GreatHearts
Irving

Remote Learning Packet

Please submit scans of written work in Google Classroom at the end of the week.

Week 8: May 18-22, 2020
Course: 9 Geometry
Teacher(s): Mr. Mooney sean.mooney@greatheartsirving.org

Weekly Plan:

Monday, May 18
☐ Review Bell Work Answer Keys
☐ Review Two-Column & Construction Answer Keys

Tuesday, May 19
☐ Bell Work 11
☐ VI.19 Reading
☐ VI.19 Two Column

Wednesday, May 20
☐ Bell Work 12
☐ VI.31 Reading
☐ VI.31 Two Column

Thursday, May 21
☐ Assessment (on Google Classroom)

Friday, May 22
☐ attend office hours
☐ catch-up or review the week's work

Statement of Academic Honesty

I affirm that the work completed from the packet is mine and that I completed it independently.

I affirm that, to the best of my knowledge, my child completed this work independently

____
Student Signature

____
Parent Signature

Dear Students,

Welcome to the last normal week of school (“normal” being a relative word in this anything-but-normal remote situation)! This week we continue in a manner very similar to previous weeks, with one important change.

On Thursday, I will be giving you an assessment, which you will complete on Google Classroom. There is no hard copy of this assessment in the packet, so all assessments must be taken in this online format. It is an assessment, so accuracy really counts here; it is, however, completely open notes and you may take as long as you like to complete it (as long as it is done by Sunday at midnight). This, I hope, takes off a lot of the pressure of it being an “assessment.”

We have some monumental proofs to cover this week. Proposition VI.31, as you will see, is particularly breath-taking in its beauty and profundity. I’d like to remind you, as we enter into this penultimate week of school, to take time to reflect on what you have accomplished in Geometry this year. Think back to when you struggled with the simplest of propositions, when you did not even understand what a proof was or why they were important. And now look at you! Masters of some of the most difficult and mathematically complex proofs in Book VI. How much more fit your minds are now for knowing Truth! It is a beautiful and wonderful thing.

I hope you enjoy coming to know the truths of these next two propositions. Have a wonderful week!

Sincerely,

Mr. Mooney

Monday, May 18

Today, as per usual, is a day to review your work from last week, with the goal of learning from your mistakes. I would like you to:

1) Review the answer keys for Bell Works 8, 9, and 10, which are simply scans of the pages which I filled in by hand.
2) Review the answer keys for VI.9, VI.11, VI.12, VI.13, and VI.16, and make all necessary corrections (including corrections to your constructions!). Please make all corrections in a different color.

Tuesday, May 19

Today I would like you to:

1) Complete Bell Work 11, which puts VI.16 into practice with proportions. (Bell Work 12 is on the same page, but you do not need to do that until tomorrow.)
2) Read the VI.19 Reading.

3) Put VI.19 into two-column notes.

Wednesday, May 20

Today, I would like you to:

1) Complete Bell Work 12 (to be found on the same page as Bell Work 11).
2) Read the VI.31 Reading.
3) Read VI.31 and write it out in two column notes.

Thursday, May 21

Today, I would like you to complete the Assessment, which is *not* in this packet, but can be found on Google Classroom. You may study as much as you like before you take it, and remember: it is an *open note test*. That means that while you are taking the test, you may look at *any of your own materials:* your own notes, your packets, any answer keys I've given you, the videos I posted on Google Classroom, etc. Please, of course, do not collaborate with anyone else or look at any of their materials, because that would be dishonest. If you have any questions about what is allowed or not allowed for taking the assessment, you are welcome to email me.

Friday, May 22

Great work! You have made it to the end of the week, the end of Book VI, and *very nearly* the end of the school year. Today you may use your time to finish up any work from earlier in the week, upload packets, and attend my Geometry Office Hours from 10:30 - 11:00 am (the link is still posted on Google Classroom).

Have a wonderful weekend!

Bell Work 8

Bell Work 8:

Part 1: Proportions in four terms

Directions: Complete the following numerical proportions by filling in the blank.

