GreatHearts Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 7: May 11-15, 2020

Course: 9 Geometry

Teacher(s): Mr. Mooney sean.mooney@greatheartsirving.org

Weekly Plan:

Monday, May 11
- ☐ Review Bell Work and Proposition Questions Answer Keys
- ☐ Review VI.5, VI.6, VI.8 answer keys and make corrections

Tuesday, May 12
- ☐ Video on Google Classroom (Optional and Encouraged)
- ☐ Bell Work 8
- ☐ VI.9 and 11 two-columns and constructions

Wednesday, May 13
- ☐ Video on Google Classroom (Optional and Encouraged)
- ☐ Bell Work 9
- ☐ VI.12 and 13 two-columns and constructions

Thursday, May 14
- ☐ Video on Google Classroom (Optional and Encouraged)
- ☐ Bell Work 10 and Note on VI.16
- ☐ VI.16 Two-Column

Friday, May 15
- ☐ attend office hours
- ☐ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____
Student Signature

____
Parent Signature

Dear Students,

I cannot believe it is already May 11th! I hope you all are doing well as we approach the end of the school year and our study of Euclid's *Elements*. This week, we will be keeping to the same routine as last week: Monday review and corrections; videos on Tuesday through Thursday; and Friday catch-up/review, Office Hours, and packet upload (as we will be doing for the rest of the *year*).

We will be continuing to make our way through Book VI this week. Unfortunately, given our limited time, we are going to begin jumping around quite a bit, skipping many good and worthwhile propositions. As always, you are encouraged to read these on your own if you have the time and the desire, but of course I will not require it. We will also begin, this week, to work a *little* bit of number into the Bell Works, so that you can get a feel for how ratio and proportion work for that non-geometrical magnitude.

I won't be so bold as to boast of no typos this time (tens of "awesome points" were justly distributed last week), but my goal will be to keep them to a minimum. Also, I hope that the videos last week helped to make the propositions more accessible to you. Obviously, nothing can replace being in the classroom, but I hope it helped us get just a little closer to that ideal.

I wish you a wonderful week. Enjoy the beautiful weather, and the beautiful propositions from Book VI!

Sincerely,
Mr. Mooney

Monday, May 11

Today, I would like you to:

1) Review the answer keys for all the Bell Work and Proposition Questions from last week, which you will find as scanned documents in this packet. You need not submit your corrections to these parts, but I encourage you to correct them nonetheless for the sake of improving your understanding.
2) Review the "answer keys" for propositions VI.5, 6, and 8, and correct your two-column notes from last week. Please scan and submit these corrected two-columns along with the rest of your packet at the end of the week (just as you did last week).

Tuesday, May 12

Today, I would like you to:

1) Complete Bell Work 8. This bell work asks you to apply the ideas that you have learned about proportions in geometrical figures to *number*. We will continue to practice this skill all week.
2) Read VI.9 and put it into two-column notes. Then, perform the construction below with compass and straightedge. I am including a *different* two-column sheet for this proposition, with space for the construction underneath it.
   VI.9 is about cutting off a "prescribed part" from a given line. Remember the definition of "part" from the beginning of Book V. Essentially, this proposition enables to take any fraction of any line. For example, given a line, take one fourth of it, or one tenth, or one forty-fifth. In Euclid's

Elements, he takes one third of the line, and we will do the same in our construction, but it is important to note that the construction applies equally well to *any* fraction.
(Optional: I am including a little "refresher" on how to do I.31, the construction of a parallel line, since it will be used in multiple constructions this week.)

3) Do the same for VI.11, writing out the two-column notes and constructing it underneath with compass and straightedge.
VI.11, as you will see, enables us to find a third proportional. As you'll recall, a third proportional is a magnitude (in this case, a line) that is the final term in a proportion in three terms. That is, it allows us to find some magnitude C, such that A:B::B:<u>C</u>.

Wednesday, May 13

Today, I would like you to:

1) Complete Bell Work 9, which looks at proportions in figures that have numerical lengths and asks you to find missing lengths.
2) Read VI.12 and put it into two-column notes. Then, perform the construction below with compass and straightedge. Again, I am including a different two-column sheet for this proposition, with space for the construction underneath it.
VI.12 enables us to find a fourth proportional. That is, it allows you to find some magnitude D, such that A:B::C:<u>D</u>.
3) Do the same for VI.13, writing out the two-column notes and constructing it underneath with compass and straightedge.
VI.13 enables us to what is called a "mean proportional." The mean proportional is some magnitude B that relates to two other magnitudes A and C as the middle term in a proportion in three terms: A:<u>B</u>::<u>B</u>:C.

