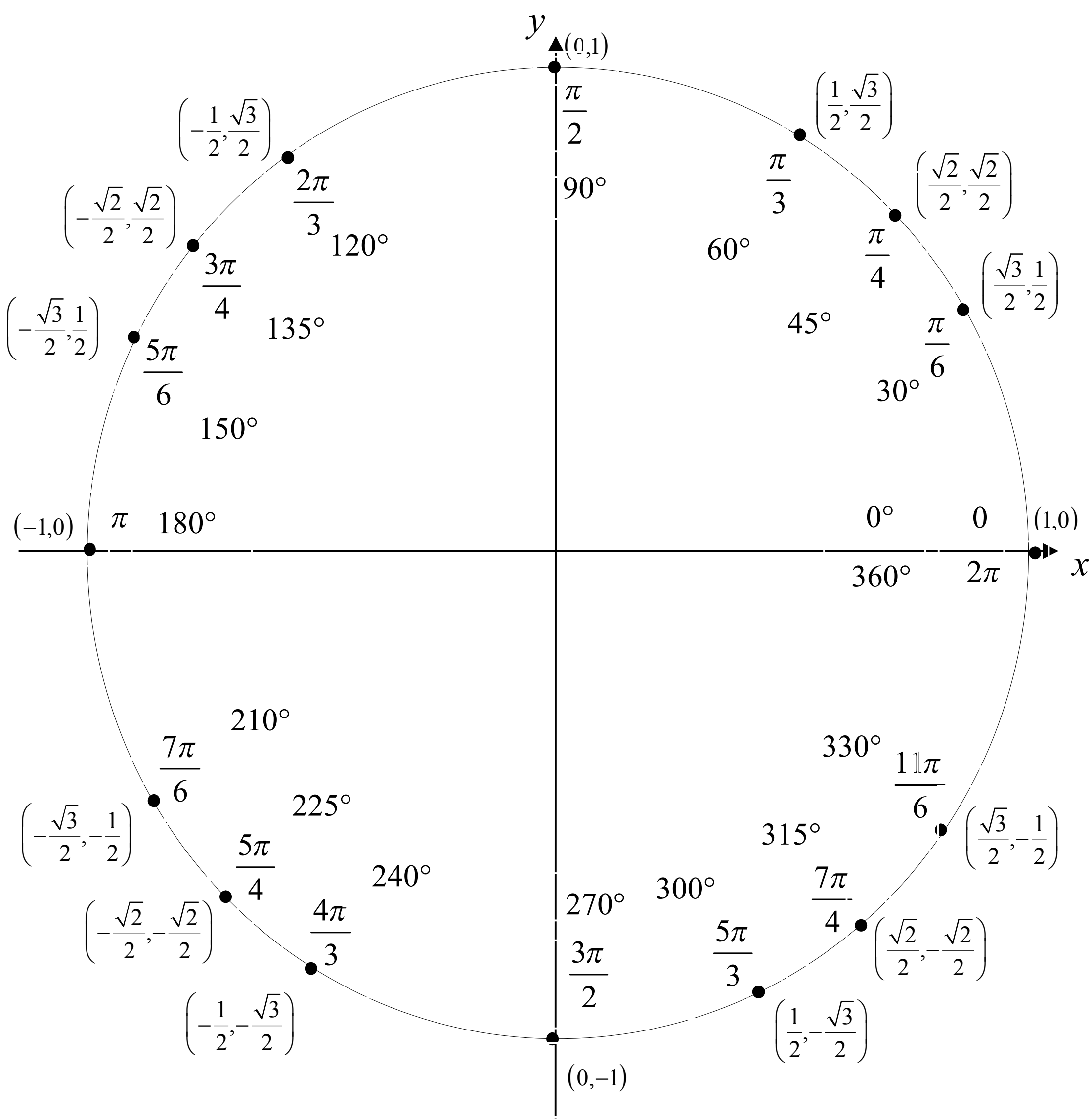
Cofunction Formulas

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta \qquad \sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta \qquad \cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

Unit Circle



For any ordered pair on the unit circle (x, y) : $\cos\theta = x$ and $\sin\theta = y$

Example

$$\cos\left(\frac{5\pi}{3}\right) = \frac{1}{2} \qquad \sin\left(\frac{5\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

Inverse Trig Functions

Definition

$y = \sin^{-1} x$ is equivalent to $x = \sin y$

$y = \cos^{-1} x$ is equivalent to $x = \cos y$

$y = \tan^{-1} x$ is equivalent to $x = \tan y$

Domain and Range

Function	Domain	Range
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1} x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

Inverse Properties

$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$

$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$

$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$

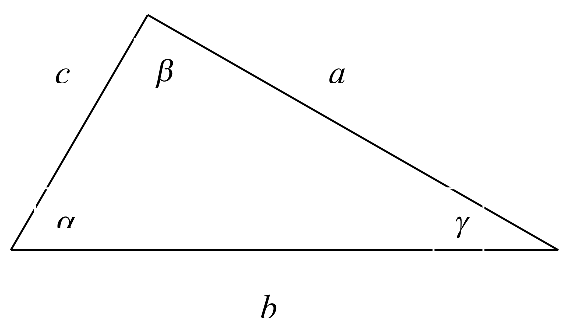
Alternate Notation

$\sin^{-1} x = \arcsin x$

$\cos^{-1} x = \arccos x$

$\tan^{-1} x = \arctan x$

Law of Sines, Cosines and Tangents



Law of Sines

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

Mollweide's Formula

$$\frac{a+b}{c} = \frac{\cos\frac{1}{2}(\alpha-\beta)}{\sin\frac{1}{2}\gamma}$$

Law of Tangents

$$\frac{a-b}{a+b} = \frac{\tan\frac{1}{2}(\alpha-\beta)}{\tan\frac{1}{2}(\alpha+\beta)}$$

$$\frac{b-c}{b+c} = \frac{\tan\frac{1}{2}(\beta-\gamma)}{\tan\frac{1}{2}(\beta+\gamma)}$$

$$\frac{a-c}{a+c} = \frac{\tan\frac{1}{2}(\alpha-\gamma)}{\tan\frac{1}{2}(\alpha+\gamma)}$$

4.1.1–5 odd – Answer Key

Precalculus
Mr. Simmons

1. (a)

$$\begin{aligned} 12^2 + 15^2 &= c^2 \\ 144 + 225 &= c^2 \\ 369 &= c^2 \\ c &= \sqrt{369} \\ &= 3\sqrt{41} \\ &\approx 19.21. \end{aligned}$$

   (b) $c = \sqrt{6.54} \approx 2.56$
   (c) $c = \sqrt{640} = 8\sqrt{10} \approx 25.30$
   (d) $b = \sqrt{204.75} \approx 14.31$
   (e) $a = \sqrt{27632} = 4\sqrt{1727} \approx 166.23$
   (f) $b = \sqrt{19.25} \approx 4.39$
   (g) $a = 3\sqrt{17} \approx 12.37$
   (h) $c = \sqrt{101} \approx 10.05$

3. No, not all triangles are possible. You cannot have a right triangle with legs of length 10 and 20 and a hypotenuse of 15, because $10^2 + 20^2 \neq 15^2$. Even outside of right triangles, not every triangle is possible. Try drawing a triangle with side lengths 5, 10, and 20.

5. (a) $c = 5$
   (b) $a = 5$
   (c) $a = 7, b = 24, c = 25$; $a = 8, b = 15, c = 17$; $a = 9, b = 40, c = 41$; $a = 11, b = 60, c = 61$; $a = 12, b = 35, c = 37$; $a = 13, b = 84, c = 85$ (and there are others).