1. 1:2 :: 5: 10
2. 3:4 :: 9: 12
3. 2:3 :: 12: 18
4. 4: 5 :: 8:10
5. 6: 16 :: 3:8
6. 5 :15 :: 3:9

Part Two: Proportions in Three Terms

Directions: For the following proportions, fill in the two blanks _with the same number_ *to make the proportion true.*

Example: 1:__ :: __:4 *What number can go in both blanks to make the proportion true? The answer is 2, because 2 is both the double of 1 and the half of four. Thus* 1:**2** :: **2**:4.

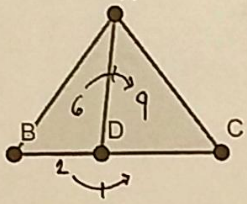
7. 2: 4 :: 4 :8
8. 3: 6 :: 6 :12
9. 1: 3 :: 3 :9
10. 1: 5 :: 5 :25

Bell Work 9

<u>Bell Work 9: Proportions in Figures (with number!)</u>

Directions: Using your knowledge of what proportions are true in each diagram, set up a proportion to find the missing length or area.

1.



Given two triangles ABD and ADC under the same height, with

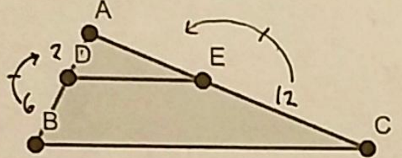
ABD = 6

ADC = 9

BD = 2

Find DC = 3

2.



Given triangle ABC, with DE//BC and

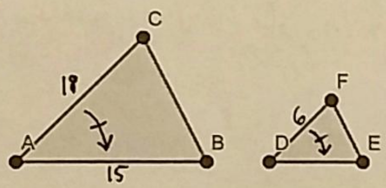
BD = 6

AD = 2

EC = 12

Find AE = 4

3.



Given similar triangles ABC and DEF, and

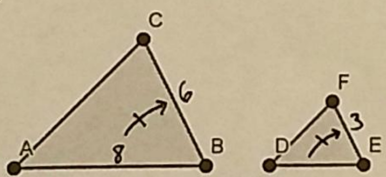
AC = 18

AB = 15

DF = 6

Find DE = 5

4.



Given similar triangles ABC and DEF, with

AB = 8

BC = 6

EF = 3

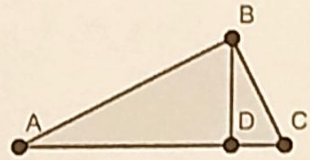
Find DE = 4

Bell Work 10

Bell Work 10: Proportions in VI.8

Part 1: For each proportion, fill in the missing line to make the proportion true by VI.8.
Given right triangle ABC, with right angle ABC and BD drawn

perpendicular to AC,

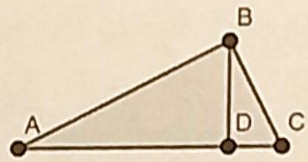


1) $AB: AC :: AD:$ AB

2) $AB: BC :: BD:$ DC

3) $AD: BD :: BD:$ DC

Part Two: Notice how in #3 above, the proportion is a proportion in three terms, with BD being the mean proportional. In each of these following problems, find the length of BD.



1) If AD = 4 and DC = 1, then BD = 2

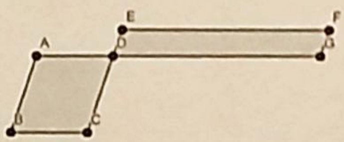
2) If AD = 9 and DC = 1, then BD = 3

3) If DC = 4 and AD = 16, then BD = 8

Note on VI.16:
Although I am calling this a note on VI.16, it is really more a note on VI.14, because VI.14 (which we skipped) is essential for understanding VI.16.

VI.14 says that "*In equal and equiangular parallelograms the sides about the equal angles are reciprocally proportional.*" (It also proves the converse of this.)

The given is fairly simple: there are two parallelograms that are equal in area, and they have all angles equal. We can imagine something like this:



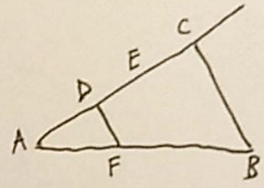
The parallelograms ABCD and DEFG have all equal angles, and their areas are equal as well. If this is the case, says VI.14, then the sides are *reciprocally proportional.* This means that $CD: DE :: DG: DA$. Notice how, in this proportion, we start the first ratio with CD, part of parallelogram ABCD, and move towards DE, part of the parallelogram DEFG. If this were a case of normal proportionality, the second ratio in the proportion would also start with a side of ABCD and move to a side of DEFG. But it does not! It starts with DG, a side of DEFG, and moves to AD, a side of ABCD. Because it the second ratio is in a "different direction" from the first, we call it "reciprocally proportional."