Thursday, May 14

Today, I would like you to:

1) Complete Bell Work 10, which looks specifically at proportions within the special case of VI.8.
2) Read the Note on VI.16, which you will find beneath today's bell work.
3) Read VI.16 and write it out in two-column notes.

Friday, May 15

Congratulations! Another week successfully completed, with good progress made in your understanding of ratio and proportion.

Use today to finish up any work you may still need to do from the week, attend my Office Hours from 10:30 - 11:00am (you can find the Zoom link on Google Classroom), and upload your packets onto Google Classroom. I hope to see you in Office Hours!

Have a wonderful weekend, filled only with what is True, Good, and Beautiful.

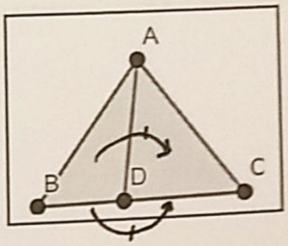
Bell Work 5 and VI.5 Questions

Bell Work 5:

Directions: Provide the resulting proportion for each. Then, mark the resulting proportion on the diagram.

1. If triangles ABD and ADC are under the same height,

   then $\triangle ABD : \triangle ADC :: BD : DC$

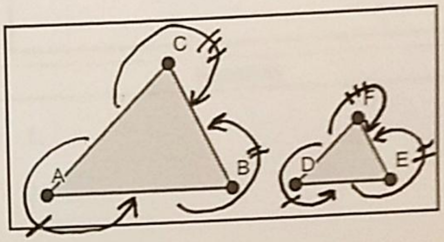


2. If triangles ABC and DEF are equiangular,

   then AC : AB :: DF : DE

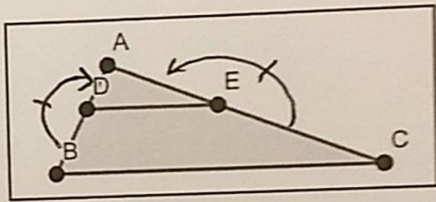
   AB : BC :: DE : EF

   and AC : BC :: DF : EF



3. If in triangle ABC, DE is parallel to BC,

   then BD : DA :: CE : EA



Note about Triangle Similarity Propositions:

Last week, you proved VI.4, which says that if two triangles are equiangular, then the sides about the equal angles will be proportional. As I said in my note on AAA Similarity last week, this would then mean that the two triangles are similar, because they are both equiangular and have proportional sides.

This week we will encounter two more similarity propositions: SSS Similarity and SAS Similarity. Like AAA Similarity (VI.4), these propositions will not make any explicit mention of similarity.

They will, however, in their "if" and "then," total up to similarity. Let me explain. VI.Def.1 has a list of requirements for two shapes to be similar: they need to have (1) all equal angels and (2) all proportional sides about those equal angles. In each Similarity proposition, the "if" portion meets half of those criteria, while the "then" portion meets the other half.

In all of these proofs, you may write as your conclusion that the triangles are similar, or you may keep to the way that Euclid wrote it. Either way, be mindful of the fact that the end result in all three of these propositions is that the triangles are similar, having both equiangularity and proportionality of sides.

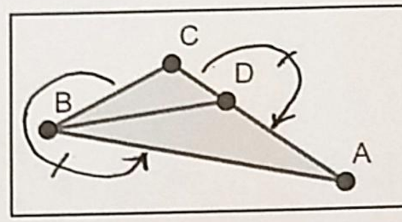
Bell Work 6 and VI.6 Questions

Bell Work 6:

Directions: Provide the resulting proportion for each. Then, <u>mark the resulting proportion</u> on the diagram.