4.2.1–5 – Answer Key

Precalculus
Mr. Simmons

1. (a) $\sin\alpha = 3/5$,
   $\cos\alpha = 4/5$,
   $\tan\alpha = 3/4$;
   $\sin\beta = 4/5$,
   $\cos\beta = 3/5$,
   $\tan\beta = 4/3$.
   (b) $\sin\alpha = 1/\sqrt{82} \approx 0.11$,
   $\cos\alpha = 9/\sqrt{82} \approx 0.99$,
   $\tan\alpha = 1/9$;
   $\sin\beta = 9/\sqrt{82} \approx 0.99$,
   $\cos\beta = 1/\sqrt{82} \approx 0.11$,
   $\tan\beta = 9$.
   (c) $\sin\alpha = 2/\sqrt{5} \approx 0.89$,
   $\cos\alpha = 1/\sqrt{5} \approx 0.45$,
   $\tan\alpha = 2$;
   $\sin\beta = 1/\sqrt{5} \approx 0.45$,
   $\cos\beta = 2/\sqrt{5} \approx 0.89$,
   $\tan\beta = 1/2$.
   (d) $\sin\alpha = 2/4.5 \approx 0.44$,
   $\cos\alpha = 4/4.5 \approx 0.89$,
   $\tan\alpha = 1/2$;
   $\sin\beta = 4/4.5 \approx 0.89$,
   $\cos\beta = 2/4.5 \approx 0.44$,
   $\tan\beta = 2$.

2. (a) The "really nice angles" are 30°, 45°, and 60°.
   (b) [Drawing of any two right triangle with acute angles 30° and 60° or both 45°. Such triangles could have a variety of side lengths, two examples being side lenths 1-1-$\sqrt{2}$ and 3-4-5.]
   (c)

α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$

3. (a) It will decrease.
   (b) Zero.
   (c) Zero.
   (d) $\cos\alpha = 7/7.04 \approx 0.99$
   (e) The length of the hypotenuse will approach the length of the adjacent side.
   (f) One.

4. (a) An example of such a triangle is one with side lengths 1-10-$\sqrt{101}$. ($\sqrt{101} \approx 10.05$.) Here, α will be 84°.
   (b) $\sin 90° = 1$; $\cos 90° = 0$.

5. Adding to the earlier table and approximating when appropriate, we have the following table.

α	$\sin\alpha$	$\cos\alpha$	$\tan\alpha$
0°	0	1	0
5°	~ 0.09	~ 0.996	~ 0.09
10°	~ 0.17	~ 0.98	~ 0.18
15°	~ 0.25	~ 0.97	~ 0.27
20°	~ 0.34	~ 0.94	~ 0.36
25°	~ 0.42	~ 0.91	~ 0.47
30°	$\frac{1}{2} = 0.5$	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{\sqrt{3}}{3} \approx 0.58$
35°	~ 0.57	~ 0.82	~ 0.70
40°	~ 0.64	~ 0.77	~ 0.84
45°	$\frac{\sqrt{2}}{2} \approx 0.71$	$\frac{\sqrt{2}}{2} \approx 0.71$	1
50°	~ 0.77	~ 0.64	~ 1.19
55°	~ 0.82	~ 0.57	~ 1.43
60°	$\frac{\sqrt{3}}{2} \approx 0.87$	$\frac{1}{2} = 0.5$	$\sqrt{3} \approx 1.73$
65°	~ 0.91	~ 0.42	~ 2.14
70°	~ 0.94	~ 0.34	~ 2.75
75°	~ 0.97	~ 0.25	~ 3.73
80°	~ 0.98	~ 0.17	~ 5.67
85°	~ 0.996	~ 0.09	~ 11.43
90°	1	0	undefined

4.3.1–14,16 – Answer Key

Precalculus
Mr. Simmons

1. (a) $\sin\alpha = 12/13$,
   $\cos\alpha = 5/13$,
   $\tan\alpha = 12/5$,
   $\csc\alpha = 13/12$,
   $\sec\alpha = 13/5$,
   $\cot\alpha = 5/12$.

   (b) $\sin\alpha = 4/5$,
   $\cos\alpha = 3/5$,
   $\tan\alpha = 4/3$,
   $\csc\alpha = 5/4$,
   $\sec\alpha = 5/3$,
   $\cot\alpha = 3/4$.

   (c) $\sin\alpha = 24/25$,
   $\cos\alpha = 7/25$,
   $\tan\alpha = 24/7$,
   $\csc\alpha = 25/24$,
   $\sec\alpha = 25/7$,
   $\cot\alpha = 7/24$.

   (d) $\sin\alpha = 1/4$,
   $\cos\alpha = \sqrt{17}/4$,
   $\tan\alpha = 1/\sqrt{17}$,
   $\csc\alpha = 4$,
   $\sec\alpha = 4/\sqrt{17}$,
   $\cot\alpha = \sqrt{17}$.

   (e) $\sin\alpha = \sqrt{69}/13$,
   $\cos\alpha = 10/13$,
   $\tan\alpha = \sqrt{69}/10$,
   $\csc\alpha = 13/\sqrt{69}$,
   $\sec\alpha = 13/10$,
   $\cot\alpha = 10/\sqrt{69}$.

   (f) $\sin\alpha = 5/\sqrt{61}$,
   $\cos\alpha = 6/\sqrt{61}$,
   $\tan\alpha = 5/6$,
   $\csc\alpha = \sqrt{61}/5$,
   $\sec\alpha = \sqrt{61}/5$,
   $\cot\alpha = 6/5$.

   (g) $\sin\alpha = 2/7$,
   $\cos\alpha = 3\sqrt{5}/7$,
   $\tan\alpha = 2/\left(3\sqrt{5}\right)$,
   $\csc\alpha = 7/2$,
   $\sec\alpha = 7/\left(3\sqrt{5}\right)$,
   $\cot\alpha = 3\sqrt{5}/2$.

   (h) $\sin\alpha = \sqrt{51}/10$,
   $\cos\alpha = 7/10$,
   $\tan\alpha = \sqrt{51}/7$,
   $\csc\alpha = 10/\sqrt{51}$,
   $\sec\alpha = 10/7$,
   $\cot\alpha = 7/\sqrt{51}$.

2. The triangle in Example 1c has opposite leg length a and hypotenuse length 1. If we call the adjacent leg length b, we have $a^2 + b^2 = 1^2$, so $b = \sqrt{1-a^2}$. So the six trig ratios are as follows:
   $\sin\alpha = a$,
   $\cos\alpha = \sqrt{1-a^2}$,
   $\tan\alpha = a/\sqrt{1-a^2}$,
   $\csc\alpha = 1/a$,
   $\sec\alpha = 1/\sqrt{1-a^2}$,
   $\cot\alpha = \sqrt{1-a^2}/a$.

3. (a) If $\tan\alpha = d$, then we can consider the triangle with leg lengths d and 1, with hypotenuse $\sqrt{d^2+1}$. Then have the following trig ratios:
   $\sin\alpha = d/\sqrt{d^2+1}$,
   $\cos\alpha = 1/\sqrt{d^2+1}$,
   $\tan\alpha = d$,
   $\csc\alpha = \sqrt{d^2+1}/d$,
   $\sec\alpha = \sqrt{d^2+1}$,
   $\cot\alpha = 1/d$.

4. (a) $\sin\alpha = a/c$,
   $\cos\alpha = b/c$,

$\tan\alpha = a/b$,
$\csc\alpha = c/a$
$\sec\alpha = c/b$
$\cot\alpha = b/a$.

5. (a) $\sin 30° = \cos 60°$
   (b) $\cos 10° = \sin 80°$
   (c) $\cot 7° = \tan 83°$
   (d) $\sec 64° = \csc 26°$
   (e) $\cos 31° = \sin 59°$
   (f) $\tan 14° = \cot 76°$
   (g) $\csc 47° = \sec 43°$
   (h) $\sin 25° = \cos 65°$
   (i) $\tan(\beta+\gamma) = \cot(90° - \beta - \gamma)$
   (j) $\sin\beta = \cos(90° - \beta)$

6. $\sin(90° - \theta) = \cos\theta$,
$\cos(90° - \theta) = \sin\theta$,
$\csc(90° - \theta) = \sec\theta$,
$\sec(90° - \theta) = \csc\theta$,
$\tan(90° - \theta) = \cot\theta$,
$\cot(90° - \theta) = \tan\theta$.

7. $\csc\theta = 1/\sin\theta$,
$\sin\theta = 1/\csc\theta$,
$\sec\theta = 1/\cos\theta$,
$\cos\theta = 1/\sec\theta$,
$\cot\theta = 1/\tan\theta$,
$\tan\theta = 1/\cot\theta$.