You will see in VI.16, that this is used on two rectangles, and the "reciprocal" part—the reverse in direction—is very important.

Answer Key

VI.9: *From a given straight line to cut off a prescribed part.*

Given: AB

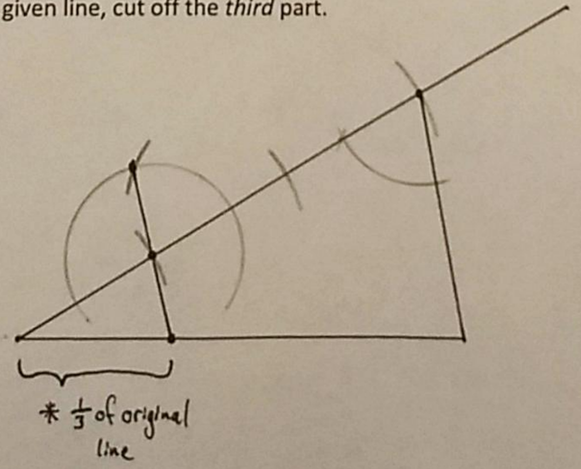


To Prove:

Cut off prescribed part (1/3) of AB

Statements	Reasons
1. Draw AC from A, any angle from AB	1. Post. 1
2. Take D on AC at random	2. —
3. Cut DE and EC = AD	3. I.3
4. Join BC	4. Post. 1
5. Through D, draw DF // BC	5. I.31
6. ∴ CD : DA :: BF : FA	6. VI.2
7. But CD is 2 × DA	7. Made it that way in step 3
8. ∴ BF is 2 × FA	8. by proportion in step 6
9. ∴ BA is 3 × AF	9. taking the ratio componendo, step 8
10. ∴ Prescribed part AF has been cut from AB	10. step 9

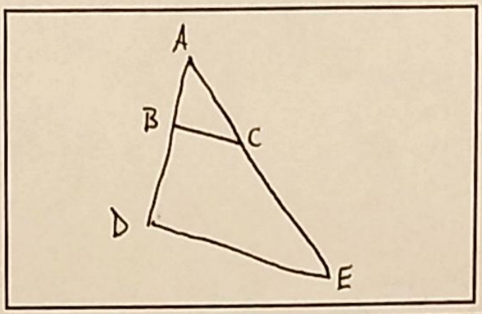
Construction: From the given line, cut off the *third* part.



Answer Key

VI.11 : *To two given straight lines to find a third proportional.*

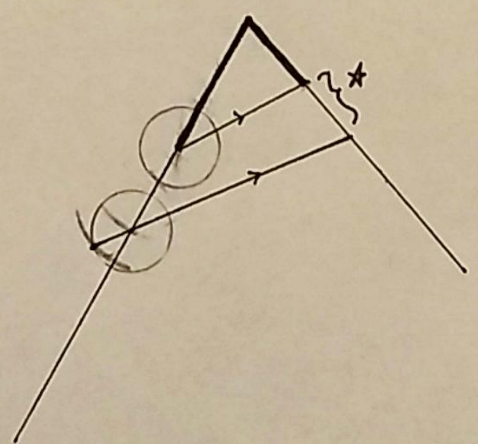
Given: BA, AC containing any angle



To Prove: Find a 3rd proportional to BA, AC

Statements	Reasons
1. Extend BA, AC → D, E	1. Post. 2
2. Cut BD = AC	2. I. 3
3. Join BC	3. Post. 1
4. Through D, draw DE // BC	4. I. 31
5. ∴ AB : BD :: AC : CE	5. VI. 2
6. But BD = AC	6. step 2
7. ∴ AB : AC :: AC : CE	7. Substitution (steps 5, 6)
8. ∴ CE is a 3rd proportional to AB, AC	8. Definition of third proportional
9.	9.
10.	10.