1. If in triangle ABC, angle CBA is bisected by BD ,

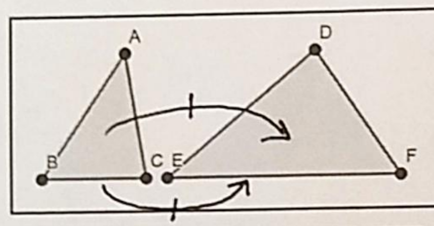
   then CD : DA :: BC : BA



2. If triangles ABC and DEF *under the same height*,

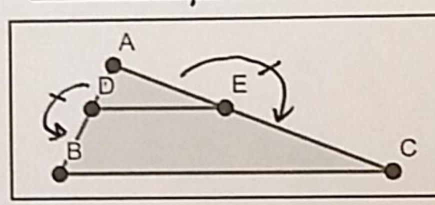
   then ΔABC : ΔEDF :: BC : EF

   (This is a bit tricky, because the diagram looks a little different than we are used to seeing.)



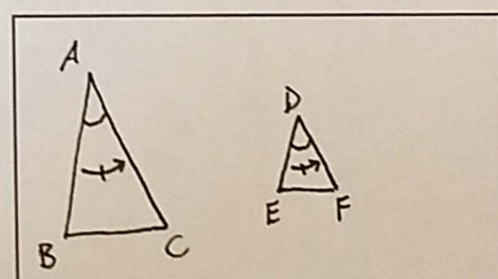
3. If in triangle ABC, DE is parallel to BC,

   then AD : DB :: AE : EC



VI.6 Questions

1. Draw the diagram in the box on the right, including *only the given parts* (i.e. do not include anything that was constructed). Mark the given.



2. When triangle DGF is constructed, it is constructed so that it will be similar to triangle ΔABC.

3. It (triangle DGF) is then proven to be congruent to triangle DEF.

4. Thus, by the end of the proof, it can be concluded that triangle DEF is equiangular with triangle ABC.

5. Since the triangles (from #4) are equiangular and have proportional sides, they would therefore be similar. [This is not included in Euclid's proof, but it is implied.]

Bell Work 7 and VI.8 Questions

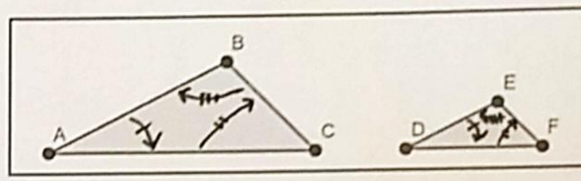
Bell Work 7:

Directions: Provide the resulting proportion for each. Then, mark the resulting proportion on the diagram.

1. Define ratio (V.Def.3): A ratio is a sort of relation in respect of size between two magnitudes of the same kind.

2. In your own words, describe what a proportion is: A proportion is a statement that two ratios are the same. (Answers may vary.)

3. If triangles ABC and DEF are equiangular,



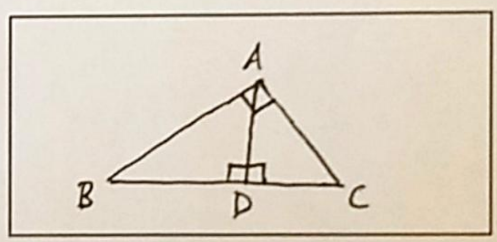
then AB : AC :: DE : DF

AC : BC :: DF : EF

and BC : AB :: EF : DE

VI.8 Questions

1. Draw the diagram in the box on the right, including *only the given parts* (i.e. do not include anything that was constructed). Mark all given information.



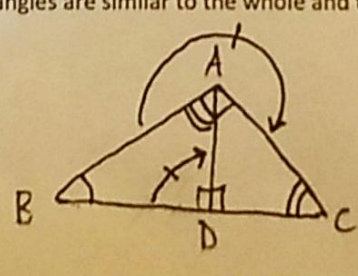
2. What are the two things proven in this proposition? (Euclid proves them in separate sections of his proof.)

   1. ΔABD & ΔADC are similar to whole ΔABC.
   2. ΔABD & ΔADC are similar to each other.

3. To prove that the two smaller triangles similar to the whole, Euclid first needs to prove that they are equiangular. To do so, he relies on the right angles, common angles (i.e. shared between the triangles), and I.32. Explain, in your own words, how he does this.

   Each of the 3 triangles has a right angle, and each of the smaller triangles shares one angle with the whole. For example, ΔABD and whole ΔABC both have a right angle and they share ∠ABD in common. Therefore, by I.32, since the angles of all Δ's add up to the same (2 rt), the third angles must be equal as well, making them similar (VI.4).

4. In the space below, redraw the diagram, and mark one true proportion that results from the fact that the two smaller triangles are similar to the whole and to each other.