8. (a) $\sin^2 30° = (1/2)^2 = 1/4$
   (b) $\cos^2 30° = \left(\sqrt{3}/2\right)^2 = 3/4$
   (c) $\tan^2 30° = \left(\sqrt{3}/3\right)^2 = 1/3$
   (d) $\cos^2 45° = \left(\sqrt{2}/2\right)^2 = 1/2$
   (e) $\tan^2 45° = 1^2 = 1$
   (f) $\sin^2 60° = \left(\sqrt{3}/2\right)^2 = 3/4$
   (g) $\cos^2 60° = (1/2)^2 = 1/4$
   (h) $\tan^2 60° = \sqrt{3}^2 = 3$
   (i) $\sin 30°^2 = \sin 900° =$ undefined (until we have the unit-circle definition of sine)
   (j) $\cos^2\alpha = (\cos\alpha)^2$

9.

α	$\sin^2\alpha$	$\cos^2\alpha$	$\tan^2\alpha$
0°	0	1	0
5°	~ 0.01	~ 0.99	~ 0.01
10°	~ 0.03	~ 0.93	~ 0.03
15°	~ 0.07	~ 0.88	~ 0.07
20°	~ 0.12	~ 0.82	~ 0.13
25°	~ 0.18	~ 0.67	~ 0.22
30°	$\frac{1}{4} = 0.25$	$\frac{3}{4} = 0.75$	$\frac{1}{3} \approx 0.33$
35°	~ 0.33	~ 0.59	~ 0.49
40°	~ 0.41	~ 0.59	~ 0.70
45°	$\frac{1}{2} = 0.5$	$\frac{1}{2} = 0.5$	1
50°	~ 0.59	~ 0.41	~ 1.42
55°	~ 0.67	~ 0.33	~ 2.04
60°	$\frac{3}{4} = 0.75$	$\frac{1}{4} = 0.25$	3
65°	~ 0.82	~ 0.18	~ 4.60
70°	~ 0.88	~ 0.12	~ 7.55
75°	~ 0.93	~ 0.07	~ 13.93
80°	~ 0.97	~ 0.03	~ 32.16
85°	~ 0.99	~ 0.01	~ 130.65
90°	1	0	undefined

10. (a) 1; 1; no maximum
    (b) 0; 0; 0
    (c) The sin and $\sin^2$ functions have the same shape, but $\sin^2$ has only positive values and is shorter. It also has hills/valleys twice as frequently. (We say that its "period" is half the length.)

11. $\sin^2\theta + \cos^2\theta = 1$

12. These expressions are all of the form $\sin^2\theta + \cos^2\theta$, so they all equal one.

13. Since $\sin^2\theta + \cos^2\theta = 1$, we have

$$\sin^2\theta = 1 - \cos^2\theta$$

and

$$\cos^2\theta = 1 - \sin^2\theta.$$

14. (a)

$$\begin{aligned}\tan\alpha\cdot\csc\alpha &= \frac{\sin\alpha}{\cos\alpha}\cdot\frac{1}{\sin\alpha}\\ &= \frac{1}{\cos\alpha}\\ &= \sec\alpha.\end{aligned}$$

(b)

$$\begin{aligned}(\sin\alpha+\cos\alpha)^2 &= \sin^2\alpha+2\sin\alpha\cos\alpha+\cos^2\alpha\\ &= 2\sin\alpha\cos\alpha+1.\end{aligned}$$

16. (a) True. Although this is not the cofunction identity (that would be $\sin\alpha = \cos(90^\circ-\alpha)$), the statement here is still true because

$$\begin{aligned}\sin\alpha &= \cos(90^\circ-\alpha)\\ &= \cos(-[90^\circ-\alpha])\\ &= \cos(\alpha-90^\circ).\end{aligned}$$

This fact is, however, dependent on the unit-circle definition of cosine, so, strictly speaking, since cosine is undefined for negative angle measures in right-triangle trigonometry, "false" would also be an acceptable answer, as long as you understood the reason.

(b) False. Counterexample: $\alpha = \beta = 30^\circ$.

(c) False. Counterexample: $\alpha = 5^\circ$

(d) False. Squares are never negative.

5.1.1–14 – Answer Key

Precalculus
Mr. Simmons

1. (a) 2
   (b) $1/3 \approx 0.33$
   (c) $20/3 \approx 6.67$
   (d) $64/3 \approx 21.33$

2. (a) 10
   (b) 9
   (c) $5/4 = 1.25$
   (d) 5

3. (a) 5
   (b) 1
   (c) $10/3 \approx 3.33$
   (d) 12

4. See 1, 2, and 3 sketched on pp. 129–30. Come to office hours if you want to see the rest sketched.

5. (a) $10\pi/9$
   (b) $5\pi/9$
   (c) $5\pi/18$
   (d) $270/\pi°$
   (e) $15°$
   (f) 4π
   (g) $36°$
   (h) $180°$

6. (a) $\pi/6$, $\pi/4$, $\pi/3$, $\pi/4$,$\pi/2$,$3\pi/2$,2π.
   (b) $\pi/2$
   (c) $120°$, $135°$, $150°$, $210°$, $225°$, $240°$, $300°$, $315°$, $330°$.
   (d) $2\pi/3$, $3\pi/4$, $5\pi/6$, $7\pi/6$, $5\pi/4$, $4\pi/3$, $5\pi/3$, $7\pi/4$, $11\pi/6$.

7. (a) $1/2$
   (b) The more pieces you divide a whole into, by necessity the smaller the pieces will be (if they are of equal size—if they are unequally sized, still the average size will be smaller).
   (c) $\pi/3$
   (d) 2
   (e) π
   (f) $15\pi/4$

8. A half rotation would be π, which, written as $6\pi/6$, is obviously greater than $5\pi/6$.

9. Quadrant IV. One way to tell quickly is that $11\pi/6$ is very nearly 2π (which fact is made obvious by writing 2π as $12\pi/6$).

10. $5\pi/3 = 300° < 315° = 7\pi/4$

11. $$\frac{5\pi}{3} = \frac{20\pi}{12} < \frac{21\pi}{12} = \frac{7\pi}{4}$$

12. (a) 24π in
    (b) 12π in
    (c) 120π in
    (d) 3 (radians)

13. (a) 10 in
    (b) 10π (radians)

14. (a) $60/(13\pi)$ ft ≈ 1.47 ft $= 17.64$ in
    (b) $80/3$ ft ≈ 26.67 ft $= 320.04$ in

§2 **Exercises**

1.) Plot the following points. You may use the same Polar Plane if you wish.
   - (A) $A(2,30°)$
   - (B) $B(1,45°)$
   - (C) $C(3,300°)$
   - (D) $D(2,150°)$
   - (E) $E(4,315°)$
   - (F) $F(2,210°)$
   - (G) $G(0.5,60°)$
   - (H) $H(3.5,135°)$
   - (I) $I(20,180°)$
   - (J) $J(1,260°)$
   - (K) $K(3.5,0°)$
   - (L) $L\left(\frac{7}{2},350°\right)$

2.) Before you start plotting points using radians, create a Polar Plane just like we did in this unit, except instead of using $30°, 45°, \ldots$ use the corresponding radian measures. Make sure each angle is labeled.

3.) Now that you have a Polar Plane with radians, plot the following points.
   - (A) $A\left(2,\frac{\pi}{2}\right)$
   - (B) $B\left(3,\frac{\pi}{3}\right)$
   - (C) $C\left(4,\frac{2\pi}{3}\right)$
   - (D) $D\left(1,\frac{5\pi}{4}\right)$
   - (E) $E(4,1)$
   - (F) $F\left(3,\frac{11\pi}{6}\right)$
   - (G) $G(2,\pi)$
   - (H) $H\left(2,\frac{3\pi}{4}\right)$
   - (I) $I\left(2,\frac{\pi}{6}\right)$
   - (J) $J\left(300,\frac{3\pi}{2}\right)$
   - (K) $K\left(5,\frac{5\pi}{3}\right)$
   - (L) $L\left(2,\frac{7\pi}{4}\right)$
   - (M) $M\left(3,\frac{7\pi}{6}\right)$

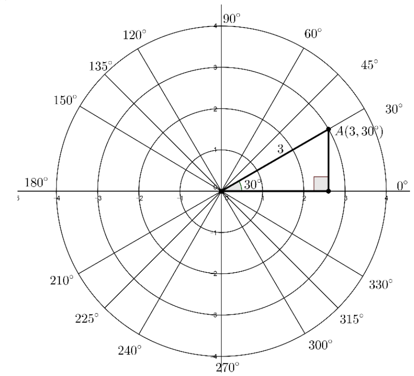
4.) Now let's work with negative angles.
   - (A) What does a negative angle represent, or tell you to do?
   - (B) Now create a Polar Plane just like you did in 2.), except this time label each angle as $-30°, -45°, \ldots$ as we started to do in the reading. Make sure each angle is labeled.
   - (C) Create another Polar Plane, except this time use negative radian measures. Again, make sure each angle is labeled.