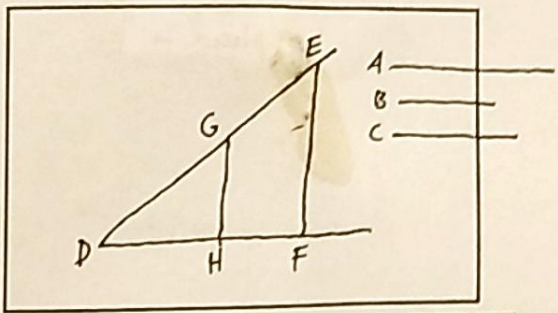
Construction: To the given two lines, construct a third proportional.



Answer Key

VI.12 : *To three given straight lines to find a fourth proportional.*

Given: A, B, C (lines)



To Prove:

Find a 4th proportional to A, B, C.

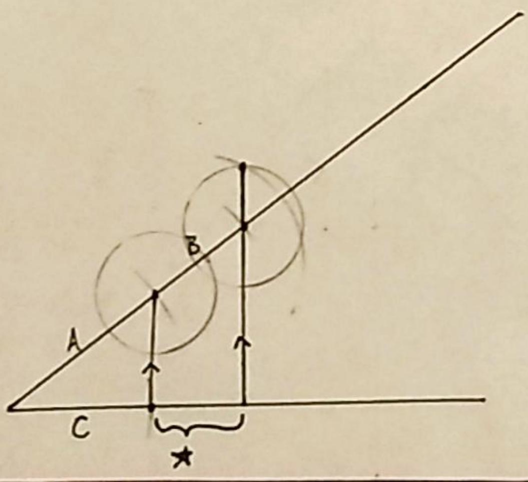
Statements	Reasons
1. Draw 2 lines DE, DF at any angle EDF	1. Post. 1
2. Cut DG = A, GE = B, DH = C	2. I. 3
3. Join GH	3. Post. 1
4. Through E, draw EF // GH	4. I. 31
5. ∴ DG : GE :: DH : HF	5. VI. 2
6. But DG = A, GE = B, DH = C	6. Step 2
7. ∴ A : B :: C : HF	7. Substitution (steps 5 & 6)
8. ∴ HF is 4th proportional to A, B, C	8. Definition of fourth proportional
9.	9.
10.	10.

Construction: To the given three lines A, B, and C, construct a fourth proportional, such that $A: B :: C: (\textit{the line you construct})$.

A

B

C

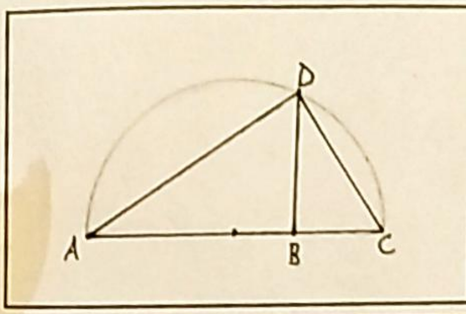


Answer Key

VI.13: *To two given straight lines to find a mean proportional.*

Given:

AB, BC

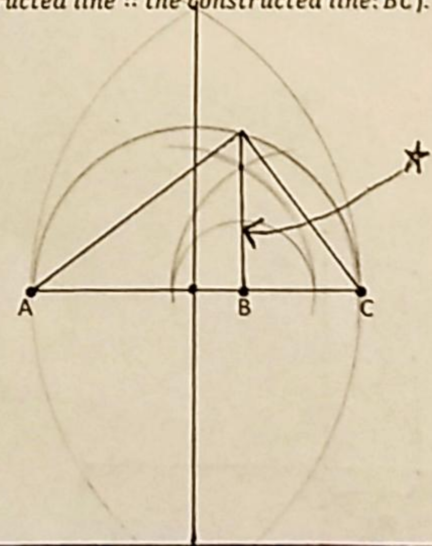


To Prove:

Find mean proportional of AB, BC

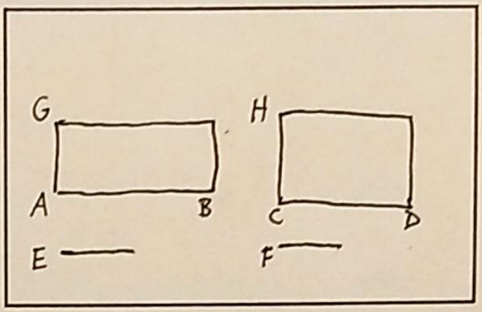
Statements	Reasons
1. Let AB, BC be in a straight line	1. — (This could be justified w/ Post. 2 + I.3)
2. Draw semicircle ADC on AC	2. Post. 3 (+ I.10)
3. From B, draw BD ⊥ AC	3. I.11
4. Join AD, DC	4. Post. 1
5. ∠ADC is rt. ∠	5. III.31
6. ∴ DB is mean proportional to AB, BC	6. VI.8. Porism (△ADC, BD ⊥ AC)
7.	7.
8.	8.
9.	9.
10.	10.