Answer Key

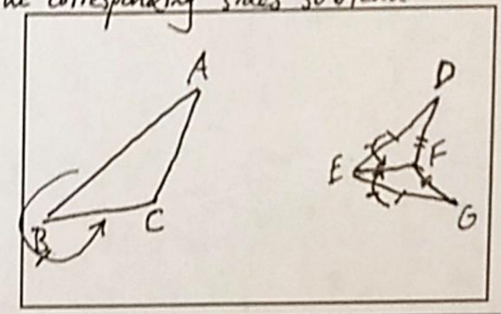
VI.5: If 2 Δ's have their sides proportional, the triangles will be equiangular, and will have those angles equal which the corresponding sides subtend.

Given:

$\triangle ABC, \triangle DEF$, $AB:BC::DE:EF$

$BC:CA::EF:FD$

$AB:CA::DE:FD$



To Prove:

$\triangle ABC$ & $\triangle DEF$ equiangular

$\angle ABC = \angle DEF$

$\angle BCA = \angle EFD$

$\angle BAC = \angle EDF$

Statements	Reasons
1. Construct at E, $\angle FEG = \angle ABC$	1. I.23
2. and at F, $\angle EFG = \angle ACB$	2. I.23
3. $\therefore \angle A = \angle G$	3. I.32
4. $\therefore \triangle ABC$ & $\triangle GEF$ are equiangular	4. Steps 1-3 (Def. of equiangular)
5. $\therefore AB:BC::GE:EF$	5. VI.4 (AAA ~)
6. But $AB:BC::DE:EF$	6. Given
7. $\therefore DE:EF::GE:EF$	7. V.11
8. $\therefore DE = GE$	8. V.9
9. $\therefore DF = GF$	9. Similar argument (steps 5-8)
10. EF is common	10. —
11. $\therefore \angle DEF = \angle GEF$	11. I.8 (SSS $\cong$)
12. $\angle DFE = \angle GFE$	12. I.4 (SAS $\cong$)
13. $\angle EDF = \angle EGF$	13. I.4 (SAS $\cong$)
14. $\therefore \angle ABC = \angle DEF$	14. C.N.1 (Steps 1 & 11)
15. $\therefore \angle ACB = \angle DFE$	15. C.N.1 (steps 2 & 12)
16. $\therefore \angle A = \angle D$	16. C.N.1 (steps 3 & 13) (or I.32)
17. $\therefore \triangle ABC$ is equiangular w/ $\triangle DEF$	17. Steps 14-16 (def of equiangular)
18.	18.
19.	19.
20.	20.

Answer Key

VI.6 : If 2 Δ's have one angle equal to one angle and the sides about the equal angles proportional, the triangles will be equiangular and will have those ∠'s = which the corresponding sides subtend.

Given:

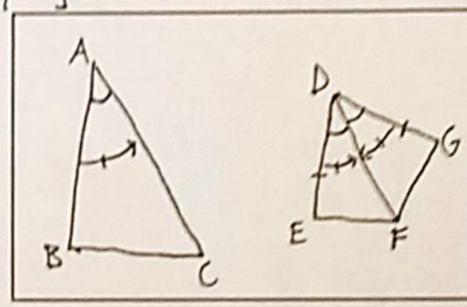
ΔABC, ΔDEF; ∠BAC = ∠EDF

BA : AC :: ED : DF

To Prove:

ΔABC equiangular w/ΔDEF ∠ABC = ∠DEF

∠ACB = ∠DFE



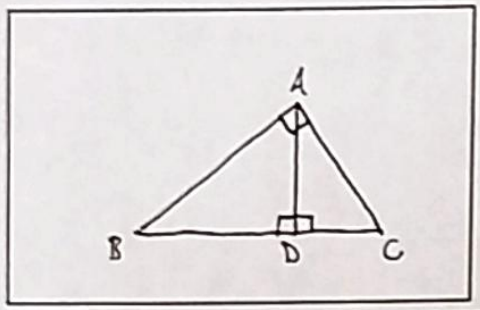
Statements	Reasons
1. Construct at D, ∠FDG = ∠BAC (or ∠EDF)	1. I.23
2. Construct at F, ∠DFG = ∠ACB	2. I.23
3. ∴ ∠B = ∠G	3. I.32
4. ∴ ΔABC is equiangular w/ ΔDEF	4. steps 1-3 (def. of equiangular)
5. ∴ BA:AC :: GD:DF	5. VI.4 (AAA ~)
6. But BA:AC :: ED:DF	6. Given
7. ∴ ED:DF :: GD:DF	7. V.11
8. ∴ ED = GD	8. V.9
9. DF is common	9. —
10. ∠EDF = ∠GDF	10. Step 1
11. ∴ ∠DFG = ∠DFE	11. I.4 (SAS ≅)
12. and ∠DGF = ∠DEF	12. I.4 (SAS ≅)
13. But ∠DFG = ∠ACB	13. step 2
14. ∴ ∠ACB = ∠DFE	14. C.N.1 (steps 11 & 13)
15. And ∠BAC = ∠EDF	15. Given
16. ∴ ∠B = ∠E	16. I.32
17. ∴ ΔABC is equiangular w/ ΔDEF	17. steps 14, 15, 16 (def. of equiangular)
18.	18.
19.	19.
20.	20.