5.) Plot the following points.
   - (A) $A(2,-45°)$
   - (B) $B\left(3,-\frac{2\pi}{3}\right)$
   - (C) $C(1,-180°)$
   - (D) $D\left(2,-\frac{7\pi}{4}\right)$
   - (E) $E(3,-300°)$
   - (F) $F\left(4,-\frac{5\pi}{6}\right)$
   - (G) $G(5,-25°)$
   - (H) $H(2,-2)$
   - (I) $I(3,-100°)$
   - (J) $J(4,-2\pi)$
   - (K) $K(3,-135°)$
   - (L) $L\left(2,-\frac{3\pi}{4}\right)$

6.) Now let's work with coterminal angles. This is incredibly important for our work in trigonometry, and was one of the main reasons we chose to work with the Polar Plane before working with the Unit Circle. Use the Polar Planes you created from previous exercises to help you.

   - (A) Give two different coterminal angles (in degrees) of the given angle measure.
     - i. $30°$.
     - ii. $45°$
     - iii. $90°$
     - iv. $135°$
     - v. $150°$
     - vi. $180°$
     - vii. $200°$
     - viii. $10°$
     - ix. $0°$
     - x. $330°$
     - xi. $240°$
     - xii. $120°$
   - (B) Give two different coterminal angles (in radians) of the given angle measure.
     - i. $\frac{\pi}{6}$
     - ii. $\frac{\pi}{4}$
     - iii. $\frac{\pi}{3}$
     - iv. $\frac{\pi}{2}$
     - v. $\frac{2\pi}{3}$
     - vi. $\frac{3\pi}{4}$
     - vii. $\frac{5\pi}{6}$
     - viii. π
     - ix. $\frac{7\pi}{6}$
     - x. $\frac{5\pi}{4}$
     - xi. $\frac{4\pi}{3}$
     - xii. $\frac{3\pi}{2}$
     - xiii. $\frac{5\pi}{3}$
     - xiv. $\frac{7\pi}{4}$
     - xv. $\frac{11\pi}{6}$
     - xvi. 2π

7.) Let us now explore how to convert a Polar point into a normal rectangular point. Consider the Figure below, of $A(3,30°)$. We will create a triangle using this point and the Pole.



Notice that if we find the lengths of the two legs of the created right triangle, then we will have found the x and y distance, and thus, the x- and y-coordinate of our point? Converting a Polar point to a rectangular one, then, amounts to finding lengths of a triangle.

But we know how to do this!

(A) Find the length of each leg of the triangle shown above.
(B) What, then, are the rectangular coordinates of the given Polar point?

8.) Using the same method above (and drawing a picture), convert the following Polar points into rectangular points.
(A) $A(1,60°)$
(B) $B(1,45°)$
(C) $C(1,90°)$
(D) $D\left(2,\frac{\pi}{6}\right)$
(E) $E\left(1,\frac{\pi}{3}\right)$
(F) Did any of your previous results seem familiar to what you've already learned? How so?
(G) Can you generalize the process of converting Polar coordinates into rectangular coordinates? It seems like there's some treasure hidden in this Exercise…

§3 The unit circle

We now embark on perhaps the most important section in Trigonometry. It is imperative that you learn and master the techniques in this section, as it will make much of Trigonometry (and, subsequently, Calculus) much easier. In this section we provide a tool to help visualize and efficiently evaluate most of the Trig functions you'll run into. Of course, as has been the case in many of these Trig sections, you must be adept at the previous lessons as well, including quickly evaluating $\sin 30°$, for example.[i]

Up to this point, we've worked with only a few Trig functions, such as $\cos 30°$. And, as we mentioned, there are many Trig functions that we simply can't evaluate with any sort of efficiency. In this section we'll learn how to evaluate a handful more – an infinite amount, actually! – quickly and efficiently.

Recall that the main reason we're able to evaluate a Trig expression like $\tan 45°$ is because we can create a special right triangle (in this case, a $45° - 45° - 90°$ triangle) which we can find the length of the sides for very quickly. Then we just have to write the

[i] Although, frankly, at this point, this warning should not be necessary.

corresponding ratio.[ii] Without these special right triangles, we would have to resort to approximation methods, which aren't very interesting to study.

But let us bring back our Polar Plane from the previous section in Figure 66. Then let us see if we can't create any other special right triangles.

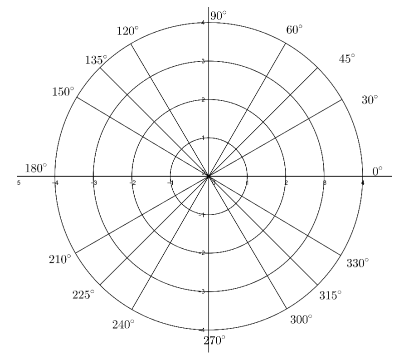


Figure 66

Can you create any special right triangles using this picture? Try making a point on one of the angles.

It turns out that not only can we make a few, but we can make many of them! We consider one example below.

Example 1

Given the point $A(2, 120°)$, create some special right triangle.

We first plot the given point. We show this in Figure 67.

[ii] We must also bring up the fact that the size of the triangle we choose does not matter.

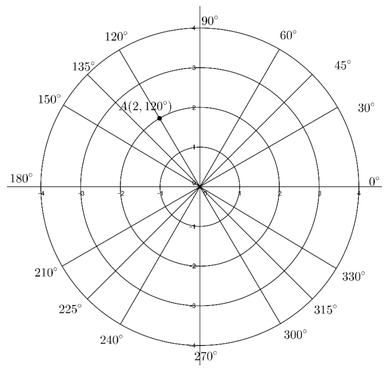


Figure 67

Two possible right triangle present themselves, and we show them both in Figure 68a and b.

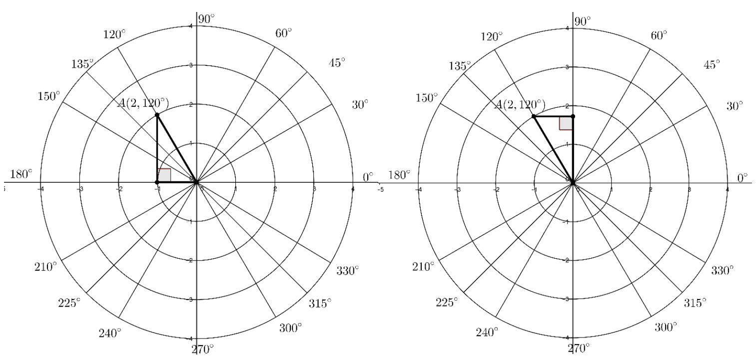


Figure 68a and b

To draw these right triangles, we just dropped a straight line down from the point the x-axis (in Figure a) and then a straight line to the right from the point to the y-axis. Right triangles are great, and this allows us to use what we've learned in previous sections. But it would be even better if they were special right triangles, right?

But that's exactly what each of them are! And it is made even more evident by drawing them on our Polar Plane like we did. In Figure 68a, for example, you can see that we have a $30° - 60° - 90°$ triangle, which we draw separately in Figure 69.

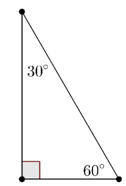


Figure 69

This is significant, and we'll demonstrate why as we proceed through this section. Hold on to this thought for a moment because we need to establish something else before making our greater point.