Construction: To the two given straight lines, find a mean proportional (such that AB: the constructed line :: the constructed line: BC).



Answer Key

VI.16 (Part 1): *If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means...*



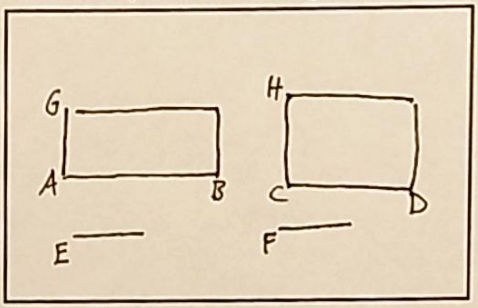
Given: AB : CD :: E : F

To Prove: Rect. AB × F = Rect. CD × E

Statements	Reasons
1. From A, C draw AG ⊥ AB, CH ⊥ CD	1. I.11
2. Cut AG = F, CH = E	2. I.3
3. Complete ▱'s BG, DH	3. — (This can be justified w/ I.31 & I.34)
4. AB : CD :: E : F	4. Given
5. E = CH and F = AG	5. step 2
6. ∴ AB : CD :: CH : AG	6. substitution (steps 4 & 5)
7. ∴ ▱BG = ▱DH	7. VI.14
8. And ▱BG is Rect. AB × F, for AG = F	8. step 2
9. And ▱DH is Rect. CD × E, for E = CH	9. step 2
10. ∴ Rect. AB × F = Rect. CD × E	10. CN.1 / subst. (steps 7, 8, 9)
11.	11.
12.	12.
13.	13.
14.	14.
15.	15.
16.	16.
17.	17.
18.	18.
19.	19.
20.	20.

Answer Key

VI.16 (Part 2): *...and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.*



Given: AB, CD, E, F

Rect. AB×F = Rect. CD×E

To Prove: AB:CD::E:F

Statements	Reasons
1. Same construction	1. Part 1 (steps 1-3)
2. Rect. AB×F = Rect. CD×E	2. Given
3. And AB×F is BG, for AG=F	3. Construction (step 2)
4. And CD×E is DH, for CH=E	4. Construction (step 1)
5. ∴ BG=DH	5. C.N.1 / subst. (steps 2-4)
6. BG is equiangular w/ DH	6. All rectangles are equiangular
7. ∴ AB:CD :: CH:AG	7. VI.14
8. But CH=E, AG=F	8. step 1
9. ∴ AB:CD :: E:F	9. Substitution (steps 7-8)
10.	10.
11.	11.
12.	12.
13.	13.
14.	14.
15.	15.
16.	16.
17.	17.
18.	18.
19.	19.
20.	20.

Bell Work 11 and 12

<u>Bell Work 11:</u>

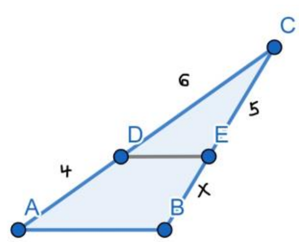
Directions: Complete the following numerical proportions using the method from VI.16: the rectangle contained by the extremes will equal the rectangle contained by the means. That is, multiply the extremes, set them equal to the product of the means, and solve for the missing term. (Note: some of your answers will be fractions/decimals).

1. $3:5 :: 6:x$ x = _______________
2. $2:3 :: x:24$ x = _______________
3. $x:9 :: 6:10$ x = _______________
4. $4:x :: 8:15$ x = _______________
5. $3:x :: x:27$ x = _______________
6. $3:x :: x:48$ x = _______________

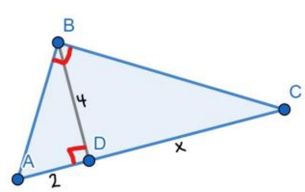
<u>Bell Work 12:</u>

Directions: Find the value of x in each diagram, based on your knowledge of the proportions that are true in the given situations.