Answer Key

VI.8: If in a right-angled Δ a perpendicular be drawn from the right angle to the base, the Δs adjoining the perpendicular are similar both to the whole and to each other.

Given:

Δ ABC, w/ ∟ BAC; AD ⊥ BC



To Prove:
Δ ABD ~ Δ ABC
Δ ADC ~ Δ ABC
Δ ABD ~ Δ ADC

Statements	Reasons
1. ∟ BAC, ∟ ADB	1. Given
2. ∴ ∠BAC = ∠ADB	2. Post. 4
3. ∠B is common to Δs ABC, ABD	3. —
4. ∴ ∠ACB = ∠BAD	4. I.32
5. ∴ Δ ABC equiangular w/ Δ ABD	5. Steps 2-4
6. ∴ BC : BA :: AB : BD :: AC : AD	6. VI.4 (AAA ~)
7. ∴ Δ ABC ~ Δ ABD	7. VI. Def. 1
8. Similarly Δ ABC ~ Δ ADC	8. Similar argument (steps 1-7)
9. ∟ BDA = ∟ ADC	9. Post. 4 (Given as ∟s)
10. ∠BAD = ∠C	10. Step 4
11. ∠B = ∠DAC	11. I.32
12. ∴ Δ ABD equiangular w/ Δ ADC	12. Steps 9-11
13. ∴ BD : DA :: AD : DC :: BA : AC	13. VI.4
14. ∴ Δ ABD ~ Δ ADC	14. VI. Def. 1
15.	15.
16.	16.
17.	17.
18.	18.
19.	19.
20.	20.

Bell Work 8

Bell Work 8:

Part 1: Proportions in four terms

Directions: Complete the following numerical proportions by filling in the blank.

1. $1:2 :: 5:$ ___

2. $3:4 :: 9:$ ___

3. $2:3 :: 12:$ ___

4. $4:$ __ $:: 8:10$

5. $6:$ __ $:: 3:8$

6. __ $:15 :: 3:9$

Part Two: Proportions in Three Terms

Directions: For the following proportions, fill in the two blanks *with the same number* to make the *proportion true.*

Example: $1:$ __ $::$ __ $:4$ *What number can go in both blanks to make the proportion true? The answer is 2, because 2 is both the double of 1 and the half of four. Thus* $1:\mathbf{2} :: \mathbf{2}:4$.

7. $2:$ __ $::$ __ $:8$

8. $3:$ __ $::$ __ $:12$

9. $1:$ __ $::$ __ $:9$

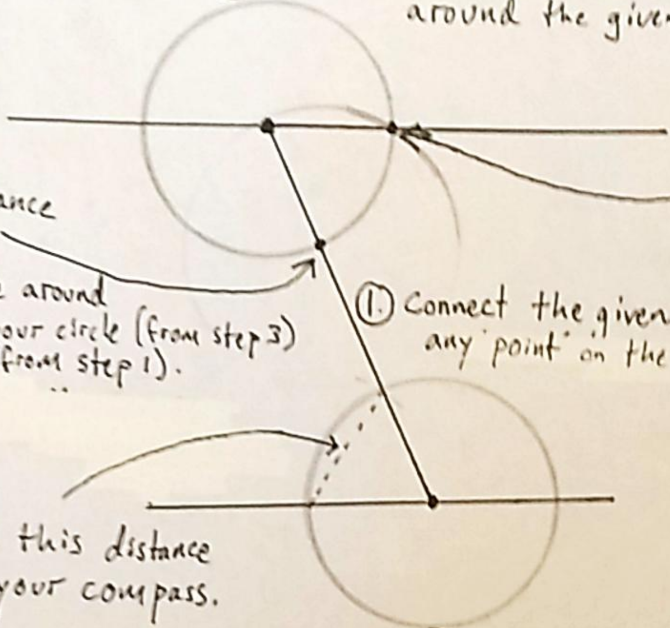
10. $1:$ __ $::$ __ $:25$

I. 31 "Refresher" (Optional)

③ Draw the same-sized circle (as in step 2) around the given point.