One of the Exercises from the previous section saw you converting Polar coordinates into rectangular coordinates. This was done by forming a right triangle (not unlike our previous Example) and then using your memorized Trig functions to find the missing lengths of the right triangle (which corresponded to the x- and y-values of the point[iii]). Let us reconsider that with some more formality and see if we can't uncover some truth.

Example 2a

Convert the Polar coordinate $(2, 45°)$ to rectangular coordinates.

Although you did this problem in the previous section, let us formalize this process. We first plot the given point, and then create a right triangle, as shown in Figure 70.

[iii] Huh? Did you miss something? There's something very interesting going on here; can you feel it?

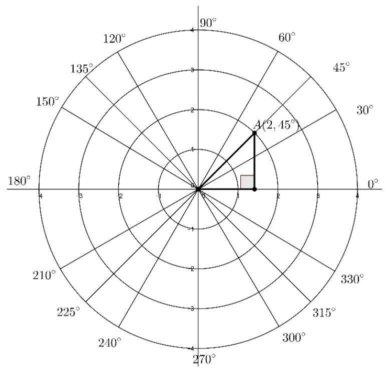


Figure 70

Now we can redraw that triangle by itself, adding in what we know, viz. the radius. We show this in Figure 71.

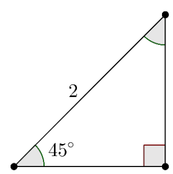


Figure 71

Recall that when we graph something on the coordinate plane, we go right some distance, and then up some distance. In the case of our picture, do you recognize that the origin is the point to the left of the right angle? Then if we travel right to the right angle, and then up, wouldn't we have just went through the process of plotting a point on the rectangular plane?[iv] Thus, if we find the length of each leg, we'll have our x- and y-coordinates, right?

So that's what we'll do. And this is quite easy, since this is an isosceles right triangle, we just divide the hypotenuse by $\sqrt{2}$, and find that each leg has a length of

[iv] That is confusing in words. Try doing what I wrote to help you see that you're just following the same procedure you've used perhaps thousands of times to plot a point. Or ask your teacher to demonstrate.

$$\frac{2}{\sqrt{2}}.$$

Let us rationalize this number so we can perhaps recognize it after we're done with it:

$$\frac{2}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{2\sqrt{2}}{2}=\sqrt{2}.$$

So each leg measures $\sqrt{2}$ in length, and this tells us that the point $A(2,45°)$ can also be written as $A(\sqrt{2},\sqrt{2})$.

This is neat, but more importantly that number $\sqrt{2}$ reminds us of a number that popped up quite a few times in the previous unit: $\frac{\sqrt{2}}{2}$. In fact, our result was twice that of $\frac{\sqrt{2}}{2}$. Why is this interesting? Because what is $\sin 45°$? And what is $\cos 45°$? And is this a coincidence??

Example 2b

Convert the Polar coordinate $B(3,30°)$ into rectangular coordinates.

Let's do one more test before trying to generalize and formalize our results. Following our previous procedure, then, we end up with Figure 72.

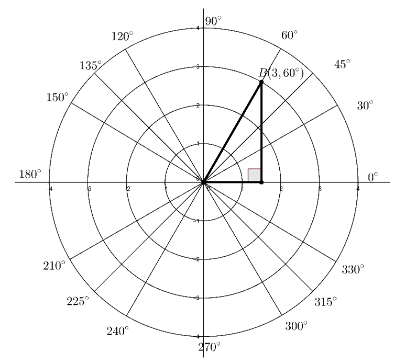


Figure 72

From here, we once again draw a right triangle, and, once again, in Figure 73 we see a special right triangle, don't we?

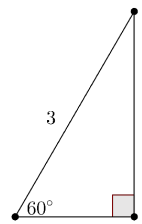


Figure 73

Again, what we're looking for are the legs of this right triangle, as that will tell us the x- and y-coordinates of the point we're looking for. Using what we learned from Unit four, we see that the short leg is 1.5 and the long leg is $1.5\sqrt{3}$. And hence our point B can be written in the rectangular plane as $B(1.5, 1.5\sqrt{3})$.

Let us again highlight the results: Remember that $\cos 60° = \frac{1}{2}$? Well, our x-coordinate is that times 3, isn't it? What about $\sin 60°$? What relationship does that have with our result of $1.5\sqrt{3}$?[v]

Isn't it interesting that the rectangular coordinates of a point from the Polar Plane keep coming up as multiples of Trig functions that we've memorized? Let us now generalize the result and make our major point of this section.

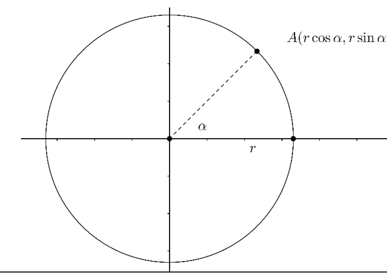
[v] You might ask how we knew about this relationship. Good question! We're not trying to teach a procedure here for you to follow, but only pointing something out, viz. that these familiar numbers keep popping up. Of course, there is a reason we chose 3 in this Example and 2 in the previous… Can you see where we get those two numbers from?

A point on a circle in rectangular coordinates

Any point on any circle is given by the rectangular coordinates

$$(r \cdot \cos\alpha\,, r \cdot \sin\alpha),$$

Where r is the radius of the circle and α is the angle of rotation from the positive x-axis to the point.



We cannot overstate how important this discovery is. So let us restate it another way. We can find the coordinates of any point on a circle using our Trig functions and the information above.

Perhaps more importantly, however, is that this establishes a relationship that we can use to find the values of other angles that we input into Trig functions. In our original definition of Trig functions, we could only use acute angles. This relationship, namely that

$$x = r \cdot \cos\alpha\,, y = r \cdot \sin\alpha,$$

Where x and y have the usual meaning as x- and y-coordinates of a point in the rectangular plane, helps us to find the output of any angle put into a Trig function. With some simple Algebra, we'll make it more clear:

$$\cos\alpha = \frac{x}{r}, \sin\alpha = \frac{y}{r}.$$

Let us now practice this concept.

Example 2a

Evaluate $\sin 120°$.

Did you memorize $\sin 120°$? Because you shouldn't have. There's a better way to deal with these angles than simply memorizing.[vi] Using our Polar Plane from before, we place some point on the $120°$ angle. We did this in Example 1, and, since the previous Example had a radius of 2, let's stick with that.[vii]

Then we can create the right triangle we saw in Figure 68a, which we redraw in Figure 74 with the radius' length added. Note that based on the way we came about our previous definition, that is, that $\cos\alpha = \frac{x}{r}$, we must use the x-axis as the base of our triangle.[viii]

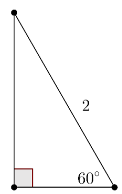


Figure 74

Then, using the relationship previously stated, viz. that

$$\sin\alpha = \frac{y}{r},$$

we can easily get our answer. Although we input $120°$, what we are working with is the $60°$ angle seen in the triangle in Figure 74.

We see that $r = 2$, because that was our chosen radius. But what is y? In this case, it's the vertical leg of the triangle drawn in Figure 74. What is the length of this leg? Hopefully you've not forgotten about $30° - 60° - 90°$ triangles, as we'll find the value of that leg using this technique. We show the lengths of the triangle in Figure 75.

[vi] Although, that being said, don't let us stop you from memorizing if that's what you're really good at.
[vii] Does the radius *need* to be 2? Excellent question! We'll cover that in Example 3.
[viii] By "base" we mean that the right angle must be located on the x-axis. This was not the case in Figure 68b. It is possible to use that Figure, but then we would need to change our definition.

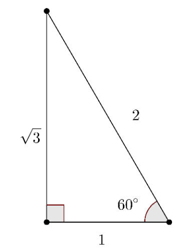


Figure 75

Now we have enough information to write our answer; we get

$$\sin 120° = \frac{\sqrt{3}}{2}.$$ [ix]

Let's do another example.

Example 2b

What is $\cos 225°$?

We follow the same procedure. This time, we need to plot a point on the Polar Plane at $225°$. What radius should we use? How about we stick with 2, for consistency's sake. We show this point, that is, $B(2, 225°)$ in Figure 76.

[ix] That seems oddly familiar... Wasn't that the same thing as $\sin 60°$...?