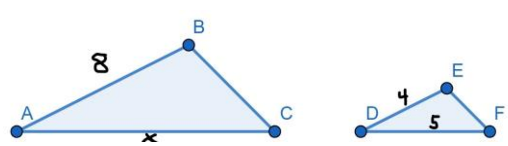
1. $\mathrm{x} =$ ____



2. $x =$ ____



3. $x =$ ____



VI.19 Reading

Part One: "Duplicate Ratios"
In VI.19, which you will be reading today, it says that if two triangles are similar, then they will be to one another in the **duplicate ratio** of their corresponding sides. The goal, right now, is simply to understand what is meant by that key term: duplicate ratio.

In V.Definition.9, it says that if three magnitudes be proportional, the first is said to have to the third the *duplicate ratio* of that which it has to the second. To understand this, let's take three magnitudes—to make it simple, let's take three numbers.

$$1:3 :: 3:9$$

The numbers 1, 3, and 9 are three proportional magnitudes. Thus, according to V.Def.9, the first (1) is said to have to the third (9), the *duplicate ratio* of that which it (1) has to the second (3).

In other words, the ratio 1:9 is the "duplicate" of 1:3. When we are thinking about number, the easiest way to think of this is "squaring" the ratio of the first to the second. That is,

$$1:9 :: (1:3)^2$$

Or, if we wrote the proportion as an equality of fractions,

$$\frac{1}{9} = \left(\frac{1}{3}\right)^2$$

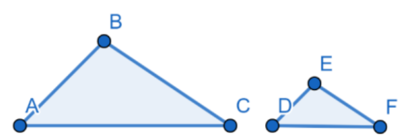
Thus, another way of stating the truth of VI.19 is: the ratios of the areas of two similar triangles will always be the square of the ratio of its corresponding sides.

Let's practice by looking at a few examples of VI.19 with number:

If EF=1, BC = 3, and triangle DEF = 1, then triangle ABC = ____

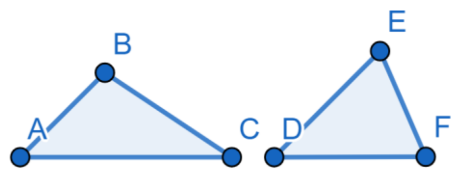
If EF=2, BC = 4, and triangle DEF = 4, then triangle ABC = ____

If triangle ABC = 25, triangle DEF =16, and BC = 5, then EF = _____



Part Two: VI.15
In last week's packet, I made a "Note on VI.16" which was really just an explanation of a proof that lack of time forced us to skip: VI.14. In this note, I'd like to explain a related proposition, VI.15, which will be used in today's VI.19.

In VI.15, it says that if two triangles have a pair of equal angles and have their sides reciprocally proportional, then the areas of the triangles will be equal. For example, if triangles ABC and DEF have equal angles ABC and DEF, and their sides are *reciprocally proportional*—that is, $AB: DE :: EF: BC$—then the areas of the triangles ABC and DEF will be equal.

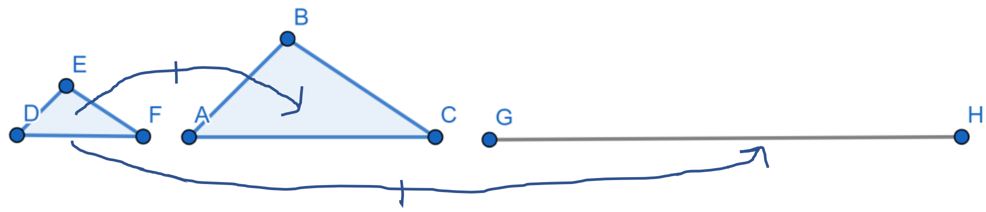


Notice especially the *reciprocal* nature of the proportion: the first ratio (AB:DE) moves from triangle ABC to triangle DEF; the second ratio (EF:BC) *reverses* that direction, moving from triangle DEF to triangle ABC.