⑤ Use that distance (from step 4) to draw a circle around the point where your circle (from step 3) cuts your line (from step 1).

⑥ Find the point where the two circles (steps 3 & 5) cut, and join it to the given point.



① Connect the given point to any point on the given line.

④ Measure this distance with your compass.

② Draw a circle of any size around the point at which the two lines meet.

⑦ The line joined in step 6 can then be extended in both directions as needed. It is parallel to the given line.

____
____
____
____

VI.9 : *From a given straight line to cut off a prescribed part.*

Given:

To Prove:

Statements	Reasons
1. ______________________________	1.
2. ______________________________	2.
3. ______________________________	3.
4. ______________________________	4.
5. ______________________________	5.
6. ______________________________	6.
7. ______________________________	7.
8. ______________________________	8.
9. ______________________________	9.
10. ______________________________	10.

Construction: From the given line, cut off the *third* part.

____

____

____

____

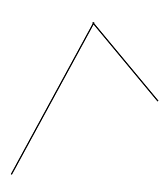
VI.11 : *To two given straight lines to find a third proportional.*

Given:

To Prove:

Statements	**Reasons**
1. ______________________________	1.
2. ______________________________	2.
3. ______________________________	3.
4. ______________________________	4.
5. ______________________________	5.
6. ______________________________	6.
7. ______________________________	7.
8. ______________________________	8.
9. ______________________________	9.
10. ______________________________	10.

Construction: To the given two lines, construct a third proportional.

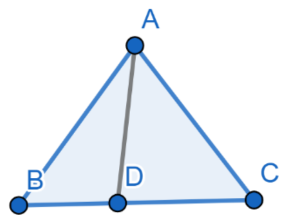


Bell Work 9

Bell Work 9: Proportions in Figures (with number!)

Directions: Using your knowledge of what proportions are true in each diagram, set up a proportion to find the missing length or area.

1.



Given two triangles ABD and ADC under the same height, with

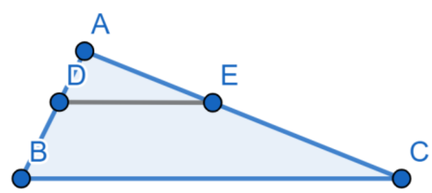
ABD = 6

ADC = 9

BD = 2

Find DC = _______

2.



Given triangle ABC, with DE//BC and

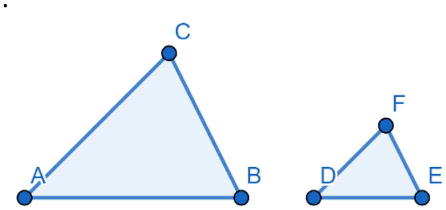
BD = 6

AD = 2

EC = 12

Find AE = _______

3.



Given similar triangles ABC and DEF, and

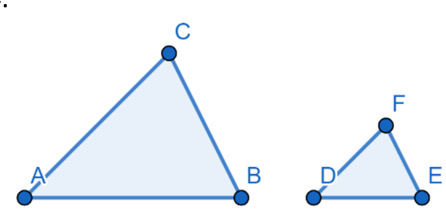
AC = 18

AB = 15

DF = 6

Find DE = _______

4.



Given similar triangles ABC and DEF, with

AB = 8

BC = 6

EF = 3

Find DE = _______

VI.12 *: To three given straight lines to find a fourth proportional.*

Given:

To Prove:

Statements	Reasons
1.	1.
2.	2.
3.	3.
4.	4.
5.	5.
6.	6.
7.	7.
8.	8.
9.	9.
10.	10.

Construction: To the given three lines A, B, and C, construct a fourth proportional, such that $A{:}B :: C{:}(\text{the line you construct})$.