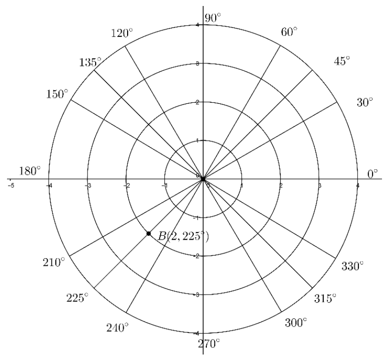


Figure 76

We now draw a right triangle using the x-axis as that base for our right angle. We show this in Figure 77.

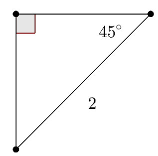


Figure 77

Since we're looking for Cosine, and, based on our findings,

$$\cos \alpha = \frac{x}{r},$$

we need to find the length of the horizontal leg.

Then we just use our knowledge of special right triangles to complete the triangle. We get Figure 78, as shown.

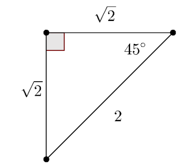


Figure 78

We can now substitute, since we know r (we chose it to be 2) and x, which is the length of the horizontal leg, which is 2. We have

$$\cos 225° = \frac{\sqrt{2}}{2}.$$

But is this correct? One way to check our result is to plot it in the coordinate plane. Our x-coordinate is given by the horizontal distance, which in this case corresponds to the horizontal leg in Figure 78.

A quick glance shows that we must be wrong. The point in Figure 76 was in Quadrant III, and this requires a negative x-value, right? We have an issue.

Or do we? There is an easy way to rectify this error: We simply make our result negative. This seems awfully artificial, and it is. But that's where our Polar Plane comes in handy. We have a picture to see that, clearly, our x-coordinate must be negative. Therefore, our result is

$$\cos 225° = -\frac{\sqrt{2}}{2}.$$

Let's do one more example before introducing you to a special circle.

Example 2c

Evaluate all six Trig functions using $\frac{5\pi}{3}$ as the input.

We are looking for $\sin \frac{5\pi}{3}, \cos \frac{5\pi}{3}$, and so on. We have radians as our input, and this is fine, since we know how to work with them. The first thing we should do is plot the point $C\left(2, \frac{5\pi}{3}\right)$ on the Polar Plane. We do this in Figure 79.

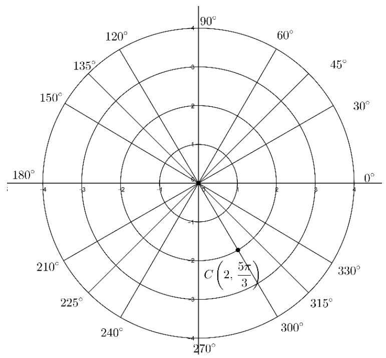


Figure 79

Do you see that $\frac{5\pi}{3}$ is equivalent to 300°?

Before proceeding, do you see how C is in Quadrant IV? Thus $x > 0$ and $y < 0$. You might want to make a note of this for each of the problems like this you do, so that you don't forget.

Now we create a right triangle with the x-axis as our base. We get Figure 80 as shown.

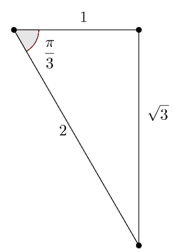


Figure 80

Verify that we've completed this special right triangle correctly.

Now all we need to do is write the appropriate ratios. We see that

$$\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$$

because Sine is opposite to hypotenuse. It must be negative since we are in Quadrant IV. We further see that

$$\cos\frac{5\pi}{3} = \frac{1}{2},$$

and, since we are in Quadrant IV, we must be positive.

What about the other Trig functions? First, recall that $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$. And since $\sin\frac{5\pi}{3}$ is negative while $\cos\frac{5\pi}{3}$ is positive, then Tangent must be negative.[x] Then we just need to find the ratio from the previous Figure, and make it negative. We get

$$\tan\frac{5\pi}{3} = -\sqrt{3}.$$

Finding the reciprocal Trig functions, like Secant, is easy. We just need to find the reciprocal of the previous three results. Note that finding a reciprocal does *not* change its sign. Therefore,

$$\csc\frac{5\pi}{3} = -\frac{2}{\sqrt{3}}, \qquad \sec\frac{5\pi}{3} = 2\,, \qquad \cot\frac{5\pi}{3} = -\frac{1}{\sqrt{3}}.$$

We leave the rationalization to the reader, if they wish.

Up to this point, we've always used a radius of 2. Just because. But certainly there has to be a better choice, right? Indeed, there is, and we call it the **unit circle**.

The unit circle
A circle centered at the origin whose radius is 1.

This simple definition has some profound implications. For example, recall that, using a circle of radius r, we have the relationship

$$\cos\alpha = \frac{x}{r}.$$

But if we have a unit circle, where $r = 1$, then we have the simpler relationship

$$\cos\alpha = x.$$

This also holds with $\sin\alpha$, of course:

$$\sin\alpha = y.$$

[x] The quotient of a negative number and a positive must be a negative number, right?

Not only is this easier to write, but it also helps us to find the Sine (or Cosine, or…) of any angle pretty easily. We show this intuition in Figure 81.

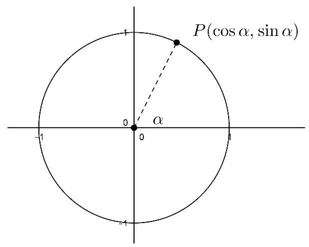


Figure 81

Any point on the circle has an x-coordinate of $\cos\alpha$ and a y-coordiante of $\sin\alpha$.

Example 3a

Evaluate $\sin 135°$.

The unit circle is a picture to place into your head so you can evaluate Trig functions like this very quickly, efficiently, and accurately. At first, you'll need to draw it out and it might take some time. But eventually it becomes second-nature, and it is indispensable in Calculus. So while someone adept at math might not need to draw it out, we will do so in each of these examples.

We draw a 135° angle on the unit circle in Figure 82.

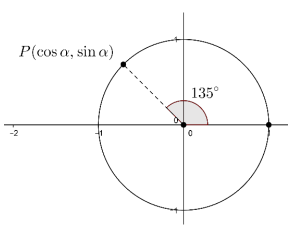


Figure 82

Since P has x- and y-coordinates of $\cos\alpha$ and $\sin\alpha$, respectively, all we need to do to evaluate $\sin 135°$ is to find the y-coordinate of P. But this amounts to the same thing we in the previous set of Examples, using the Polar Plane. The only difference is that our

radius will always be 1.[xi] Thus we need to make a right triangle with the x-axis as our base as shown in Figure 83.

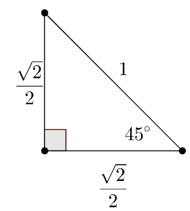


Figure 83

If you have difficulties seeing what type of special right triangle you get, use your Polar Plane. It will be more evident since each angle is marked. Also note we rationalized the denominators on each leg.

Now we just use substitute. We get that

$$\sin 135° = \frac{\sqrt{2}}{2}.$$

And yes, our result should be positive. Another way to see that our result must be positive is because, if we start at the origin, we had to travel up to get to P, correct? And isn't up a positive direction?

Example 3b

Evaluate $\cos\frac{11\pi}{6}$.

We first draw an angle of $\frac{11\pi}{6}$ on our unit circle in Figure 84.

[xi] As opposed to whatever we want. Why choose 1, again?

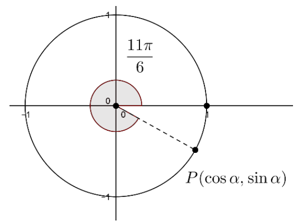


Figure 84

Again, use your Polar Plane to help you find $\frac{11\pi}{6}$. However, you'll want to have a very firm grasp of radians so don't completely rely on you Polar Plane.

Again, we now create a right triangle using the x-axis as our base. We get the special right triangle shown in Figure 85.

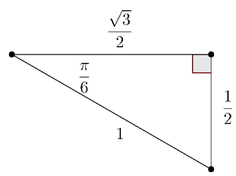


Figure 85

Then we just substitute, knowing what ratio we get with the Cosine function. Hence

$$\cos\frac{11\pi}{6} = \frac{1}{2}.$$

This, also, should be positive, since we went to the right to get to P.