VI.19 Reading

Part Three: The Porism

Since, as VI.19 proves, that similar triangles are to one another in the duplicate ratio of their corresponding sides, it follows that the ratio of triangle DEF to triangle ABC will be the same as the ratio of side DF to a third proportional to sides DF and AC. That is:

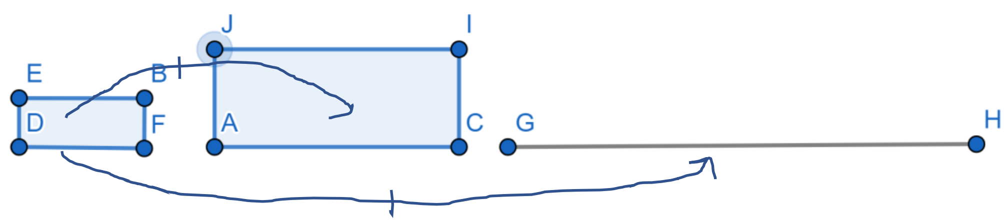


$$\Delta DEF : \Delta ABC :: DF : GH$$

The Porism explicitly states that if three straight lines be proportional ($DF : AC :: AC : GH$), then the first (DF) will be to the third (GH) as the any figure built on the first (for example, ΔDEF) will be to any similar and similarly described figure on the second (for example, ΔABC).

It is key to note that this is true of *any similar figures* described on the first and second of three proportional lines.

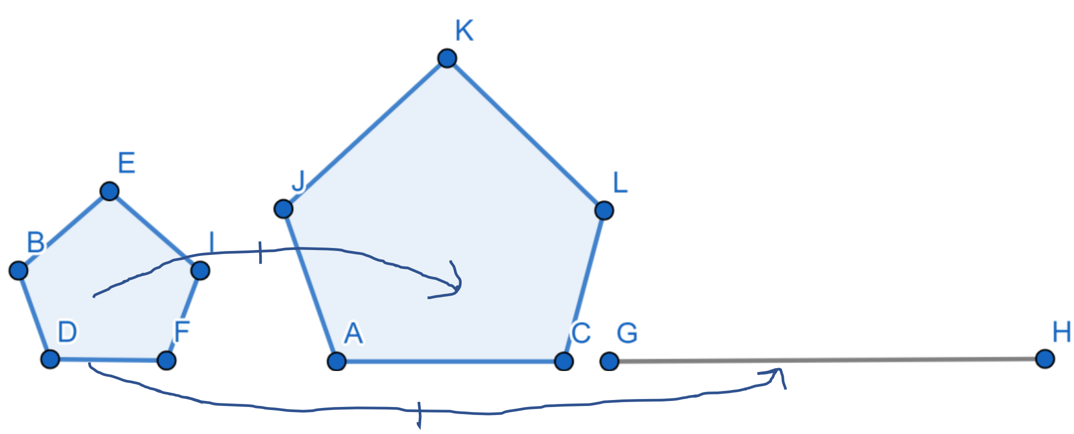
For example, if DF, AC, and GH are proportional ($DF : AC :: AC : GH$), and DEBF and AJIC are similar,



then

$$DEBF : AJIC :: DF : GH$$

Or again, if the three lines are again proportional, and the two pentagons are similar,



then

$$DBEIF : AJKLC :: DF : GH$$

____
____
____
____

VI.19: *Similar triangles are to one another in the duplicate ratio of their corresponding sides.*

Given:

To Prove:

Statements	Reasons
1. ________________________	1.
2. ________________________	2.
3. ________________________	3.
4. ________________________	4.
5. ________________________	5.
6. ________________________	6.
7. ________________________	7.
8. ________________________	8.
9. ________________________	9.
10. ________________________	10.
11. ________________________	11.
12. ________________________	12.
13. ________________________	13.
14. ________________________	14.
15. ________________________	15.
16. ________________________	16.
17. ________________________	17.
18. ________________________	18.
19. ________________________	19.
20. ________________________	20.

VI.31 Reading

Part One: V.24

In order to understand VI.31 today, we need one more truth from Book V that we did not cover when we studied Book V earlier. It is this:

> *If a first magnitude have to a second the same ratio as a third has to a fourth, and also a fifth have to the second the same ratio as a sixth to the fourth, then the first and fifth added together will have to the second the same ratio as the third and sixth have to the fourth.*

Now, all of the numbering can make this enunciation difficult to understand. Let's use letters instead. Consider six magnitudes: A, B, C, D, E, F.