A

B

C

____

____

____

____

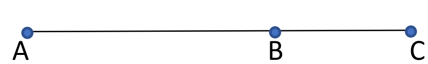
VI.13 : *To two given straight lines to find a mean proportional.*

Given:

To Prove:

Statements	**Reasons**
1. ____________________________	1.
2. ____________________________	2.
3. ____________________________	3.
4. ____________________________	4.
5. ____________________________	5.
6. ____________________________	6.
7. ____________________________	7.
8. ____________________________	8.
9. ____________________________	9.
10. ____________________________	10.

Construction: To the two given straight lines, find a mean proportional (such that $AB: \textit{the constructed line} :: \textit{the constructed line}: BC$).

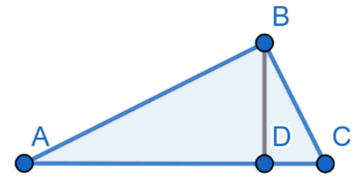


Bell Work 10

<u>Bell Work 10: Proportions in VI.8</u>

Part 1: For each proportion, fill in the missing line to make the proportion true by VI.8.
Given right triangle ABC, with right angle ABC and BD drawn

perpendicular to AC,

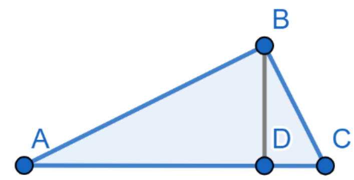


1) $AB : AC :: AD :$ ____

2) $AB : BC :: BD :$ ____

3) $AD : BD :: BD :$ ____

Part Two: Notice how in #3 above, the proportion is a proportion in three terms, with BD being the mean proportional. In each of these following problems, find the length of BD.



1) If AD = 4 and DC = 1, then BD = ___________

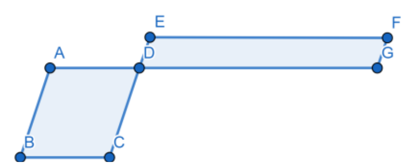
2) If AD = 9 and DC = 1, then BD = ___________

3) If DC = 4 and AD = 16, then BD = ___________

<u>Note on VI.16:</u>
Although I am calling this a note on VI.16, it is really more a note on VI.14, because VI.14 (which we skipped) is essential for understanding VI.16.

VI.14 says that "*In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional."* (It also proves the converse of this.)

The given is fairly simple: there are two parallelograms that are equal in area, and they have all angles equal. We can imagine something like this:



The parallelograms ABCD and DEFG have all equal angles, and their areas are equal as well. If this is the case, says VI.14, then the sides are *reciprocally proportional*. This means that $CD : DE :: DG : DA$. Notice how, in this proportion, we start the first ratio with CD, part of parallelogram ABCD, and move towards DE, part of the parallelogram DEFG. If this were a case of normal proportionality, the second ratio in the proportion would also start with a side of ABCD and move to a side of DEFG. But it does not! It starts with DG, a side of DEFG, and moves to AD, a side of ABCD. Because it the second ratio is in a "different direction" from the first, we call it "reciprocally proportional."

You will see in VI.16, that this is used on two rectangles, and the "reciprocal" part—the reverse in direction—is very important.

____

____

____

____

VI.16 (Part 1): *If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means…*

Given:

To Prove:

Statements	Reasons
1. ____________________________	1.
2. ____________________________	2.
3. ____________________________	3.
4. ____________________________	4.
5. ____________________________	5.
6. ____________________________	6.
7. ____________________________	7.
8. ____________________________	8.
9. ____________________________	9.
10. ____________________________	10.
11. ____________________________	11.
12. ____________________________	12.
13. ____________________________	13.
14. ____________________________	14.
15. ____________________________	15.
16. ____________________________	16.
17. ____________________________	17.
18. ____________________________	18.
19. ____________________________	19.
20. ____________________________	20.

____
____
____
____

VI.16 (Part 2): *…and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.*

Given:

To Prove:

Statements	Reasons
1. ______________________	1.
2. ______________________	2.
3. ______________________	3.
4. ______________________	4.
5. ______________________	5.
6. ______________________	6.
7. ______________________	7.
8. ______________________	8.
9. ______________________	9.
10. ______________________	10.
11. ______________________	11.
12. ______________________	12.
13. ______________________	13.
14. ______________________	14.
15. ______________________	15.
16. ______________________	16.
17. ______________________	17.
18. ______________________	18.
19. ______________________	19.
20. ______________________	20.