Be patient and resilient as you learn the unit circle. Mastery will come, but only with practice and perseverance. Once you master the unit circle, Trigonometry becomes your plaything.

§3 **Exercises**

1.) Plot the following points on the Polar Plane, then create a right triangle where the x-axis serves as the base.
   (A) $A(2,135°)$
   (B) $B(3,300°)$
   (D) $D\left(3,\frac{\pi}{6}\right)$

GreatHearts Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020
Course: Spanish III
Teacher(s): Ms. Barrera anna.barrera@greatheartsirving.org
Supplemental links: www.spanishdict.com www.lingt.com/barreratumble

Weekly Plan:

Monday, May 11
☐ Capítulo 5 - Part I Read about the life and customs of Spain.
☐ Capítulo 5 - Reading comprehension of vocabulary classification and

Tuesday, May 12
☐ Capítulo 5 - Part II Read about the life and customs of Spain.
☐ Capítulo 5 - Reading comprehension by completing the statements and definition of festivals.

Wednesday, May 13
☐ Capítulo 5 - Listening Activity: Spanish storytime from the book titled Vida o muerte en el Cusco.
☐ Capítulo 5 - Speaking Activity: Listen and answer questions in Spanish.

Thursday, May 14
☐ Capítulo 5 - Grammar: The present perfect and pluperfect tenses.
☐ Capítulo 5 - Grammar: The present perfect Subjunctive.

Friday, May 15
☐ attend office hours
☐ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____
Student Signature

____
Parent Signature

Monday, May 11

Capítulo 5 - Part I - Read about the life and customs of Spain. Comprehension of the reading using matching, vocabulary classification and matching.

I. **Reading Handout:** ***La vida y las costumbres de España.*** Reading comprehension. **Exercise A**. Indicate for each of the 15 words if they belong under alimentation, beverages, garments, religious festivals or social customs. **Exercise B**. Match the 10 vocabulary words with the words on the right side of the column.

Tuesday, May 12

Capítulo 5- Part II - Read about the life and customs of Spain. Comprehension of the reading by completing the statements and definition of festivals.

I. **Reading Handout:** ***La vida y las costumbres de España.*** Reading comprehension. **Exercise C**. Complete the following 10 sentences with the appropriate vocabulary. **Exercise D.** Explain each of these 10 religious holidays. For example; **romería** Cada pueblo tiene su santo patrón, en el dia del santo se va a la tumba del santo a hacer picnics religiosas. Esos picnics se llaman romerías.

Wednesday, May 13

Capítulo 5 - Listening Activity: Storytime in Spanish from the book titled Vida o muerte en el Cusco. Speaking Activity: Listen and answer questions in Spanish.

I. Storytime: **Listen** to **chapters 2 and 3** of *Vida o muerte en el Cusc*o. Video link in google classroom.

II. **Speaking Assignment**: Answer questions in Spanish about chapter 2 and 3. Questions are in lingt.

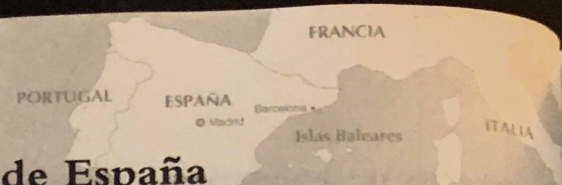
Thursday, May 14

Capítulo 5 - The present perfect and pluperfect tenses. The present perfect Subjunctive.

I.Grammar lesson: **Textbook - Present Perfect grammar review p. 222. Pluperfect grammar p. 225**. **Present Perfect Subjunctive p. 235**. I have included a written example of how all three of these tenses are used and their similarities and differences. Translate the following sentences into either the present perfect, pluperfect or present perfect subjunctive tense. Remember to go back to those pages I listed above to review the explanation of how to form and use the tense. Also, look at my examples. And if you still don't understand then email me.

1. They had not lived there in years. 2. They haven’t washed their hands yet.
3. I did not believe that they had left. 4. Have you seen the film?
5. We hope that everything has gone well. 6. I doubt that they have written the letter.
7. Maria has traveled to Ecuador several times. 8. They have never written to her.

Chapter 37



La vida y las costumbres de España

Hoy día la vida diaria de España es muy semejante a la vida diaria de los Estados Unidos. Sin embargo, cada país tiene algo de particular. En España existen muchas costumbres y tradiciones interesantes.

LA CASA Y LA FAMILIA

Las casas de las ciudades grandes se parecen a las de las otras ciudades del mundo. Hay edificios de apartamentos y casas particulares. Muchos españoles viven en su condominio o piso que forma parte de un edificio alto.

En los pueblos pequeños, las ciudades antiguas y en las partes antiguas de las ciudades grandes las casas están en calles estrechas. Generalmente, estas casas son de un solo piso, con balcones, ventanas con rejas y pintorescos patios interiores. En muchas casas, las paredes están cubiertas de azulejos.

LOS NOMBRES Y LOS APELLIDOS

Los nombres españoles son diferentes a los de los anglosajones, y esta costumbre de los nombres se extiende a todos los países hispanos. Además del nombre de pila, los españoles e hispanos llevan dos apellidos (el apellido del padre seguido del apellido de la madre): por ejemplo, Rafael *Hernández Silva*. Este nombre se archiva bajo «H» porque se considera más importante el apellido del padre.

Cuando una mujer se casa, ella retiene el apellido de su padre y añade el «de» seguido por el apellido de su esposo. Por ejemplo, si Rafael se casa con Marisa Trujillo Rodríguez, su nombre sería Marisa Trujillo de Hernández. Ella no usa ninguno de los apellidos maternos.

Por lo general, el nombre de pila de la mayoría de los españoles (y de los hispanoamericanos) es el nombre de un santo. Además de su propio cumpleaños, celebran el día del santo.

TIPOS PINTORESCOS

La tuna es un grupo de músicos ambulantes que ha sido parte de la vida estudiantil universitaria desde el siglo XVI. Los tunos usan un traje tradicional de color negro y una capa. Tocan música romántica y alegre. Sus instrumentos incluyen el laúd, la bandurría, el requinto, la guitarra y la pandereta.

Los gitanos viven en el sur de España, principalmente cerca de las ciudades de Sevilla y Granada. Conservan muchas de sus tradiciones dentro de las que la música y el baile son muy importantes. Hablan un idioma llamado romani.

COSTUMBRES

La vida social es muy importante para los españoles. Son muy sociales y amigables. Les gusta pasar mucho tiempo charlando con los amigos en los cafés y en los bares. Cenan mucho más tarde que en nuestro país, y por consiguiente, se acuestan más tarde.

En muchas ciudades hay un **ateneo,** un club intelectual donde se reúnen grupos literarios y científicos.

La tertulia es una reunión informal con el propósito de charlar y divertirse. Muchas veces no termina hasta después de la medianoche.

La siesta es la tradición de acostarse por la tarde durante las horas de mayor calor. Las tiendas y las oficinas se cierran, y los trabajadores regresan a sus casas para comer, descansar o dormir la siesta. Después de la siesta, las tiendas y oficinas vuelven a abrir y quedan abiertas hasta muy tarde. La tradición de la siesta ha ido desapareciendo en las ciudades grandes.

La lotería está dirigida por el gobierno y es muy popular. Se usan las ganancias de la lotería para el beneficio de las personas pobres y los niños huérfanos. El sorteo tiene lugar tres veces al mes y hay muchos premios. El premio mayor se llama «el premio gordo».

«Pelar la pava» es una tradición antigua que se usaba cuando el novio cortejaba a la novia. El novio le hablaba a la novia por medio de la reja. El estaba de pie afuera y la mujer estaba sentada dentro de la casa al otro lado de la reja. Hoy día muchas de las costumbres de cortejar a las mujeres están desapareciendo, especialmente en las ciudades grandes.

COMIDAS Y BEBIDAS

La tradición de comer tres comidas al día existe tanto en España como en los Estados Unidos. Principalmente, la diferencia está en el horario. En España existe la costumbre de comer más tarde. Se toma el desayuno a eso de las ocho de la mañana y generalmente consiste en café con leche o chocolate y pan con mantequilla y mermelada o bollos. Se toma la comida a eso de las dos de la tarde y no se toma la cena hasta después de las ocho o nueve de la noche. Estas dos comidas son comidas completas. Pueden consistir en sopa, ensalada, carne con arroz o verduras y postre. Alrededor de las seis de la tarde los españoles toman la merienda, algo ligero como un sándwich. Se acostumbra comer bocados que llaman «tapas» cuando toman el aperitivo antes de la cena.