If $A : B :: C : D$
and $E : B :: F : D$

---

then $(A + E) : B :: (C + F) : D$

Do you see the pattern? B and D are the consequents in every proportion, while the antecedents of the proportions in the "if" change and then are added together in the antecedents of the "then."

Let's try an example. Fill in the missing conclusion below:

If $AB : CD :: EF : GH$
and $KL : CD :: MN : GH$

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then ______________________

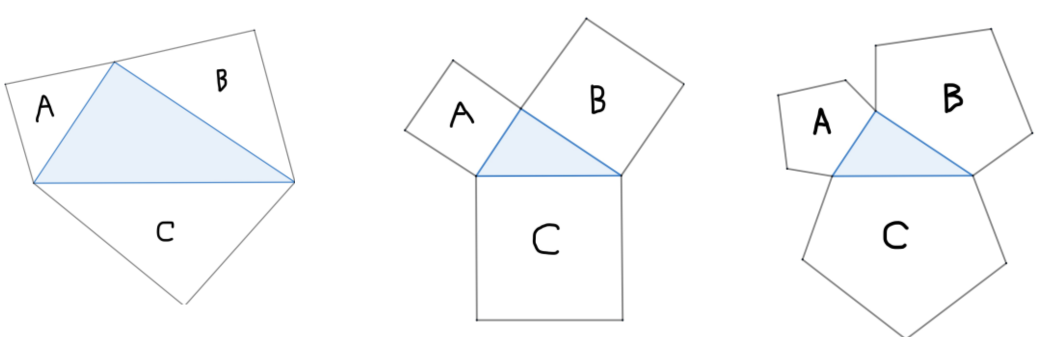
Part Two: Significance of VI.31

This proposition, VI.31, is the penultimate (second-to-last) proposition of the last book that we will study together. Do you remember the penultimate proposition of the *first* book?

It turns out—and perhaps this is no accident, but rather Euclid's love of symmetry—that this penultimate proposition of Book VI is the generalization of the penultimate proposition of Book I. That is, VI.31 is the generalization of I.47—the Pythagorean Theorem.

You remember, of course, that the Pythagorean Theorem states that in any right triangle the *square* on the hypotenuse is equal to the sum of the *squares* on the legs.

VI.31 proves that this is true *not only of squares, but of every similar and similarly described figure on the sides of a right triangle!* That is, $C = A + B$ in *every case shown below!*



… and so on!

VI.31 Reading

As long as the figures are similar (and similarly described), it does not matter what they look like, nor how many sides they have: it will always be true that the figures on the legs will add up in area to the figure on the hypotenuse! Who could have guessed it? It turns out that the Pythagorean Theorem is only a special case of a much broader and more universal truth![1] Incredible!

Do you remember, at the very beginning of the year, when we proved things that you thought were obvious, and you wondered why we were proving it. I tried to explain, at those times, how we needed to establish a firm foundation, proving even the most obvious things, because they would eventually build up to the most amazing and incredible truths, ones that are very far from obvious. I think it is fitting that we encounter this incredible truth here, as our study of Euclid's *Elements* begins to draw to a close. From the great heights of this proposition, let's take a moment to look back with awe and wonder at how far we have come, one proposition at a time.

[1] Thus, the proof of VI.31 is not only a shorter and simpler, and perhaps more elegant alternative proof of I.47, but it also proves much more than I.47, extending its reach to every similar shape (not just squares).

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VI.31: *In right-angled triangles the figure on the side subtending the right angle is equal to the similar and similarly described figures on the sides containing the right angle.*

Given:

To Prove:

Statements	Reasons
1. ____________________	1.
2. ____________________	2.
3. ____________________	3.
4. ____________________	4.
5. ____________________	5.
6. ____________________	6.
7. ____________________	7.
8. ____________________	8.
9. ____________________	9.
10. ____________________	10.
11. ____________________	11.
12. ____________________	12.
13. ____________________	13.
14. ____________________	14.
15. ____________________	15.
16. ____________________	16.
17. ____________________	17.
18. ____________________	18.
19. ____________________	19.
20. ____________________	20.