España tiene una gastronomía variada, interesante y deliciosa. Entre los platos tradicionales está el **puchero** que es considerado quizás el plato nacional de España. Es un guisado que se sirve casi a diario, especialmente entre los campesinos. Se llama también olla o cocido.

El plato más conocido de España es el **arroz con pollo.** En Valencia añaden mariscos y otros ingredientes al arroz con pollo y lo llaman paella.

Las bebidas que toman los españoles incluyen el **café,** el **té,** la **leche** y los **refrescos.** La **horchata** es una bebida tradicional hecha de almendras, agua y azúcar. La horchata se toma fría en el verano como refresco. El **chocolate caliente** es una bebida popular del desayuno y de la merienda. Lo preparan muy espeso y lo toman muy caliente con **panecillos, bizcochos** o **churros.**

LA ROPA

La ropa de los españoles se parece a la del resto de Europa. Sin embargo, existen prendas tradicionales. La **mantilla** es un pañuelo grande de seda y encajes que la mujer lleva en la cabeza. Debajo de la **mantilla** se usa un peine alto, ricamente adornado, que se llama **peineta.** El **mantón** es un chal grande, ricamente bordado. Sirve de adorno o de abrigo. La **boina** es una gorra de lana redonda parecida al «beret» francés. Las **alpargatas** son una especie de sandalia hecha de lona. Son comunes entre los trabajadores en muchas partes de España.

FIESTAS RELIGIOSAS

Puesto que la mayoría de los españoles practican la religión católica, las fiestas religiosas son muy importantes en el país.

1. La **Navidad** se celebra el 25 de diciembre y es la fiesta más importante del año. Desde principios de diciembre en cada casa se preparan los **Nacimientos** (grupos de figuras que representan el nacimiento de Jesucristo). La **Nochebuena** (Christmas Eve) la gente asiste a la **misa del gallo** (midnight mass) en las iglesias. Grupos de personas caminan por las calles cantando villancicos (Christmas carols). También se acostumbra dar regalos, llamados aguinaldos, a las personas que han servido a la familia durante el año (el cartero, los criados y otros). Los niños reciben sus regalos el 6 de enero, que se llama el **Día de los Reyes Magos.** Los Reyes Magos corresponden a nuestro Santa Claus.
2. El **Carnaval** es un período de tres días de diversión antes del Miércoles de Ceniza, que comienza la **Cuaresma** (Lent). La Cuaresma son los cuarenta días que siguen. Termina el domingo de Resurrección, la **Pascua Florida.** La **Semana Santa,** que precede a la Pascua Florida, se celebra con mucha solemnidad y devoción, sobre todo en Sevilla. El **Viernes Santo** hay procesiones religiosas en muchos pueblos y ciudades.

   Cada pueblo tiene su santo patrón, cuyo día se celebra con una fiesta. La noche anterior se celebra una **verbena** (evening festival). El día del santo hay **romerías** (religious picnics) a la tumba del santo.
3. El **Día de los Difuntos o Muertos** (All Souls' Day) se celebra el 2 de noviembre, en memoria de los muertos.

FIESTAS NACIONALES

1. El **dos de mayo** es la fiesta nacional de España. Conmemora un suceso patriótico, el comienzo de la resistencia contra los franceses en 1808.
2. El **Día de la Raza,** que se celebra el 12 de octubre, corresponde a nuestro «Columbus Day». Se celebra esta fiesta en todo el mundo hispano.

DEPORTES Y DIVERSIONES

1. La **corrida de toros** es un espectáculo muy típico de España. Generalmente hay corridas los domingos por la tarde y los días de fiesta. Tiene lugar en la plaza de toros. La corrida comienza con un desfile de todos los participantes por la arena, mientras se escucha la música de pasadobles.

   La corrida tiene tres partes o lo que llaman suertes. Los picadores entran, montados a caballo en la primera suerte. Llevan picas largas. Los banderilleros entran a pie en la segunda suerte. Ellos llevan banderillas. En la tercera suerte el matador (el torero final), armado de una espada de acero muy fina, y llevando una pequeña muleta roja, exhibe su arte y su valor. Ejecuta varios pases con la muleta, hasta que el momento ideal llega para matar al toro.
2. El **jai-alai** es un juego de pelota de origen vasco que se juega con una pelota dura en un

gran frontón de tres paredes. Es semejante al «handball» pero el jugador usa una cesta larga y estrecha, que está atada a la muñeca, para tirar y coger la pelota.

3. El **fútbol** es muy popular no sólo en España, sino también en el resto de Europa y en Latinoamérica. Se puede decir que es el deporte nacional de la mayoría de esos países.

EXERCISE A. **Identifique cada una de las siguientes palabras como alimento, bebida, prenda de vestir, fiesta religiosa, o costumbre social.**

1. boina ____________________
2. verbena ____________________
3. paella ____________________
4. tertulia ____________________
5. cocido ____________________
6. romería ____________________
7. horchata ____________________
8. el Día de los Difuntos ____________________
9. alpargatas ____________________
10. arroz con pollo ____________________
11. mantilla ____________________
12. Carnaval ____________________
13. merienda ____________________
14. peineta ____________________
15. chocolate ____________________

EXERCISE B. **A la izquierda de cada expresión de la lista A, escriba la letra de la palabra o expresión de la lista B que tenga relación con ella.**

A	B
______ 1. apellido	a. premio gordo
______ 2. pelar la pava	b. frontón
______ 3. lotería	c. sandalias
______ 4. Nochebuena	d. músicos ambulantes
______ 5. banderillero	e. reja
______ 6. jai-alai	f. nombre
______ 7. día del santo patrón	g. alimento
______ 8. puchero	h. Navidad
______ 9. tuna	i. corrida de toros
______ 10. alpargatas	j. verbena

EXERCISE C. **Complete las frases siguientes.**

1. Antonio Moreno y Villa está casado con Luisa Gómez y Vega, y tienen un hijo, Juan. El nombre completo del hijo es Juan ____________________ .
2. Un alimento popular en Valencia, hecho de arroz, pollo y mariscos se llama ____________________ .
3. La fiesta nacional de España se celebra ____________________ .
4. Los niños españoles reciben regalos el 6 de enero, el Día de los ____________________ .
5. El torero que va montado a caballo se llama el ____________________ .
6. El 12 de octubre se celebra ____________________ .
7. En vez de celebrar su cumpleaños, los niños españoles celebran su ____________________ .
8. ____________________ es el nombre que se da a un club científico y literario.
9. La horchata es una bebida fría que se hace de ____________________ .
10. El jai-alai es un deporte que se juega en un ____________________ .

EXERCISE D. **Explique cada uno de los siguientes.**

1. Día de los Muertos ____________________
2. matador ____________________
3. Semana Santa ____________________
4. azulejos ____________________
5. verbena ____________________
6. Pascua Florida ____________________
7. romería ____________________
8. villancicos ____________________
9. nacimiento ____________________
10. Misa del Gallo ____________________

Past participles p. 222

abrir → abierto
decir → dicho
escribir → escrito
morir → muerto
poner → puesto
resolver → resuelto
romper → roto
ser → sido
ver → visto

1. * Present Perfect → Yo he bailado (Present) (Past) → I have danced (Present)
2. Pluperfect → Yo había bailado (Past) (Past) → I had danced. (Past)
3. * Present Perfect Subjunctive [Me alegro de que] hayas bailado conmigo. (Present) (Past)
   [I am glad that] you have danced with me. (present) (past)

Do you notice that # 1 and # 3 are both using the present past. The only difference that in my sentence (#3) I added an expression of emotion prompting the use of the subjunctive. More examples.

not Subjunctive { No he reparado la bicicleta todavía
{ I haven't repaired the bicycle yet.

Subjunctive { Me alegro de que no hayas reparado la . . .
{ I am glad that you haven't repaired the . . .